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CONGRESS OF ARTS AND SCIENCE

UNIVERSAL EXPOSITION ST. LOUIS 1904

IN EIGHT VOLUMES

VOLUME I

International

CONGRESS OF ARTS AND SCIENCE

UNIVERSAL EXPOSITION, ST. LOUIS, 1904

EDITED BY

HOWARD J. ROGERS, A.M., LL.D.

DIRECTOR OF CONGRESSES

VOLUME I

HISTORY OF THE CONGRESS

By the Editor

SCIENTIFIC PLAN OF THE CONGRESS

By Professor Hugo Münsterberg

PHILOSOPHY AND MATHEMATICS

BOSTON AND NEW YORK
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Universal Exposition, 1904.

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CONGRESS OF ARTS AND SCIENCE

THE HISTORY OF THE CONGRESS

BY HOWARD J. ROGERS A.M., LL.D.

THE forces which bring to a common point the thousandfold energies of a universal exposition can best promote an international congress of ideas. Under national patronage and under the spur of international competition the best products and the latest inventions of man in science, in literature, and in art are grouped together in orderly classification. Whether the motive underlying the exhibits be the promotion of commerce and trade, or whether it be individual ambition, or whether it be national pride and loyalty, the resultant is the same. The space within the boundaries of the exposition is a forum of the nations where equal rights are guaranteed to every representative from any quarter of the globe, and where the sovereignty of each nation is recognized whenever its flag floats over a national pavilion or an exhibit area. The productive genius of every governed people contends in peaceful rivalry for world recognition, and the exposition becomes an international clearing-house for practical ideas.

For the demonstration of the value of these products men thoroughly skilled in their development and use are sent by the various exhibitors. The exposition by the logic of its creation thus gathers to itself the expert representatives of every art and industry. For at least two months in the exposition period there are present the members of the international jury of awards, selected specially by the different governments for their thorough knowledge, theoretical and practical, of the departments to which they are assigned, and selected further for their ability to impress upon others the correctness of their views. The renown of a universal exposition brings, as visitors, students and investigators bent upon the solution of problems and anxious to know the latest contributions to the facts and the theories which underlie every phase of the world's development.

The material therefore is ready at hand with which to construct the framework of a conference of parts, or a congress of the whole of any subject. It was a natural and logical step to accompany the study of the exhibits with a debate on their excellence, an analysis of their growth, and an argument for their future. Hence the congress. The exposition and the congress are correlative terms. The former concentres the visible products of the brain and hand of man; the congress is the literary embodiment of its activities.

Yet it was not till the Paris Exposition of 1889 that the idea of a series of congresses, international in membership and universal in scope, was fully developed. The three preceding expositions, Paris, 1878, Philadelphia, 1876, and Vienna, 1873, had held under their auspices many conferences and congresses, and indeed the germ of the congress idea may be said to have been the establishment of the International Scientific Commission in connection with the Paris Exposition of 1867; but all of these meetings were unrelated and sometimes almost accidental in their organization, although many were of great scientific interest and value.

The success of the series of seventy congresses in Paris in 1889 led the authorities of the World's Columbian Exposition in 1893 to establish the World's Congress Auxiliary designed "to supplement the exhibit of material progress by the Exposition, by a portrayal of the wonderful achievements of the new age in science, literature, education, government, jurisprudence, morals, charity, religion, and other departments of human activity, as the most effective means of increasing the fraternity, progress, prosperity, and peace of mankind." The widespread interest in this series of meetings is a matter easily within recollection, but they were in no wise interrelated to each other, nor more than ordinarily comprehensive in their scope.

It remained for the Paris Exposition of 1900 to bring to a perfect organization this type of congress development. By ministerial decree issued two years prior to the exposition the conduct of the department was set forth to the minutest detail. One hundred twenty-five congresses, each with its separate secretary and organizing committee, were authorized and grouped under twelve sections corresponding closely to the exhibit classification. The principal delegate, M. Gariel, reported to a special commission, which was directly responsible to the government. The department was admirably conducted and reached as high a degree of success as a highly diversified, ably administered, but unrelated system of international conferences could. And yet the attendance on a majority of these congresses was disappointing, and in many there was scarcely any one present outside the immediate circle of those concerned in its development. If this condition could prevail in Paris, the home of arts and letters, in the immediate centre of the great constituency of the University and of many scientific circles and learned societies, and within easy traveling distance of other European university and literary centres, it was fair to presume that the usefulness of this class of congress was decreasing. It certainly was safe to assume, on the part of the authorities of the St. Louis Exposition of 1904, that such a series could not be a success in that city, owing to its geographical position and the limited number of university and scientific circles within a reasonable traveling distance. Something more than a repetition of the stereotyped form of conference was admitted to be necessary in order to arouse interest among scholars and to bring credit to the Exposition.

This was the serious problem which confronted the Exposition of St. Louis. No exposition was ever better fitted to serve as the groundwork of a congress of ideas than that of St. Louis. The ideal of the Exposition, which was created in time and fixed in place to commemorate a great historic event, was its educational influence. Its appeal to the citizens of the United States for support, to the Federal Congress for appropriations, and to foreign governments for coöperation, was made purely on this basis. For the first time in the history of expositions the educational influence was made the dominant factor and the classification and installation of exhibits made contributory to that principle. The main purpose of the Exposition was to place within reach of the investigator the objective thought of the world, so classified as to show its relations to all similar phases of human endeavor, and so arranged as to be practically available for reference and study. As a part of the organic scheme a congress plan was contemplated which should be correlative with the exhibit features of the Exposition, and whose published proceedings should stand as a monument to the breadth and enterprise of the Exposition long after its buildings had disappeared and its commercial achievements grown dim in the minds of men.

DEVELOPMENT OF THE CONGRESS

The Department of Congresses, to which was to be intrusted this difficult task, was not formed until the latter part of 1902, although the question was for a year previous the subject of many discussions and conferences between the President of the Exposition, Mr. Francis; the Director of Exhibits, Mr. Skiff; the Chief of the Department of Education, Mr. Rogers; President Nicholas Murray Butler of Columbia University, and President William R. Harper of Chicago University. To the disinterested and valuable advice of the two last-named gentlemen during the entire history of the Congress the Exposition is under heavy obligations. During this period proposals had been made to two men of international reputation to give all their time for two years to the organization of a plan of congresses which should accomplish the ultimate purpose of the Exposition authorities. Neither one, however, could arrange to be relieved of the pressure of his regular duties, and the entire scheme of supervision was consequently changed. The plan adopted was based upon the idea of an advisory board composed of men of high literary and scientific standing who should consider and recommend the kind of congress most worthy of promotion, and the details of its development.

In November, 1902, Howard J. Rogers, LL.D., was appointed Director of Congresses, and the members of the Advisory (afterwards termed Administrative) Board selected as follows: —

CHAIRMAN: NICHOLAS MURRAY BUTLER, PH.D., LL.D., President Columbia University.

WILLIAM R. HARPER, PH.D., LL.D., President University of Chicago.

HONORABLE FREDERICK W. HOLLS, A.M., LL.B., New York.

R. H. JESSE, PH.D., LL.D., President University of Missouri.

HENRY S. PRITCHETT, PH.D., LL.D., President Massachusetts Institute of Technology.

HERBERT PUTNAM, LITT.D., LLD., Librarian of Congress.

FREDERICK J. V. SKIFF, A.M., Director of Field Columbian Museum.

The action of the Executive Committee of the Exposition, approved by the President, was as follows: —

> There shall be appointed by the President of the Exposition Company a Director of Congresses who shall report to the President of the Exposition Company.
>
> There shall be appointed by the President of the Exposition Company an Advisory Board of seven persons, the chairman to be named by the President, who shall meet at the call of the Director of Congresses, or the Chairman of the Advisory Board.
>
> The expenses of the members of the Advisory Board while on business of the Exposition shall be a charge against the funds of the Exposition Company.
>
> The duties of the said Advisory Board shall be: to consider and make recommendations to the Director of Congresses on all matters submitted to them; to determine the number and the extent of the congresses; the emphasis to be placed upon special features; the prominent men to be invited to participate; the character of the programmes; and the methods for successfully carrying out the enterprise.
>
> There shall be set aside from the Exposition funds for the maintenance of the congresses the sum of two hundred thousand dollars ($200,000).

The standing Committee on Congresses from the Exposition board of directors was shortly afterwards appointed and was composed of five of the most prominent men in St. Louis: —

CHAIRMAN: HON. FREDERICK W. LEHMANN, Attorney at Law.

BRECKENRIDGE JONES, Banker.

CHARLES W. KNAPP, Editor of *The St. Louis Republic.*

JOHN SCHROERS, Manager of the *Westliche Post.*

A. F. SHAPLEIGH, Merchant.

To this committee were referred for consideration by the President all matters of policy submitted by the Director of Congresses. This committee had jurisdiction over all congress matters, including not only the Congress of Arts and Science, but also the many miscellaneous congresses and conventions, and a great part of the success of the congresses is due to their broad-minded and liberal determination of the questions laid before them.

IDEA OF THE CONGRESS OF ARTS AND SCIENCE

It is impossible to ascribe the original idea of the Congress of Arts and Science to any one person. It was a matter of slow growth from the many conferences which had been held for a year by men of many occupations, and as finally worked out bore little resemblance to the original plans under discussion. The germ of the idea may fairly be said to have been contained in Director Skiff's insistence to the Executive Committee of the Exposition that the congress work stand for something more than an unrelated series of independent gatherings, and that some project be authorized which would at once be distinctive and of real scientific worth. To support this view Director Skiff brought the Executive Committee to the view of expending $200,000, if need be, to insure the project. Starting from this suggestion many plans were brought forward, but one which seems to belong of right to the late Honorable Frederick W. Holls, of New York City, contained perhaps the next recognizable step in advance. This thought was, briefly, that a series of lectures on scientific and literary topics by men prominent in their respective fields be delivered at the Exposition and that the Exposition pay the speakers for their services. This point was thoroughly discussed by Mr. Holls and President Butler, and the next step in the evolution of the Congress was the idea of bringing these lecturers together at the Exposition at about the same time or all during one month. At this stage Professor Hugo Münsterberg, who was the guest of Mr. Holls and an invited participant in the conference, made the important suggestion that such a series of unrelated lectures, even though given by most eminent men, would have little or no scientific value, but that if some relation, or underlying thought, could be introduced into the addresses, then the best work could be done, which would be of real value to the scientific world. He further stated that only in this case would scientific leaders be likely to favor the plan of a St. Louis congress, as they would feel attracted not so much through the honorariums to be given for their services as through the valuable opportunity of developing such a contribution to scientific thought. Subsequently Professor Münsterberg was asked by Mr. Holls to formulate his ideas in a manner to be submitted to the Exposition authorities. This was done in a communication under date of October 20, 1902, which contained logically presented the foundation of the plan afterwards worked out in detail. At this juncture the Department of Congresses was organized, as has been stated, the Director named, and the Administrative Board appointed, and on December 27, 1902, the first meeting of the Director with the Administrative Board took place in New York City.

A thorough canvass of the subject was made at this meeting and as a result the following recommendations were made to the Exposition authorities: —

(1) That the sessions of this Congress be held within a period of four weeks, beginning September 15, 1904.

(2) That the various groups of learned men who may come together be asked to discuss their several sciences or professions with reference to some theme of universal human interest, in order that thereby a certain unity of interest and of action may be had. Under such a plan the groups of men who come together would thus form sections of a single Congress rather than separate congresses.

(3) As a subject which has universal significance, and one likely to serve as a connecting thread for all of the discussions of the Congress, the theme "The Progress of Man since the Louisiana Purchase" was considered by the Administrative Board fit and suggestive. It is believed that discussions by leaders of thought in the various branches of pure and applied science, in philosophy, in politics, and in religion, from the standpoint of man's progress in the century which has elapsed, would be fruitful, not only in clearing the thoughts of men not trained in science and in government, but also in preparing the way for new advances.

(4) The Administrative Board further recommends that the Congress be made up from men of thought and of action, whose work would probably fall under the following general heads: —

a. The Natural Sciences (such as Astronomy, Biology, Mathematics, etc.).

b. The Historical, Sociological, and Economic group of studies (History, Political Economy, etc.).

c. Philosophy and Religion.

d. Medicine and Surgery.

e. Law, Politics, and Government (including development and history of the colonies, their government, revenue and prosperity, arbitration, etc.).

f. Applied Science (including the various branches of engineering).

(5) The Administrative Board recommends further referring to a special committee of seven the problem of indicating in detail the method in which this plan can best be carried out. To this committee is assigned the duty of choosing the general divisions of the Congress, the various branches of science and of study in these divisions, and of recommending to the Administrative Board a detailed plan of the sections in which, in their judgment, those who come to the Congress may be most effectively grouped, with a view not only to bring out the central theme, but also to represent in a helpful way and in a suggestive manner the present boundary of knowledge in the

various lines of study and investigation which the committee may think wise to accept.

These recommendations were transmitted by the Director of Congresses to the Committee on Congresses, approved by them, and afterwards approved by the Executive Committee and the President. The first four recommendations were of a preliminary character, but the fifth contained a distinct advance in the formation of a Committee on Plan and Scope which should be composed of eminent scientists capable of developing the fundamental idea into a plan which should harmonize with the scientific work in every field. The committee selected were as follows: —

Dr. Simon Newcomb, Ph.D., LL.D., Retired Professor of Mathematics, U. S. Navy.

Prof. Hugo Münsterberg, Ph.D., LL.D., Professor of Psychology, Harvard University.

Prof. John Bassett Moore, LL.D., ex-assistant Secretary of State, and Professor of International Law and Diplomacy, Columbia University.

Prof. Albion W. Small, Ph.D., Professor of Sociology, University of Chicago.

Dr. William H. Welch, M.D., LL.D., Professor of Pathology, Johns Hopkins University.

Hon. Elihu Thomson, Consulting Engineer General Electric Company.

Prof. George F. Moore, D.D., LL.D., Professor of Comparative Religion, Harvard University.

In response to a letter from President Butler, Chairman of the Administrative Board, giving a complete résumé of the growth of the idea of the Congress to that time, all of the members of the committee, with the exception of Mr. Thomson, met at the Hotel Manhattan on January 10, 1903, for a preliminary discussion. The entire field was canvassed, using the recommendations of the Administrative Board and the aforementioned letter of Professor Münsterberg's to Mr. Holls as a basis, and an adjournment taken until January 17 for the preparation of detailed recommendations.

The Committee on Plan and Scope again met, all members being present, at the Hotel Manhattan on January 17, and arrived at definite conclusions, which were embodied in the report to the Administrative Board, a meeting of which had been called at the Hotel Manhattan for January 19, 1903. The report of the Committee on Plan and Scope is of such historic importance in the development of the Congress that it is given as follows, although many points were afterwards materially modified: —

New York, January 19, 1903.

President Nicholas Murray Butler,
Chairman Administrative Board of World's Congress at
The Louisiana Purchase Exposition:

Dear Sir, — The undersigned, appointed by your Board a committee on the scope and plan of the proposed World's Congress, at the Louisiana Purchase Exposition, have the honor to submit the following report: —

The authority under which the Committee acted is found in a communication addressed to its members by the Chairman of the Administrative Board. A subsequent communication to the Chairman of the Committee indicated that the widest scope was allowed to it in preparing its plan. Under this authority the Committee met on January 10, 1903, and again on January 17. The Committee was, from the beginning, unanimous in accepting the general plan of the Administrative Board, that there should be but a single congress, which, however, might be divided and subdivided, in accord with the general plan, into divisions, departments, and sections, as its deliberations proceed.

PLANS OF THE CONGRESS

As a basis of discussion two plans were drawn up by members of the Committee and submitted to it. The one, by Professor Münsterberg, started from a comprehensive classification and review of human achievement in advancing knowledge, the other, by Professor Small, from an equally comprehensive review of the great public questions involved in human progress.

Professor Münsterberg proposed a congress having the definite task of bringing out the unity of knowledge with a view of correlating the scattered theoretical and practical scientific work of our day. This plan proposed that the congress should continue through one week. The first day was to be devoted to the discussion of the most general problem of knowledge in one comprehensive discussion and four general divisions. On the second day the congress was to divide into several groups and on the remaining days into yet more specialized groups, as set forth in detail in the plan.

The plan by Professor Small proposed a congress which would exhibit not merely the scholar's interpretation of progress in scholarship, but rather the scholar's interpretation of progress in civilization in general. The proposal was based on a division of human interests into six great groups: —

I. The Promotion of Health.
II. The Production of Wealth.
III. The Harmonizing of Human Relations.
IV. Discovery and Spread of Knowledge.
V. Progress in the Fine Arts.
VI. Progress in Religion.

The plan agreed with the other in beginning with a general discussion and then subdividing the congress into divisions and groups.

As a third plan the Chairman of the Committee suggested the idea of a congress of publicists and representative men of all nations and of all civilized peoples, which should discuss relations of each to all the others and throw light on the question of promoting the unity and progress of the race.

After due consideration of these plans the Committee reached the conclusion that the ends aimed at in the second and third plans could be attained by taking the first plan as a basis, and including in its subdivisions, so far as was deemed advisable, the subjects proposed in the second and third plans. They accordingly adopted a resolution that "Mr. Münsterberg's plan be adopted as setting forth the general object of the Congress and defining the scope of its work, and that Mr. Small's plan be communicated to the General Committee as containing suggestions as to details, but without recommending its adoption as a whole."

DATE OF THE CONGRESS

Your Committee is of opinion that, in view of the climatic conditions at St. Louis during the summer and early autumn, it is desirable that the meeting of this general Congress be held during the six days beginning on Monday, September 19, 1904, and continuing until the Saturday following. Special associations choosing St. Louis as their meeting-place may then convene at such other dates as may be deemed fit; but it is suggested that learned societies whose field is connected with that of the Congress should meet during the week beginning September 26.

The sectional discussions of the Congress will then be continued by these societies, the whole forming a continuous discussion of human progress during the last century.

PLAN OF ADDRESSES

The Committee believe that in order to carry out the proposed plan in the most effective way it is necessary that the addresses be prepared by the highest living authorities in each and every branch. In the last subdivisions, each section embraces two papers; one on the history of the subject during the last one hundred years and the other on the problems of to-day.

The programme of papers suggested by the Committee as embraced in Professor Münsterberg's plan may be summarized as follows: —

On the first day four papers will be read on the general subject, and four on each of the four large divisions, twenty in all. On the second day those four divisions will be divided into twenty groups, or departments, each of which will have four papers referring to the divisions and relations of the sciences, eighty in all. On the last four days, two papers in each of the 120 sections, 240 in all, thus making a total of 340 papers.

In view of the fact that the men who will make the addresses should not be expected to bear all the expense of their attendance at the Congress, it seems advisable that the authorities of the Fair should provide for the expenses necessarily incurred in the journey, as well as pay a small honorarium for the addresses. The Committee suggest, therefore, that each American invited be offered $100 for his traveling expenses and each European $400. In addition to this that each receive $150 as an honorarium. Assuming that one half of those invited to deliver addresses will be Americans and one half Europeans, this arrangement will involve the expenditure of $136,000. This estimate will be reduced if the same person prepares more than one address. It will also be reduced if more than half of the speakers are Americans, and increased in the opposite case.

As the Committee is not advised of the amount which the management of the Exposition may appropriate for the purpose of the Congress, it cannot, at present, enter further into details of adjustment, but it records its opinion that the sum suggested is the least by which the ends sought to be attained by the Congress can be accomplished To this must be added the expenses of administration and publication.

All addresses paid for by the Congress should be regarded as its property, and be printed and published together, thus constituting a comprehensive work exhibiting the unity, progress, and present state of knowledge.

This plan does not preclude the delivery of more than one address by a single scholar. The directors of the Exposition may sometimes find it advisable to ask the same scholar to deliver two addresses, possibly even three.

The Committee recommends that full liberty be allowed to each section of the Congress in arranging the general character and programme of its discussions within the field proposed.

As an example of how the plan will work in the case of any one section, the Committee take the case of a neurologist desiring to profit by those discussions which relate to his branch of medicine. This falls under C of the four main divisions as related to the physical sciences. His interest on the first day will therefore be centred in Division C, where he may hear the general discussion of the physical sciences and the relations to the other sciences. On the second day he will hear four papers in Group 18 on the subjects embraced in the general science of anthropology; one on its fundamental conceptions; one on its methods and two on the relation of anthropology to the sciences most closely connected with it. During the remaining four days he will meet with the representatives of medicine and its related subjects, who will divide into sections, and listen to four papers in each section. One paper will consider the progress of that section in the last one hundred years, one paper will be devoted to the problems of to-day, leaving room for such contributions and discussions as may seem appropriate during the remainder of the day.

COÖPERATION OF LEARNED SOCIETIES INVOKED

In presenting this general plan, your Committee wishes to point out the difficulty of deciding in advance what subjects should be included in every section. Therefore, the Committee deems it of the utmost importance to secure the advice and assistance of learned societies in this country in perfecting the details of the proposed plan, especially the selection of speakers and the programme of work in each section. It will facilitate the latter purpose if such societies be invited and encouraged to hold meetings at St. Louis during the week immediately preceding, or, preferably, the week following the General Congress. The selection of speakers should be made as soon as possible, and, in any case, before the end of the present academic year, in order that formal invitations may be issued and final arrangements made with the speakers a year in advance of the Congress.

CONCLUDING SUGGESTIONS

With the view of securing the coöperation of the governments and leading scholars of the principal countries of Western and Central Europe in the proposed Congress, it seems advisable to send two commissioners to these countries for this purpose. It seems unnecessary to extend the operations of this commission outside the European continent or to other than the leading countries. In other cases arrangements can be made by correspondence.

It is the opinion of the Committee that an American of world-wide reputation as a scholar should be selected to preside over the Congress.

All which is respectfully submitted.

(Signed) Simon Newcomb,
Chairman;
George F. Moore,
John B. Moore,
Hugo Münsterberg,
Albion W. Small,
William H. Welch,
Elihu Thomson,
Committee.

The Administrative Board met on January 19 to receive the report of the Committee on Plan and Scope which was presented by Dr. Newcomb. Professor Münsterberg and Professor John Bassett Moore were also present by invitation to discuss the details of the scheme. In the afternoon the Board went into executive session, and the following recommendations were adopted and transmitted by the Director of Congresses to the Committee on Congresses of the Exposition and to the President and Executive Committee, who duly approved them.

To the Director of Congresses: —

The Administrative Board have the honor to make the following recommendations in reference to the Department of Congresses: —

(1) That there be held in connection with the Universal Exposition of St. Louis in 1904, an International Congress of Arts and Science.

(2) That the plan recommended by the Committee on Plan and Scope for a general congress of Arts and Science, to be held during the six days beginning on Monday, September 19, 1904, be approved and adopted, subject to such revision in point of detail as may be advisable, preserving its fundamental principles.

(3) That Simon Newcomb, LL.D., of Washington, D. C., be named for President of the International Congress of Arts and Science, provided for in the foregoing resolution.

(4) That Professor Münsterberg, of Harvard University, and Professor Albion W. Small, of the University of Chicago, be invited to act as Vice-Presidents of the Congress.

(5) That the Directors of the World's Fair be requested to change the name of this Board from the "Advisory Board" to the "Administrative Board of the International Congress of Arts and Science."

(6) That the detailed arrangements for the Congress be intrusted to a committee consisting of the President and two Vice-Presidents already named, subject to the general oversight and control of the Administrative Board, and that the Directors of the Exposition be requested to make appropriate provision for their compensation and necessary expenses.

(7) That it be recommended to the Directors of the World's Fair that appropriate provision should be made in the office of the Department of Congresses for an executive secretary and such clerical assistance as may be needed.

(8) That the following payment be recommended to those scholars who accept invitations to participate and do a specified piece of work, or submit a specified contribution in the International Congress of Arts and Science: For traveling expenses for a European scholar, $500. For traveling expenses for an American scholar, $150.

(9) That provision be made for the publication of the proceedings of the Congress in suitable form to constitute a permanent memorial of the work of the World's Fair for the promotion of science and art, under competent editorial supervision.

(10) That an appropriation of $200,000 be made to cover expenses of the Department of Congresses, of which sum $130,000 be specifically appropriated for an International Congress of Arts and Science, and the remainder to cover all expenses connected with the publication of the proceedings of said International Congress of Arts and Science, and the expenses for promotion of all other congresses.

In addition to the foregoing recommendations, Professor Münsterberg was requested at his earliest convenience to furnish each member with a revised plan of his classification, which would reduce as far as possible the number of sections into which the Congress was finally to be divided.

With the adjournment of the Board on January 19 the Congress may be fairly said to have been launched upon its definite course, and such changes as were thereafter made in the programme did not in any wise affect the principle upon which the Congress was based, but were due to the demands of time, of expediency, and in some cases to the accidents attending the participation. The organization of the Congress and the personnel of its officers from this time on remained unchanged, and the history of the meeting is one of steady and progressive development. The Committee on Plan and Scope were discharged of their duties, with a vote of thanks for the laborious and painstaking work which they had accomplished and the thoroughly scientific and novel plan for an international congress which they had recommended.

It was determined by the Administrative Board to keep the services of three of the members of the Committee on Plan and Scope, who should act as a scientific organizing committee and who should also be the presiding officers of the Congress. The choice for President of the Congress fell without debate to the dean of American scientific circles, whose eminent services to the Government of the United States and whose recognized position in foreign and domestic scientific circles made him particularly fitted to preside over such an international gathering of the leading scientists of the world, Dr. Simon Newcomb, retired Professor of Mathematics, United States Navy. Professor Hugo Münsterberg, of Harvard University, and Professor Albion W. Small, of the University of Chicago, were designated as the first and second Vice-Presidents respectively.

The work of the succeeding spring, with both the Organizing Committee and the Administrative Board, was devoted to the perfecting of the programme and the selection of foreign scientists to be invited to participate in the Congress. The theory of the development of the programme and its logical bases are fully and forcibly treated by Professor Münsterberg in the succeeding chapter, and therefore will not be touched upon in this record of facts. As an illustration of the growth of the programme, however, it is interesting to compare its form, which was adopted at the next meeting of the Organizing Committee on February 23, 1903, in New York City, with its final form as given in the completed programme presented at St. Louis in September, 1904 (pp. 47–49). No better illustration can be given of the immense amount of labor and painstaking adjustment, both to scientific and to physical conditions, and of the admirable adaptability of the original plan to the exigencies of actual practice. At the meeting of February 23, 1903, which was attended by all of the members of the Organizing Committee and by President Butler of the Administrative Board, it was determined that the number of Departments should be sixteen, with the following designations: —

A. NORMATIVE SCIENCES

1. Philosophical Sciences.
2. Mathematical Sciences.

B. HISTORICAL SCIENCES

3. Political Sciences.
4. Legal Sciences.
5. Economic Sciences.
6. Philological Sciences.
7. Pedagogical Sciences.
8. Æsthetic Sciences.
9. Theological Sciences.

C. PHYSICAL SCIENCES

10. General Physical Sciences.
11. Astronomical Sciences.
12. Geological Sciences.
13. Biological Sciences.
14. Anthropological Sciences.

D. MENTAL SCIENCES

15. Psychological Sciences.
16. Sociological Sciences.

SECTIONS

1. *a* Metaphysics.
    b Logic.
    c Ethics.
    d Æsthetics.
2. *a* Algebra.
    b Geometry.
    c Statistical Methods.
3. *a* Classical Political History of Asia.
    b Classical Political History of Europe.
    c Medieval Political History of Europe.
    d Modern Political History of Europe.
    e Political History of America.
4. *a* History of Roman Law.
    b History of Common Law.
    aa Constitutional Law.
    bb Criminal Law.
    cc Civil Law.
    dd History of International Law.
5. *a* History of Economic Institutions.
    b History of Economic Theories.
    c Economic Law.
    aa Finance.
    bb Commerce and Transportation.
    cc Labor.
6. *a* Indo-Iranian Languages.
    b Semitic Languages.
    c Classical Languages.
    d Modern Languages.
7. *a* History of Education.
    aa Educational Institutions.
8. *a* History of Architecture.
    b History of Fine Arts.
    c History of Music.
    d Oriental Literature.
    e Classical Literature.
    f Modern Literature.
    aa Architecture.
    bb Fine Arts.
    cc Music.
9. *a* Primitive Religions.
    b Asiatic Religions.
    c Semitic Religions.
    d Christianity.
    aa Religious Institutions.
10. *a* Mechanics and Sound.
    b Light and Heat.
    c Electricity.
    d Inorganic Chemistry.
    e Organic Chemistry.
    f Physical Chemistry.
    aa Mechanical Technology.
    bb Optical Technology.
    cc Electrical Technology.

SECTIONS — *continued*

10. *dd* Chemical Technology.
11. *a* Theoretical Astronomy.
b Astrophysics.
12. *a* Geodesy.
b Geology.
c Mineralogy.
d Physiography.
e Meteorology.
aa Surveying.
bb Metallurgy.
13. *a* Botany.
b Plant Physiology.
c Ecology.
d Bacteriology.
e Zoölogy.
f Embryology.
g Comparative Anatomy.
h Physiology.
aa Agronomy.
bb Veterinary Medicine.
14. Anthropological Sciences:
a Human Anatomy.
b Human Physiology.
c Neurology.
d Physical Chemistry.
e Pathology.
f Raceomatology.
aa Hygiene.
bb Contagious Diseases.
cc Internal Medicine.
dd Surgery.
ee Gynecology.
ff Ophthalmology.
gg Therapeutics.
hh Dentistry.
15. Psychological Sciences:
a General Psychology.
b Experimental Psychology.
c Comparative Psychology.
d Child Psychology.
e Abnormal Psychology.
16. Sociological Sciences:
a Social Morphology.
b Social Psychology.
c Laws of Civilization.
d Laws of Language and Myths.
e Ethnology.
aa Social Technology.

It was also resolved, that the discussion of subjects falling under the first four divisions should be held in the forenoon of each of the four days, from Wednesday until Saturday, and those relating to the three divisions of Practical Science in the afternoon of the same days. The programme was thus rearranged by the addition of the following: —

E. UTILITARIAN SCIENCES

17. Medical Sciences:
a Hygiene.
b Sanitation.
c Contagious Diseases.
d Internal Medicine.
e Psychiatry.
f Surgery.
g Gynecology.
h Ophthalmology.
i Otology.
j Therapeutics.
k Dentistry.
18. Practical Economic Sciences:
a Extractive Productions of Wealth.
b Transportation.
c Commerce.
d Postal Service.
e Money and Banking.
19. Technological Sciences:
a Mechanical Technology.
b Electrical Technology.
c Chemical Technology.
d Optical Technology.
e Surveying.
f Metallurgy.
g Agronomy.
h Veterinary Medicine.

F. REGULATIVE SCIENCES

20. Practical Political Sciences:
    - *a* Internal Practical Politics.
    - *b* National Practical Politics.
    - *c* Tariff.
    - *d* Taxation.
    - *e* Municipal Practical Politics.
    - *f* Colonial Practical Politics.
21. Practical Legal Sciences:
    - *a* International Law.
    - *b* Constitutional Law.
    - *c* Criminal Law.
    - *d* Civil Law.
22. Practical Social Sciences:
    - *a* Treatment of the Poor.
    - *b* Treatment of the Defective.
    - *c* Treatment of the Dependent.
    - *d* Treatment of Vice and Crime.
    - *e* Problems of Labor.
    - *f* Problems of the Family.

G. CULTURAL SCIENCES

23. Practical Educational Sciences:
    - *a* Kindergarten and Home.
    - *b* Primary Education.
    - *c* Universities and Research — Secondary.
    - *d* Moral Education.
    - *e* Æsthetic Education.
    - *f* Manual Training.
    - *g* University.
    - *h* Libraries.
    - *i* Museums.
    - *j* Publications.
24. Practical Æsthetic Sciences:
    - *a* Architecture.
    - *b* Fine Arts.
    - *c* Music.
    - *d* Landscape Architecture.
25. Practical Religious Sciences:
    - *a* Religious Education.
    - *b* Training for Religious Service.
    - *c* Missions.
    - *d* Religious Influence.

The programme was again thoroughly revised at the meeting of the Organizing Committee on April 9, 1903, at Hotel Manhattan, and as thus amended was submitted to the Administrative Board at a meeting held in New York on April 11. A careful consideration of the programme at this meeting, and a final revision made at the meeting of the Administrative Board at the St. Louis Club April 30, 1903, brought it practically into its final shape, with such minor changes as were found necessary in the latter days of the Congress due to the unexpected declinations of foreign speakers at the last moment. The continuous and exacting work done in perfecting the programme by each member of the Organizing Committee and by the Chairman of the Administrative Board deserves special mention, and was productive of the best results by its logical appeal to the scientific world. The programme as finally worked out in orderly detail, shortened in many departments by various exigencies, may be found on pages 47 to 49 of this volume.

PARTICIPATION AND SUPPORT

The general plan of the Congress having been determined and the programme practically perfected by May 1, 1903, two most important questions demanded the attention of the Administrative Board: first, the participation in the Congress, both foreign and domestic;

second, the support of the scientific public. At a meeting of the Board held in New York City April 11, 1903, these points were given full consideration. It was determined that the list of speakers both foreign and domestic should be made up on the advice of men of letters and of scientific thought in this country, and accordingly there was sent to the officers of the various scientific societies in the United States, to heads of university departments and to every prominent exponent of science and art in this country, a printed announcement and tentative programme of the Congress, and a letter asking advice as to the scientists best fitted in view of the object of the Congress to prepare an address. From the hundreds of replies received in response to this appeal were made up the original lists of invited speakers, and only those were placed thereon who were the choice of a fair majority of the representatives of the particular science under selection. The Administrative Board reserved to itself the full right to reject any of these names or to change them so as to promote the best interests of the Congress, but in nearly every instance it would be safe to say that the person selected was highly satisfactory to the great majority of his fellow scientists in this country. Many changes were unavoidably made at the last moment to meet the situation caused by withdrawals and declinations, but the list of second choices was so complete, and in many cases there was such a delicate balance between the first and second choice, that there was no difficulty in keeping the standard of the programme to its original high plane.

It was early determined that the seven Division speakers and the forty-eight Department speakers, which occupied the first two days of the programme, should be Americans, and that these Division and Department addresses should be a contribution of American scholarship to the general scientific thought of the world. This decision commended itself to the scientific public both at home and abroad, and it was so carried out. It was further determined that the Division and Department speakers and the foreign speakers should be selected during the summer of 1903, and that the American participation in the Section addresses should be determined after it was definitely known what the foreign participation would be. In view of the importance of the Congress, it was deemed inadvisable to attempt to interest foreign scientific circles by correspondence, and it was further decided to pay a special compliment to each invited speaker by sending an invitation at the hands of special delegates. Arrangements were therefore made for Dr. Newcomb and Professors Münsterberg and Small to proceed to Europe during the summer of 1903, and to present in person to the scientific circles of Europe and to the scientists specially desired to deliver addresses the complete plan and scope of the Congress and an invitation to participate.

INVITATIONS TO FOREIGN SPEAKERS

The members of the Organizing Committee, armed with very strong credentials from the State Department to the diplomatic service abroad, sailed in the early summer of 1903 to present the invitation of the Exposition to the selected scientists. Dr. Newcomb sailed May 6, Professor Münsterberg May 30, and Professor Small June 6. A general interest in the project had at this time become aroused, and there was assured a respectful hearing. Both the President of the United States and the Emperor of Germany expressed their warm interest in the plan, and the State Department at Washington gave to the Congress both on this occasion and on succeeding occasions its effective aid. The Director of Congresses wishes to express his obligations both to the late Secretary Hay and to Assistant-Secretary Loomis for their valuable suggestions and courteous coöperation in all matters relating to the foreign participation. Strong support was also given the Committee and the plan of the Congress by Commissioner-General Lewald of Germany, and Commissioner-General Lagrave of France. Throughout the entire Congress period, both of these energetic Commissioners-General placed themselves actively at the disposition of the Department in promoting the attendance of scientists from their respective countries.

Geographically the division between the three members of the Organizing Committee gave to Dr. Newcomb, France; to Professor Münsterberg, Germany, Austria, and Switzerland; and to Professor Small, England, Russia, Italy, and a part of Austria. It was also agreed that Dr. Newcomb should have special oversight of the departments of Mathematics, Physics, Astronomy, Biology, and Technology; Professor Münsterberg, special charge of Philosophy, Philology, Art, Education, Psychology, and Medicine; and that Professor Small should look after Politics, Law, Economics, Theology, Sociology, and Religion. The Committee worked independently of each other, but met once during the summer at Munich to compare results and to determine their closing movements.

The public and even the Exposition authorities have probably never realized the delicacy and the extremely careful adjustment exercised by the Organizing Committee in their summer's campaign. Scientists are as a class sensitive, jealous of their reputations, and loath to undertake long journeys to a distant country for congress purposes. The amount of labor devolving upon the Committee to find the scientists scattered over all Europe; the careful and painstaking presentation to each of the plan of the Congress; the appeal to their scientific pride; the hearing of a thousand objections, and the answering of each; the disappointments incurred; the substitutions made necessary at the last moment; — all sum up a task of the greatest difficulty and of enormous labor. The remarkable success with which the mission was crowned stands out the more prominently in view of these conditions. When the Committee returned in the latter part of September, they had visited every important country of Europe, delivered more than one hundred fifty personal invitations, and for the one hundred twenty-eight sections had secured one hundred seventeen acceptances.

At a meeting of the Administrative Board, which met with the Organizing Committee on October 13, 1903, a full report of the European trip was received and ways and means considered for insuring the attendance from abroad. A list of the foreign acceptances was ordered printed at once for general distribution, and the Chairman of the Administrative Board was requested to address a letter to each of the foreign scientists confirming the action of the special delegates and giving additional information as to the length of addresses, and rules and details governing the administration of the Congress.

DEATH OF FREDERICK W. HOLLS

The number of the Administrative Board was decreased during the summer by the sudden death of the Hon. Frederick W. Holls, on July 23, 1903. Mr. Holls had been intensely interested in the development of the Congress from its earliest days, and was very instrumental in determining the form in which it was finally promoted. His great influence abroad as a member of the Hague Conference, and his high standing in legal and literary circles in this country, rendered him one of the most prominent members of the Board. A resolution of regret at his untimely death was spread upon the minutes of the Administrative Board at the meeting in October, and it was decided that his place upon the Board should remain unfilled.

DOMESTIC PARTICIPATION

At this same meeting of October 13, active measures were taken to forward the American participation in the Congress. The necessity was now very evident that our strongest men of science must be induced to take part, in order to compare favorably with the leading minds which Europe was sending. The Organizing Committee were instructed to consult the American scientific societies and associations regarding the selection of American speakers, and also in reference to presiding officials for each section. Six weeks was considered sufficient for this task, and the Committee were asked to submit to the Administrative Board at a meeting in New York, on December 3 and 4, their recommendations for American speakers.

An immense amount of detailed labor, in the way of correspondence, now devolved upon the Organizing Committee as well as upon the Director of Congresses, and a branch office was established in

Washington equipped with clerks and stenographers under the charge of Dr. Newcomb, who devoted the greater portion of his time for the next six months to the many details connected with the selection of foreign and American speakers and chairmen. The meeting of the Administrative Board in New York in December, and a similar meeting with the Organizing Committee held at the St. Louis Club on December 28, were given over entirely to perfecting the personnel of the programme. Great care was exerted in selecting the chairmen of the departments and sections, inasmuch as they must be men of international reputation and conceded strength. For the secretaryships younger men of promise and ability were selected, chiefly from university circles. Both the chairmen and secretaries served without compensation.

The work of the late winter was a continuance of the perfecting of details, and at a meeting of the Administrative Board held in New York in February, 1904, a final approval was given to the programme and the speakers. The imminent approach of the Exposition and the work of the college commencement season made it impossible for further general meetings, and on June 1 the Organizing Committee was constituted a committee with power to fill vacancies in the programme or to amend the programme as circumstances might demand. All suggestions with reference to details were to be made directly to the Director of Congresses, upon whom devolved from this time forward the entire executive control of the Congress.

ASSEMBLY HALLS

The highly diversified nature of the Congress and the holding of one hundred twenty-eight section meetings in four days' time rendered necessary a large number of meeting-places centrally located. The Exposition was fortunate in having the use of the new plant of the Washington University, nine large buildings of which had been erected. Many of these buildings contained lecture halls and assembly rooms, seating from one hundred fifty to fifteen hundred people. Sixteen halls were necessary to accommodate the full number of sections running at any one time, and of this number twelve were available in the group of University Buildings; the other four were found in the lecture halls of the Education Building, Mines and Metallurgy Building, Agriculture Building, and the Transportation Building. The opening exercises, at which the entire Congress was assembled, was held in Festival Hall, capable of seating three thousand people. In the assignment of halls care was taken so far as possible to assign the larger halls to the more popular subjects, but it often happened that a great speaker was of necessity assigned to a smaller hall. Two of the halls also proved bad for speaking owing to the traffic of the Intramural Railway, and there was lacking in nearly all of the halls that academic peace and quiet which usually surrounds gatherings of a scientific nature. This, however, was to be expected in an exposition atmosphere, and was readily acquiesced in by the speakers themselves, and very little objection was heard to the halls as assigned. Every one seemed to recognize the fact that the immediate value of the meeting lay in the commingling and fellowship, and that the addresses, of which one could hear at most only one in sixteen, could not be judged in the proper light until their publication.

SUPPORT OF THE SCIENTIFIC PUBLIC

A strong effort was made by the Organizing Committee to secure the attendance of an audience which should not only in its proportions be complimentary to the eminence of the speakers, but also be thoroughly appreciative of the addresses and conversant with the topic under discussion. Letters were therefore sent to all of the prominent scientific societies in the United States, asking that wherever possible the meetings of the society be set for the Congress week in St. Louis, and wherever this was not possible that the societies send special delegates to attend the Congress, and urge their membership to make an effort to be present. Personal letters were also sent to the leading members of the different professions and sciences, to the faculties of universities and colleges, urging them to attend, and pointing out the necessity of the support of the American scientific public.

Special invitations were also sent in the name of the Organizing Committee to the leading authorities of the various subjects under discussion in the Congress, asking them to contribute a ten-minute paper to any section in which they were particularly interested. The result of this careful campaign, in addition to the general exploitation which the Congress received, was such a flattering attendance of American scientists, as to be both a compliment to the European speakers and a benefit to scientific thought. Many societies, such as the American Neurological Association, American Philological Association, American Mathematical Society, Physical and Chemical Societies of America, American Astronomical Society, Germanic Congress, American Electro-Therapeutic Association, held their annual meetings during the week of the Congress, although the date rendered it impossible for the majority of the associations to meet at that time. The eighth International Geographic Congress adjourned from Washington to St. Louis to meet with the Congress of Arts and Science. In response to the special invitations, two hundred forty-seven ten-minute addresses were promised and one hundred two actually read.

RECEPTION OF FOREIGN GUESTS

Every effort was made by the Department of Congresses to assist the foreign speakers in their traveling arrangements and to make matters as easy and comfortable as possible. A letter of advice was mailed to each speaker prior to his departure, carefully setting forth the conditions of American travel, routes to be followed, reception committees to be met, and other essential details. The official badge of the Congress was also mailed, so that those wearing them might be easily identified by the reception committees both in New York and St. Louis. Nine tenths of the speakers came by the way of New York, and in order to facilitate the clearance of their baggage and to provide for their fitting entertainment in New York, a special reception committee was formed composed of the following members: —

F. P. Keppel, Columbia University, New York City, Chairman.

Prof. Herbert V. Abbott, New York.
R. Arrowsmith, New York.
C. William Beebe, New York.
George Bendelari, New York.
Edward W. Berry, Passaic.
J. Fuller Berry, Old Forge,
Rev. H. C. Birckhead, New York.
Dr. James H. Canfield, New York.
Rev. G. A. Carstenson, New York.
Prof. H. S. Crampton, New York.
Sanford L. Cutler, New York.
Dr. Israel Davidson, New York.
William H. Davis, New York.
Prof. James C. Egbert, New York.
Dr. Haven Emerson, New York.
Prof. T. S. Fiske, New York.
J. D. Fitz-Gerald, II, Newark.
W. D. Forbes, Hoboken.
Clyde Furst, Yonkers.
William K. Gregory, New York.
George C. O. Haas, New York.
Prof. W. A. Hervey, New York.
Carl Herzog, New York.
Robert Hoguet, New York.
Dr. Percy Hughes, Brooklyn.
Prof. A. V. W. Jackson, New York.
Albert J. W. Kern, New York.
Prof. Charles F. Kroh, Orange.
Dr. George F. Kunz, New York.
Prof. L. A. Lousseaux, New York.
Frederic L. Luqueer, Brooklyn.
R. A. V. Minckwitz, New York.
Charles A. Nelson, New York.
Dr. Harry B. Penhollow, New York.
Prof. E. D. Perry, New York.
John Pohlman, New York.
Dr. Ernest Richard, New York.
Dr. K. E. Richter, New York.
Edward Russ, Hoboken.
Prof. C. L. Speranza, Oak Ridge.
Prof. Francis H. Stoddard, New York.
Dr. Anthony Spitzka, Goodground.
Harvey W. Thayer, Brooklyn.
Prof. H. A. Todd, New York.
Dr. E. M. Wahl, New York.
Prof. F. H. Wilkens, New York.

To each foreign speaker was extended the courtesies of the Century and the University clubs while remaining in New York City. Mention should also be made of the assistance of the Treasury Department and of the courtesy of Collector of the Port, Hon. N. N. Stranahan, through whom special privileges of the Port were extended to the members of the Congress. The work of the reception committee was most satisfactorily and efficiently performed, and was highly appreciated by the foreign guests. Special acknowledgment is due Mr. F. P. Keppel, of Columbia University, for his painstaking and efficient management of the affairs of the committee in New York. Many of the speakers proceeded singly to St. Louis, stopping at various places, but the great majority went directly to the University of Chicago, where they were entertained during the week preceding the Congress by President Harper and Professor Small, of the University of Chicago. The arrivals at St. Louis were made on Saturday the 17th and Sunday the 18th of September. Many of the participants had arrived at earlier dates, and fully twenty of the speakers were members of the International Jury of Awards for their respective countries, and had been in St. Louis since September 1, the beginning of the Jury work.

A reception committee similar to that in New York was also formed at St. Louis from the members of the University Club, and their duties were to meet all incoming trains and conduct the members of the Congress personally to their stopping-places, and assist them in all matters of detail. This committee was comprised of the following members, nearly all of the University Club, who performed their work efficiently and enthusiastically to the great satisfaction of the Exposition and to the thorough appreciation of the foreign guests: —

V. M. Porter, Chairman,	St. Louis.
E. H. Angert,	St. Louis.
Gouverneur Calhoun,	St. Louis.
W. M. Chauvenet,	St. Louis.
H. G. Cleveland,	St. Louis.
Mr. M. B. Clopton,	St. Louis.
Walter Fischel,	St. Louis.
W. L. R. Gifford,	St. Louis.
E. M. Grossman,	St. Louis.
L. W. Hagerman,	St. Louis.
Louis La Beaume,	St. Louis.
Carl H. Lagenburg,	St. Louis.
Sears Lehmann,	St. Louis.
G. F. Paddock,	St. Louis,
T. G. Rutledge,	St. Louis.
Luther Ely Smith,	St. Louis.
J. Clarence Taussig,	St. Louis.
C. E. L. Thomas,	St. Louis.
W. M. Tompkins,	St. Louis.
G. T. Weitzel,	St. Louis.
Tyrrell Williams,	St. Louis.

The itinerary of the foreign speakers after leaving St. Louis at the end of the Congress took them on appointed trains to Washington, where they were given an official reception by President Roosevelt and a reception by Dr. Simon Newcomb, President of the Congress. From here they proceeded to Harvard University, Cambridge, Mass., where they were given a reception by Prof. Hugo Münsterberg, and were entertained as guests of Harvard University. Thence the great majority of the speakers returned to New York, where they were the guests of Columbia University, and were given a farewell dinner by the Association of Old German Students. Many of the speakers, however, visited other portions of the country before returning to Europe.

The foreign speakers while in St. Louis were considered the guests of the Exposition Company, and were relieved from all care and expense for rooms and entertainment. Those who were accompanied by their wives and daughters were entertained by prominent St. Louis families, and those who came singly were quartered in the dormitory of the Washington University, which was set aside for this purpose during the week of the Congress. The dormitory arrangement proved a very happy circumstance, as nearly one hundred foreign and American scientists of the highest rank were thrown in contact, much after the fashion of their student days, and thoroughly enjoyed the novelty and fellowship of the plan. The dormitory contained ninety-six rooms newly fitted up with much care and with all modern conveniences. Light breakfasts were served in the rooms, and special service provided at the call of the occupants. The situation of the dormitory also in the Exposition grounds in close proximity to the assembly halls was highly appreciated, and although at times there were minor matters which did not run so smoothly, the almost unanimous expression of the guests of the Exposition was one of delight and appreciation of the arrangements. Special mention ought in justice to be made to those residents of St. Louis who sustained the time-honored name of the city for hospitality and courtesy by entertaining those foreign members of the Congress who were accompanied by the immediate members of their family. They were as follows: —

Dr. C. Barck
Dr. William Bartlett
Judge W. F. Boyle
Mr. Robert Brookings
Mrs. J. T. Davis
Dr. Samuel Dodd
Mr. L. D. Dozier
Dr. W. E. Fischel
Mr. Louis Fusz
Mr. August Gehner
Dr. M. A. Goldstein
Mr. Charles H. Huttig
Dr. Ernest Jonas
Mr. R. McKittrick Jones
Mr. F. W. Lehmann
Dr. Robert Luedeking
Mr. Edward Mallinckrodt
Mr. George D. Markham
Mr. Thomas McKittrick
Mr. Theodore Meier
Dr. S. J. Niccolls
Dr. W. F. Nolker
Dr. S. J. Schwab
Dr. Henry Schwartz
Mr. Corwin H. Spencer
Dr. William Taussig
Mr. G. H. Tenbroek
Dr. Herman Tuholske
Hon. Rolla Wells
Mr. Edwards Whitaker
Mr. Charles Wuelfing
Mr. Max Wuelfing.

DETAIL OF THE CONGRESS

The immense amount of detail work which devolved upon the Department in the matter of preparing halls for the meetings, receiving guests, providing for their comfort, issuing the programmes, managing the detail of the receptions, banquets, invitations, etc., providing for registration, payment of honorariums, and furnishing information on every conceivable topic, rendered necessary the formation of a special bureau which was placed in charge of Dr. L. O. Howard of Washington, D. C., as Executive Secretary. Dr. Howard's long experience as Secretary of the American Association for the Advancement of Science rendered him particularly well qualified to assume this laborious and thankless task. By mutual arrangement the Director of Congresses and the Executive Secretary divided the field of labor. The Director had, in addition to the general oversight of the Congress, special supervision of the local reception committee, the entertainment of the guests, official banquets and entertainments, and all financial details. The Executive Secretary took entire charge of the programme, assignment of rooms in the dormitory, care and supervision of the dormitory, assignment of halls for speakers, registration books and bureau of information. Dr. Howard arrived on September 1 to begin his duties, and remained until September 30.

WEEK OF THE CONGRESS

The opening session of the Congress was set for Monday afternoon, September 19, at 2.30 o'clock in Festival Hall. The main programme of the Congress began Tuesday morning. The sessions were held in the mornings and afternoons, the evenings being left free for social affairs. The list of functions authorized in honor of the Congress of Arts and Science were as follows: —

Monday evening, September 19, grand fête night in honor of the guests of the Congress, with special musical programme about the Grand Basin and lagoons, boat rides and lagoon fête; this function was unfortunately somewhat marred by inclement weather. It was the only evening free in the entire week, however, for members of the Congress to witness the illuminations and decorative evening effects.

Banquet given by the St. Louis Chemical Society at the Southern Hotel to members of the chemical sections of the Congress.

Tuesday evening, September 20, general reception by the Board of Lady Managers to the officers and speakers of the Congress and officials of the Exposition.

Wednesday afternoon, September 21, garden fête given to the members of the Congress at the French National Pavilion by the Commissioner-General from France. The gardens of the miniature Grand Trianon were never more beautiful than on this brilliant afternoon, and the presence of the Garde Républicaine band and the entire official representation of the Exposition, lent a color and spirit to the affair unsurpassed during the Exposition period.

Wednesday evening, reception by the Imperial German Commissioner-General to the officers and speakers of the Congress and the officials of the Exposition, at the German State House. The magnificent hospitality which characterized this building during the entire Exposition period was fairly outdone on this occasion, and the function stands prominent as one of the brilliant successes of the Exposition period.

Thursday evening, September 22, Shaw banquet at the Buckingham Club to the foreign delegates and officers of the Congress. Through the courtesy of the trustees of Shaw's Garden and of the officers of Washington University, the annual banquet provided for men of science, letters, and affairs, by the will of Henry B. Shaw, founder of the Missouri Botanical Gardens, was given during this week as a compliment to the noted foreign scientists who were the guests of the city of St. Louis.

Friday evening, September 23, official banquet given by the Exposition to the speakers and officials of the Congress and the officials of the Exposition, in the banquet hall of the Tyrolean Alps.

Saturday evening, September 24, banquet at the St. Louis Club given by the Round Table of St. Louis, to the foreign members of the Congress. The Round Table is a literary club which meets at banquet six times annually for discussion of topics of interest to the literary and scientific world.

Banquet given by the Imperial Commissioner-General from Japan to the Japanese delegation to the Congress and to the Exposition officials and Chiefs of Departments.

Dinner given by Commissioner-General from Great Britain to the English members of the Congress.

OPENING OF THE CONGRESS

The assembling of the Congress on the afternoon of September 19, in the magnificent auditorium of Festival Hall which crowned Cascade Hill and the Terrace of States, was marked with simple ceremonies and impressive dignity. The great organ pealed the national hymns of the countries participating and closed with the national anthem of the United States. In the audience were the members of the Congress representing the selected talent of the world in their field of scientific endeavor, and about them were grouped an audience drawn from every part of the United States to promote by their presence the success of the Congress and to do honor to the noted personages who were the guests of the Exposition and of the Nation. On the stage were seated the officials of the Congress, the honorary vice-presidents from foreign nations, and the officials of the Exposition.

At the appointed hour the Director of Congresses, Dr. Howard J. Rogers, called the meeting to order, and outlined in a few words the object of the Congress, welcomed the foreign delegates, and presented the members, both foreign and American, to the President of the Exposition, Hon. David R. Francis.

The President spoke as follows: —

What an ambitious undertaking is a universal exposition! But how worthy it is of the highest effort! And, if successful, how far-reaching are its results, how lasting its benefits! Who shall pass judgment on that success? On what evidence, by what standards shall their verdicts be formed? The development of society, the advancement of civilization, involve many problems, encounter many and serious difficulties, and have met with deplorable reactions which decades and centuries were required to repair. The proper study of mankind is man, and any progress in science that ignores or loses sight of his welfare and happiness, however admirable and wonderful such progress may be, disturbs the equilibrium of society.

The tendency of the times toward centralization or unification is, from an economic standpoint, a drifting in the right direction, but the piloting must be done by skillful hands, under the supervision and control of far-seeing minds, who will remember that the masses are human beings whose education and expanding intelligence are constantly broadening and emphasizing their individuality. A universal exposition affords to its visitors, and those who systematically study its exhibits and its phases, an unequaled opportunity to view the general progress and development of all countries and all races. Every line of human endeavor is here represented.

The conventions heretofore held on these grounds and many planned to be held — aggregating over three hundred — have been confined in their deliberations to special lines of thought or activity. This international congress of arts and sciences is the most comprehensive in its plan and scope of any ever held, and is the first of its kind. The lines of its organization, I shall leave the Director of Exhibits, who is also a member of the administrative board of this congress, to explain. You who are members are already advised as to its scope, and your almost universal and prompt acceptance of the invitations extended to you to participate, implies an approval which we appreciate, and indicates a willingness and a desire to coöperate in an effort to bring into intelligent and beneficial correlation all branches of science, all lines of thought. You need no argument to convince you of the eminent fitness of making such a congress a prominent feature of a universal exposition in which education is the dominant feature.

The administrative board and the organizing committee have discharged their onerous and responsible tasks with signal fidelity and ability, and the success that has rewarded their efforts is a lasting monument to their wisdom. The management of the Exposition tenders to them, collectively and individually, its grateful acknowledgments. The membership in this congress represents the world's elect in research and in thought. The participants were selected after a careful survey of the entire field; no limitations of national boundaries or racial affiliations have been observed. The Universal Exposition of 1904, the city of St. Louis, the Louisiana territory whose acquisition we are celebrating, the entire country, and all participating in or visiting this Exposition are grateful for your coming, and feel honored by your presence.

We are proud to welcome you to a scene where are presented the best and highest material products of all countries and of every civilization, participated in by all peoples, from the most primitive to the most highly cultured — a marker in the progress of the world, and of which the International Congress of Arts and Science is the crowning feature.

May the atmosphere of this universal exposition, charged as it is with the restless energies of every phase of human activity and permeated by that ineffable sentiment of universal brotherhood engendered by the intelligent sons of God, congregating for the friendly rivalries of peace, inspire you with even higher thoughts — imbue you with still broader sympathies, to the end that by your future labors you may be still more helpful to the human race and place your fellow men under yet deeper obligations.

Director Frederick J. V. Skiff was then introduced by the President as representing the Division of Exhibits, whose untiring labors had filled the magnificent Exposition palaces surrounding the Festival Hall with the visible products of those sciences and arts, the theory, progress, and problems of which the Congress was assembled to consider.

Mr. Skiff spoke as follows: —

The division of exhibits of the Universal Exposition of 1904 has looked forward to this time, when the work it has performed is to be reviewed and discussed by this distinguished body. I do not, of course, intend to convey the idea that the international congress is to inspect or criticise the exhibitions, but I do mean to say that the deliberations of this organization are contemporaneous with and share the responsibility for the accomplishments of which the exhibitions made are the visible evidences.

The great educational yield of a universal exposition comes from the intellectual more than from the mechanical processes. It is the material condition of the times. It is as well the duty of the responsible authorities to go yet further and record the thoughts and theories, the investigations, experiments, and observations of which these material things are the tangible results.

A congress of arts and science, whose membership is drawn from all educational as well as geographical zones, not only accounts for and analyzes the philosophy of conditions, but points the way for further advance along the lines consistent with demonstration. Its contribution to the hour is at once a history and a prophecy.

The extent to which the deliberations and utterances of this congress may regulate the development of society or give impulse to succeeding generations, it is impossible to estimate, but not unreasonable to anticipate. The plans of the congress matured in the minds of the best scholars; the classification of its purpose, the scope, the selection of its distinguished participants, gave to the hopes and ambitions of the management of the Exposition inspiration of a most exalted degree. At first these ambitions were — not without reason — regarded as too high. The plane upon which the congress had been inaugurated, the aim, the broad intent, seemed beyond the merits, if not beyond the capacity, of this hitherto not widely recognized intellectual centre. But the courage of the inception, the loftiness of the purpose, appealed so profoundly to the toilers for truth and the apostles of fact, that we find gathered here to-day in the heart of the new Western continent the great minds whose impress on society has rendered possible the intellectual heights to which this age has ascended and now beckon forward the students of the world to limitless possibilities.

While international congresses of literature, science, art, and industry have been accomplished by previous expositions, yet to classify and select the topics in sympathy with the classification and installation of the exhibits material is a step considerably in advance of the custom. The men who build an exposition must by temperament, if not by characteristic, be educators. They must be in sympathy with the welfare of humanity and its higher destiny. The exhibitions at this Exposition are not the haphazard gatherings of convenient material, but the outcome of a plan to illustrate the productiveness of mankind at this particular time, carefully digested, thoroughly thought out, and conscientiously executed. The exhibit, therefore, in each of the departments of the classification, as well as in the groups of the different departments, are of such character, and so arranged as to reflect the best that the world can do along departmental lines, and the best that different peoples can do along group lines. The congresses accord with the exhibits, and the exhibits give expression to the congresses.

Education has been the keynote of this Exposition. Were it not for the educational idea, the acts of government providing vast sums of money for the upbuilding of this Exposition would have been impossible. This congress reflects one idea vastly outstripping others, and that is, in the unity of thought in the universal concert of purpose. It is the first time, I believe, that there has been an international gathering of the authorities of all the sciences, and in that respect the congress initiates and establishes the universal brotherhood of scholars.

A thought uncommunicated is of little value. An unrecorded achievement is not an asset of society. The real lasting value of this congress will consist of the printed record of its proceedings. The delivery of the addresses, reaching and appealing to, as must necessarily be the case, a very limited number of people, can be considered as only a method of reaching the lasting and perpetual good of civilization.

In just the degree that this Exposition in its various divisions shall make a record of accomplishments, and lead the way to further advance, this enterprise has reached the expectations of its contributors and the hopes of its promoters. This congress is the peak of the mountain that this Exposition has builded on the highway of progress. From its heights we contemplate the past, record the present, and gaze into the future.

This universal exposition is a world's university. The International Congress of Arts and Science constitutes the faculty; the material on exhibition are the laboratories and the museums; the students are mankind.

That in response to invitation of the splendid committee of patriotic men, to whom all praise is due for their efforts in this crowning glory of the Exposition, so eminent a gathering of the scholars and savants of the world has resulted, speaks unmistakably for the fraternity of the world, for the sympathy of its citizenship, and for the patriotism of its people.

In reply to these addresses of the officials of the Exposition, the honorary Vice-Presidents for Great Britain, France, Germany, Russia, Austria, Italy, and Japan made brief responses in behalf of their respective countries.

Sir William Ramsay of London spoke in the place of Hon. James Bryce, extending England's thanks for the courtesy which had been shown her representatives and declaring that England, particularly in the scientific field, looked upon America as a relative and not as a foreign country.

France was represented by Professor Jean Gaston Darboux, Perpetual Secretary of the Academy of Sciences of Paris, who spoke as follows: —

MR. PRESIDENT, LADIES AND GENTLEMEN, — My first word will be to thank you for the honor which you have been so courteous as to pay my country in reserving for her one of the vice-presidencies of the Congress. Since the time of Franklin, who received at the hands of France the welcome which justice and his own personal genius and worth demanded, most affectionate relations have not ceased to unite the scientists of France and the scientists of America. The distinction which you have here accorded to us will contribute still further to render these relations more intimate and more fraternal. In choosing me among so many of the better fitted delegates sent by my country, you have without doubt wished to pay special honor to the Académie des Sciences and to the Institut de France, which I have the honor of representing in the position of Perpetual Secretary. Permit me therefore to thank you in the name of these great societies, which are happy to count in the number of their foreign associates and of their correspondents so many of the scholars of America. In like manner as the Institut de France, so the Congress which opens to-day seeks to unite at the same time letters, science, and arts. We shall be happy and proud to take part in this work and contribute to its success.

Germany was represented by Professor Wilhelm Waldeyer, of the University of Berlin, who replied as follows: —

MR. PRESIDENT, HONORED ASSEMBLAGE, — The esteemed invitation which has been offered to me in this significant hour of the opening of the Congress of Arts and Science to greet the members of this congress, and particularly my esteemed compatriots, I have had no desire to decline. I have been for a fortnight under the free sky of this mighty city — so I must express myself, since enclosing walls are unknown in the United States — and this fact, together with the hospitality offered me in such delightful manner by the Chairman of the Committee on Congresses, Mr. Frederick W. Lehmann, has almost made me a St. Louis man. Therefore I may perhaps take it upon myself to greet you here.

I confess that I arrived here with some misgiving — some doubts as to whether the great task which was here undertaken under most difficult circumstances could be accomplished with even creditable success. These doubts entirely disappeared the first time I entered the grounds of the World's Fair and obtained a general view of the method, beautiful as well as practical, by which the treasures gathered from the whole world were arranged and displayed. I trust you, too, will have a like experience; and will soon recognize that a most earnest and good work is here accomplished.

And I must remark at this time that we Germans may indeed be well satisfied here; the unanimous and complete recognition which our coöperation in this great work has received is almost disconcerting.

What can be said of the whole Exposition with reference to its extent and the order in which everything is arranged, I may well say concerning the departments of science, especially interesting to us. In this hour in which the Congress of Arts and Science is being opened, we shall not express any thanks to those who took this part of the work upon their shoulders — a more difficult task indeed than all the others, for here the problem is not to manage materials, but heads and minds. And as I see here assembled a large number of German professors — I, too, belong to the profession — of whom it is said, I know not with how much justice, that they are hard to lead, the labors of the Directors and Presidents of the Congress could not have been, and are not now, small. Neither shall we to-day prophesy into what the Congress may develop. The greater number of speakers cannot expect to have large audiences, but even to-day we can safely say this: the imposing row of volumes in which shall be given to posterity the reviews here to be presented concerning the present condition, and future problems of the sciences and arts as they appear to the scientific world at the beginning of the twentieth century, will provide a monumental work of lasting value. This we may confidently expect. The thanks which we to-day do not wish to anticipate in words, let us show by our actions to our kind American hosts, and especially to the directors of the World's Fair and of this Congress. With exalted mind, forgetting all little annoyances which may and will come, let us go forward courageously to the work, and let us do our best. Let us grasp heartily the open hand honestly extended to us.

May this Congress of Arts and Science worthily take part in the great and undisputed success which even to-day we must acknowledge the World's Fair at St. Louis.

For Austria Dr. Theodore Escherich, of the University of Vienna, responded as follows: —

In the name of the many Austrians present at the Congress I express the thanks of my compatriots to the Committee which summoned us, for their invitation and the hospitality so cordially extended. . . .

I congratulate the authorities upon the idea of opening this Congress. How many world-expositions have already been held without an attempt having been made to exhibit the spirit that has created this world of beautiful and useful things? It was reserved for these men to find the form in which the highest results of human thought — Science — represented in the persons of her representatives, could be incorporated in the compass of the World's Fair. The conception of this International Congress of all Sciences in its originality and audacity, in its universality and comprehensive organization, is truly a child of the "young-American spirit." . . .

After this Congress has come to a close and the collection of the lectures delivered, an unparalleled encyclopædia of human knowledge, both in extent and content, will have appeared. We may say that this Fair has become of epochal importance, not alone for trade and manufactures, but also for science. These proud palaces will long have disappeared and been forgotten when this work, a *monumentum aere perennius*, shall still testify to future generations the standard of scientific attainment at the beginning of the twentieth century.

Short acknowledgments were then made for Russia by Dr. Oscar Backlund, of the Astronomical Observatory at Pulkowa, Russia, and for Japan by Prof. Nobushige Hozumi, of the Imperial University at Tokio, Japan.

The last of the Vice-Presidents to respond to the addresses of welcome was Signor Attilio Brunialti, Councilor of State for Italy, who after a few formal words in English broke into impassioned eloquence in his native tongue, and in brilliant diction and graceful periods expressed the deep feeling and profound joy which Italy, the mother of arts, felt in participating in an occasion so historic and so magnificent. Signor Brunialti said in part: —

I thank you, gentlemen, for the honor you have paid both to my country and myself by electing me a Vice-President of this great scientific assembly. Would that I could thank you in words in which vibrate the heart of Rome, the scientific spirit of my land, and all that it has given to the world for the progress of science, literature, and art. You know Italy, gentlemen, you admire her, and therefore it is for this also that my thanks are due to you. What ancient Rome has contributed to the common patrimony of civilization is also reflected here in a thousand ways, and a classical education, held in such honor, by a young and practical people such as yours, excites our admiration and also our astonishment. By giant strides you are reviving the activity of Italy at the epoch of the Communes, when all were animated by unwearying activity and our manufactures and arts held the first place in Europe. I have already praised here the courageous spirit which has suggested the meeting of this Congress — a Congress that will remain famous in the annals of science. Many things in your country have aroused in me growing surprise, but nothing has struck me more, I assure you, than this homage to science which is pushing all the wealthy classes to a noble rivalry for the increase of education and mental cultivation.

You have already large libraries and richly endowed universities, and every kind of school, where the works of Greece and Rome are perhaps even more appreciated and adapted to modern improvements than with us old classical nations.

Full of energy, activity, and wealth, you have before you perpetual progress, and what, up to this, your youth has not allowed you to give to the world, you will surely be able to give in the future. Use freely all the treasures of civilization, art, and science that centuries have accumulated in the old world, and especially in my beloved Italy; fructify them with your youthful initiation and with your powerful energy. By so doing you will contribute to peace, and then we may say with truth that we have prepared your route by the work of centuries; and like unto those who from old age are prevented from following the bold young man of Longfellow in his course, we will accompany you with our greetings and our alterable affection.

By my voice, the native country of Columbus, of Galileo, of Michelangelo and Raphael, of Macchiavelli and Volta, salutes and with open arms hails as her hopeful daughter young America, — thanking and blessing her for the road she has opened to the sons of Italy, workmen and artists, to civilization, to science, and to modern research and thought.

The Chairman of the Administrative Board, President Nicholas Murray Butler, of Columbia University, was prevented by illness in his family from being present at the Congress, and in place of the address to have been delivered by him on the idea and development of the Congress and the work of the Administrative Board, President William R. Harper, of the University of Chicago, spoke on the same subject as follows: —

I have been asked within a few hours by those in authority to present to you on behalf of the Administrative Board of this International Congress a statement concerning the origin and purpose of the congress. It is surely a source of great disappointment to all concerned that the chairman of the board, President Butler, is prevented from being present.

Many of us recall the fact that at the Paris Exposition of 1889 the first attempt was made to do something systematic in the way of congresses. This attempt was the natural outcome of the opinion which had come to exist that so splendid an opportunity as was afforded by the coming together of leaders in every department of activity should not be suffered to pass by unimproved. What could be more natural in the stimulating and thought-provoking atmosphere of an exposition than the proposal to make provision for a consideration and discussion of some of the problems so closely related to the interests represented by the exposition?

The results achieved at the Paris Exposition of 1889 were so striking as to lead those in charge of the World's Columbian Exposition in Chicago, 1893, to organize what was called the World's Congress Auxiliary, including a series of congresses, in which, to use the language of the original decree, "the best workers in general science, philosophy, literature, art, agriculture, trade, and labor were to meet to present their experiences and results obtained in all those various lines of thought up to the present time." Seven years later, in connection with the Paris Exposition of 1900, there was held another similar series of international congresses. The general idea had in this way slowly but surely gained recognition.

The authorities of the Universal Exposition at St. Louis, from the first, recognized the desirability of providing for a congress which should exceed in its scope those that had before been attempted. In the earliest days of the preparation for this Exposition Mr. Frederick J. V. Skiff, the Director of the Field Columbian Museum, my nearest neighbor in the city of Chicago, took occasion to present this idea, and particularly to emphasize the specific point that something should be undertaken which not only might add dignity and glory to the great name of the Exposition, but also constitute a permanent and valuable contribution to the sum of human knowledge. After a consideration of the whole question, which extended over many months, the committee on international congresses resolved to establish an administrative board of seven members, to which should be committed the responsibility of suggesting a plan in detail for the attainment of the ends desired. This Board was appointed in November, 1902, and consisted of President Nicholas Murray Butler, of Columbia University, New York; President R. H. Jesse, of the University of Missouri; President Henry S. Pritchett, of the Massachusetts Institute of Technology; Dr. Herbert Putnam, Librarian of Congress; Mr. Frederick J. V. Skiff, of the Field Columbian Museum, Chicago; Frederick G. Holls, of New York City, and the present speaker.

This Board held several meetings for the study of the questions and problems involved in the great undertaking. Much valuable counsel was received and considered. The Board was especially indebted, however, to Prof. Hugo Münsterberg of Harvard University for specific material which he placed at their disposal — material which, with modification, served as the basis of the plans adopted by the Board, and recommended to the members of the Exposition.

At the same time the Administrative Board recommended the appointment of Dr. Howard J. Rogers as the Director of Congresses, and nominated Prof. Simon Newcomb of the United States Navy to be President of the Congress, and Professors Hugo Münsterberg of Harvard University and Albion W. Small of the University of Chicago to be Vice-Presidents of the Congress; the three to constitute the Organizing Committee of the Congress. This Organizing Committee was later empowered to visit foreign countries and to extend personal invitations to men distinguished in the arts and sciences to participate in the Congress. The reception accorded to these, our representatives, was most cordial. Of the 150 invitations thus extended, 117 were accepted; and of the 117 learned savants who accepted the invitation, 96 are here in person this afternoon to testify by their presence the interest they have felt in this great concourse of the world's leaders. I am compelled by necessity this afternoon to omit many points of interest in relation to the origin and history of the undertaking, all of which will be published in due time.

After many months of expectancy we have at last come together from all the nations of the world. But for what purpose? I do not know that to the statement already published in the programme of the Congress anything can be added which will really improve that statement. The purpose, as it has seemed to some of us, is threefold:

In the first place, to secure such a general survey of the various fields of learning, with all their "subdivisions and multiplication of specialties," as will at the same time set forth their mutual relations and connections, and likewise constitute an effort toward the unification of knowledge. This idea of unity has perhaps been uppermost in the minds of all concerned with the work of organizing the Congress.

In the second place, to provide a platform from which might be presented the various problems, a solution of which will be expected of the scholarship of the future. This includes a recognition of the fundamental principles and conception that underlie these mutual relations, and therefore serve necessarily as the basis of all such future work. Here again the controlling idea is that of unity and law, in other words, universal law.

In the third place, to bring together in person and spirit distinguished investigators and scholars from all the countries of the world, in order that by contact of one with another a mutual sympathy may be promoted, and a practical coöperation may be effected among those whose lifework leads them far apart. Here, still again, unity of result is sought for.

As we now take up the work of this convention, which already gives sure promise of being notable among the conventions that have called together men of different nations, let us confidently assure ourselves that the great purpose which has throughout controlled in the different stages of its organization will be realized; that because the Congress has been held, the nations of the earth will find themselves drawn more closely together; that human thought will possess a more unified organization and human life a more unified expression.

Following these addresses of welcome and of response came the first paper of the specific programme, designed to be introductory to the division, department, and section addresses of the week. This address, which will be found in full in its proper place, on pages 135 to 147 of this volume, was given by Dr. Simon Newcomb, President of the Congress and Chairman of the Organizing Committee, whose labors for fifteen months were thus brought to a brilliant conclusion.

At the close of Dr. Newcomb's address the assembly was dismissed by a few words of President Francis, in which he placed at the disposition of the members of the Congress the courtesies and privileges of the Exposition, and expressed the hope and belief that their presence and the purpose for which they were assembled, would be the crowning glory of the Universal Exposition of 1904.

On Tuesday, September 20, the seven division addresses and the twenty-four department addresses were given, all the speakers being Americans: Royce, in Normative Science; Wilson, in Historical Science; Woodward, in Physical Science; Hall, in Mental Science; Jordan, in Utilitarian Science; Lowell, in Social Regulation; and Harris, in Social Culture, treating the main divisions of science and their applications, each dwelling particularly on the scope of the great field included in his address and the unification of the work therein. The forty-eight department speakers divided the field of knowledge, one address in each department giving the fundamental conceptions and methods, the other the history and development of the work of the department during the last century.

With Wednesday the international participation began, and in the one hundred twenty-eight sections into which the departments were divided one half of the speakers were drawn, so far as circumstances permitted, from foreign scientific circles. With the exception of the last two sections, Religious Influence Personal, and Religious Influence Social, the work of the Congress closed on Saturday afternoon. These two sections having four speakers each were placed, one on Sunday morning and one on Sunday afternoon, in Festival Hall, and passes to the grounds given upon application to any one desiring to attend. Large numbers availed themselves of the privilege, and the closing hours of the Congress were eminently suitable and worthy of its high success. At the end of the afternoon session in Festival Hall, Vice-President of the Congress, Dr. Albion W. Small, reviewed in a few words the work of the week, its meaning to science, its possible effect upon American thought, and then formally announced the Congress closed.

OFFICIAL BANQUET

The official banquet given by the Exposition to all participants, members, and officials of the Congress, on Friday evening, at the Tyrolean Alps banquet hall, proved a charming conclusion to the labors of the week. No better place could be imagined for holding it, within the grounds of an exposition, than the magnificently proportioned music and dining hall of the "Alps." A room 160 feet by 105 feet, capable of seating fifteen hundred banqueters; the spacious, oval, orchestral stage at the south end; the galleries and boxes along the sides of the hall done in solid German oak; the beautiful and impressive mural decorations, the work of the best painters of Germany; the excellence of the cuisine, and the thoroughly drilled corps of waiters, rendered the physical accessories of a banquet as nearly perfect as possible in a function so extensive.

The banquet was the largest held during the Exposition period, eight hundred invitations being issued and nearly seven hundred persons present. The music was furnished by the famous Garde Républicaine Band of France, as the Exposition orchestra was obliged to fill its regular weekly assignment at Festival Hall. The decorations of the hall, the lights and flowers, the musical programme, the galleries and boxes filled with ladies representing the official and social life of the Exposition, and the distinguished body of the Congress, formed a picture which appealed to the admiration and enthusiasm of every one alike. No attempt was made to assign seats to the banqueters outside the speakers' table, and little coteries and clusters of scientists, many of whom were making acquaintances and intellectual alliances during this week which would endure for a lifetime, were scattered about the hall, giving an interest and an animation to the scene quite beyond the powers of description. In one corner were Harnack, Budde, Jean Réville, and Cuthbert Hall, chatting as animatedly as though their religious theories were not as far apart as the poles; in another, Waldeyer, Escherich, Jacobi, Allbutt, and Kitasato formed a medical group, the counterpart of which would be hard to find unless in another part of this same hall; still again were Erdmann, Sorley, Ladd, Royce, and Creighton as the centre of a group of philosophers of world renown. So in every part of the picture which met the eye were focused the leaders of thought and action in their respective fields. The *tout ensemble* of the Congress was here brought out in its strongest effect, as, with the exception of the opening exercises at Festival Hall at which time many had not arrived, it was the only time when the entire membership was together. The banquet coming at the close of the week was also fortunate, as by this time the acquaintances made, and the common incidents and anecdotes experienced, heightened the enjoyment of all.

The toastmaster of the banquet and presiding officer, Hon. David R. Francis, was never in a happier vein than when he assumed the gavel and proposed the health of the President of the United States and the rulers of all nations represented at the board.

President Francis said: —

MEMBERS OF THE INTERNATIONAL CONGRESS OF ARTS AND SCIENCE:

On the façade at the base of the Louisiana Monument, which is the central feature of this Exposition picture, is a group of Livingston, Monroe, and Marbois. It represents the signing of the treaty, which by peaceful negotiation transferred an empire from France to the United States. Upon the inscription are the words of Livingston, "We have lived long and accomplished much, but this is the crowning act of our lives."

It is that transfer of an empire which this Exposition is held to commemorate. And paraphrasing the words of Livingston, permit me to say that I have presided over many dinners, but this is the crowning act of my career.

In opening the deliberations of the International Congress of Arts and Science, I made the statement that a Universal Exposition is an ambitious undertaking. I stated also that the International Congress of Arts and Science is the crowning feature of this Exposition. I did not venture the assertion then which I have the presumption to make now, that the most difficult task in connection with this Universal Exposition was the assembling of an International Congress of Arts and Science. I venture to make the statement now, because I feel that I am justified in doing so by the success which up to the present has attended your deliberations. Any congregation of the leaders of thought in the world is a memorable occasion. This is the first systematic one that has ever been attempted. Whether it proves successful or not, it will be long remembered in the history of the civilized countries that have participated in it. If it be but the precursor of other like assemblages it will still be long remembered, and in that event it will be entitled to unspeakable credit if it accomplishes anything toward the realization of the very laudable objects which prompted its assembling.

The effort to unify all human knowledge and to establish the inter-relations thereof is a bold conception, and requires the courage that characterizes the people who live in the western section of the United States. If it be the last effort of the kind it will still be remembered, and this Universal Exposition, if it had done nothing else to endear it to cultured people of this and other countries, will not be forgotten. The savants assembled by the call of this Exposition have pursued their respective lines of thought and research, prompted by no desire other than one to find a solution of the problem which confronts humanity. By bringing you together and making an effort to determine and establish the relations between all lines of human knowledge, we have certainly made an advance in the right direction. If your researches, if the results of your studies, can be utilized by the human race, then we who have been the instruments of that great blessing will be entitled to credit secondary only to the men who are the discoverers of the scientific knowledge whose relations we are endeavoring to establish. The Management of the Universal Exposition of 1904 salutes the International Congress of Arts and Science. We drink to the perpetuation of that organization, and I shall call upon its distinguished President, Professor Newcomb, to respond to the sentiment.

Dr. Newcomb in a few words thanked the members of the Congress for their participation, which had made possible the brilliant success of the enterprise, portrayed its effect and the influence of its perpetuation, and then extended to all the invitation from the President of the United States to attend the reception at the White House on the following Tuesday.

In responding to these toasts the senior Honorary Vice-President, Hon. James Bryce, of Great Britain, spoke in matchless form and held the attention of the vast hall closely while he portrayed in a few words the chief glories of England in the field of science, and the pride the English nation felt in the glorious record made by her eldest daughter, the United States. Mr. Bryce spoke extemporaneously, and his remarks cannot be given in full.

For Germany, Commissioner-General Lewald responded in an eloquent address, in which, after thanking the Exposition and the American Government for the high honor done the German nation in selecting so large a percentage of the speakers from German scientific circles, he enlarged upon the close relations which had existed between German university thought and methods and American thought and practice, due to the vast number of American students who had pursued their post-graduate courses in the universities of Germany. He dwelt upon the pride that Germany felt in this sincerest form of tribute to German supremacy in scientific thought, and of the satisfaction which the influence in this country of German-trained students afforded. He described at length the great exhibit made by German universities in the education department of the Exposition, and pointed to it as demonstrating the supremacy of German scientific thought and accurate methods. Dr. Lewald closed with a brilliant peroration, in which he referred to the immense service done for the cause of science in the last fifty years of German history and to the patronage and support of the Emperor, not only to science in general, but to this great international gathering of scientific experts, and drank to the continued cordial relations of Germany and America through its university circles and scientific endeavors.

For the response from France, Prof. Gaston Darboux was delegated by Commissioner-General Gerald, who was unable to be present on account of sickness. In one of the most beautiful and polished addresses of the evening, Professor Darboux spoke in French, of which the following is a translation: —

GENTLEMEN, — Graciously invited to respond in the name of the delegates of France who have accepted the invitation of the American Government, I consider it my duty in the first place to thank this great nation for the honor which it has paid to us, and for the welcome which it has extended to us. Those of you who are doing me the honor to listen, know of that disagreeable feeling of isolation which at times the traveler in the midst of a strange people experiences; — that feeling I know only from hearsay. We have not had a moment of time to experience it. They are accustomed in Europe to portray the Americans as exclusively occupied with business affairs. They throw in our faces the famous proverb,' Business is Business,' and give it to us as the rule of conduct for Americans. We are able to testify entirely to the contrary, since the inhabitants of this beautiful country are always seeking to extend to strangers a thousand courtesies. Above all, we have encountered no one who has not been anxious to go out of his way to give to us, even before we had asked it, such information as it was necessary for us to have. And what shall I say of the welcome which we have received here at the hands of our American confrères, — Monsieur the President of the Exposition, Monsieur the Director of Congresses and other worthy colaborers? The authorities of the Exposition and the inhabitants of St. Louis have rivaled each other in making our stay agreeable and our ways pleasant in the heart of this magnificent Exposition, of which we shall ever preserve the most enchanting memory.

We should have wished to see in a more leisurely manner, and to make acquaintance with the attractions without number with which the Exposition literally swarms (men of letters and men of science love at times to disport themselves) and to study the exhibits classified in a method so exact in the palaces of an architecture so original and so impressive. But Monsieur Newcomb has not permitted this. The Congress of which he is the illustrious President offers so much in the way of attractions, — of a kind a little rigorous it is true, — and so much of work to be accomplished, that to our very great regret we have had to refuse many invitations which it would have been most agreeable to accept. The Americans will pardon us for this, I am sure; they know better than any one else the value of time, but they know also that human strength has some limits, especially among us poor Europeans, for I doubt whether an American ever knows the meaning of fatigue.

Messieurs, the Congress which is about to terminate to-morrow has been truly a very great event. It is the first time, I believe, that there has been seen assembled in one grand international reunion that which our great minister, Colbert, had in mind, and that which we have realized for the first time in our Institut de France, — the union of letters, science, and arts. That this union shall maintain itself in the future is the dearest wish of my heart.

Science is a unit, even as the Universe. The aspects which it presents know neither boundaries of states nor the political divisions established between peoples. In all civilized countries they calculate with the same figures, they measure with the same instruments, they employ the same classifications, they study the same historic facts, economics, and morals. If there exists among the different nations some differences in methods, these differences are slight. They are a benefit at the same time as well as a necessity. For the doing of the immense amount of work of research imposed on that part of humanity which thinks, it is necessary that the subjects of study should not be identically the same, or better, if they are identical, that the difference between the points of view from which they are considered in the different countries contribute to our better knowledge of their nature, their results, and their applications. It is necessary then that each people preserve their distinctive genius, their particular methods which they use to develop the qualities they have inherited. In exactly the same way that it is important in an orchestra that each instrument play in the most perfect manner, and with the timbre which accords with its nature, the part which is given to it, so in science as in music, the harmony between the players is a necessary condition, which each one ought to exert himself to realize. Let us endeavor then in scientific research to execute in the most perfect manner that part of the task which fate has devolved upon us, but let us endeavor also to maintain that accord which is a necessary condition to the harmony which will alone be able in the future to assure the progress of humanity.

Gentlemen, in this international reunion it would not be fitting that I dwell upon the services which my country has been able to render to science; and on the other hand it would be difficult for me to say to you exactly what part America is called upon to take in this concert of civilized nations; but I am certain that the part will be worthy of the great nation which has given to itself a constitution so liberal and which in so short a space of time has known how to conquer, and measure in value, a territory so immense that it extends from ocean to ocean. I lift my glass to the honor of American science; I drink to the future of that great nation, for which we, as well as all other Frenchmen, hold so much of common remembrance, so much of close and living sympathy, and so much of profound admiration. I am the more happy to do this in this most beautiful territory of Louisiana, which France in a former age ceded freely to America.

Perhaps the treat of the evening was the response made in behalf of the Empire of Japan by Professor Hozumi, of the Faculty of Law of the University of Tokio.

Unfortunately this response was not preserved in full, but Professor Hozumi dwelt with much feeling on the world-wide significance of the Congress and the common plane upon which all nations might meet in the pursuit of science and the manifold applications of scientific principles. He paid a beautiful tribute to the educational system of the United States and to the great debt which Japan owed to American scholars and to American teachers for their aid in establishing modern educational principles and methods in the Empire of Japan. The impetus given to scientific study in Japan by the Japanese students trained in American universities was also earnestly dwelt upon, and the close relations which had always existed between Japanese and American students and instructors feelingly described. In the field of science Japan was yet young, but she had shown herself a close and apt pupil, and her period of initiative and original research was at hand. In bacteriology, in medicine, in seismology, oceanography, and other fields, Japan has made valuable contributions to science and established the right to recognition in an international gathering of this nature. It was with peculiar and grateful pride and pleasure that the Japanese Government had sent its delegation to this Congress of selected experts in response to the invitation of the American Government. Near the close of his address Professor Hozumi made a gracious and happy allusion, based upon the conflict with Russia, in which he said that of all places where men meet, and of all places sunned by the light of heaven, this great Congress, built on the high plane of the brotherhood of science and the fellowship of scholars, was the only place where a Japanese and a Russian could meet in mutual accord, with a common purpose, and clasp hands in unity of thought. This chivalrous and beautiful idea, given here so imperfectly from memory, brought the great assembly to its feet in rounds of cheers. In closing, Professor Hozumi expressed the earnest belief that the benefits of science from a gathering of this nature would quickly be felt, by a closer coöperation in the application of theory and practical principles and a simultaneous advance in all parts of the world.

The closing response of the evening for the foreign members was made for Italy by Signor Attilio Brunialti, whose brilliant eloquence at many times during the week had won the admiration of the members of the Congress. Under the inspiration of this assemblage he fairly surpassed himself, and the following translation of his remarks but poorly indicates the grace and brilliant diction of the original: —

I have had the good fortune to be present in this wonderful country at three international Congresses, that of science, the peace parliament, and the geographic. I wish to record the impression they have excited in my mind, already so favorably inclined by your never-to-be-forgotten and gracious reception. You must, please, allow me to address you in my own language, because the Latin tongue inspires me, because I wish to affirm more solemnly my nationality, and also, because I cannot express my feelings well in a language not familiar to me. My country, the land of Columbus, of Galileo, the nation that more than all others in Europe is an element of peace, is already in itself the synthesis of the three Congresses. And I can call to mind that this land is indebted to geography for the fact of its being made known to the world, because the immortal Genoese pointed it out to people fighting in the old world for a small territory, and opened to mortals new and extensive countries destined to receive the valiant and the audacious of the entire world and to rise like yours to immortal glory.

Thus the poet can sing, —

L' avanza, l' avanza
Divino straniero,
Conosci la stanza
Che i fati ti diero;
Se lutti, se lagrime
Ancora rinterra
L' giovin la terra.

Thus Columbus of old could point out to men — who run down each other, disputing even love for fear that man may become a wolf for man — the vast and endless wastes awaiting laborers, and give to man the treasures of the fruitful land. 'T is in the name of peace that I greet modern science in all its forms, and I say to you chemists: "Invent new means of destruction;" and to you mechanics and shipbuilders: "Give us invulnerable men-of-war and such perfect cannons, that your own progress may contribute to make war rarer in the world." Then will men, amazed at their own destructive progress, be drawn together by brotherly love, by the development of common knowledge and sympathy, and by the study of geography be led to know that there is plenty of room for every one in the world to contribute to progress and civilization.

Americans! these sentiments are graven in your country; in point of fact, it is a proof of the harmony that reigns in this Congress between guests come from all parts of the world, that I, an Italian, am allowed to address you in my own language on American ground, near the Tyrolean Alps, greeted by the music of the Républicaine French Garde, united in eternal bonds of friendship by the two great goddesses of the modern world, — Science and Peace.

The last speaker of the evening was Hon. Frederick W. Lehmann, Chairman of the Exposition Committee on Congresses, who in eloquent periods set forth the ambition of the city of St. Louis and the

Exposition of 1904 in creating a Congress of intellect on the same high plane that had characterized the educational ideals of the Exposition, and the intense satisfaction which the officials of the Congress felt in its brilliant outcome, and the possibilities which it promised for an unequaled contribution to scientific literature.

At the close of these addresses the members of the Congress and the spectators in the gallery sang, in full chorus and under the lead of the Garde Républicaine Band, the various national anthems, closing with "The Star Spangled Banner."

PUBLICATION OF THE REPORT

In accordance with the recommendation of the Administrative Board to the Committee on Congresses, the Executive Committee appointed Dr. Howard J. Rogers, Director of Congresses, editor of the proceedings of the Congress of Arts and Science. The Congress records were removed from St. Louis to Albany, New York, the home of the Director, from which place the publication has been prepared. Upon collecting the papers it was found that they could be divided logically, and with a fair degree of similarity in size, into eight volumes, each of which should cover a definite and distinct portion of the programme. These are as follows: —

Volume 1. History of the Congress, Scientific Plan of the Congress, Philosophy, Mathematics.
Volume 2. Political and Economic History, History of Law, History of Religion.
Volume 3. History of Language, History of Literature, History of Art.
Volume 4. Physics, Chemistry, Astronomy, Sciences of the Earth.
Volume 5. Biology, Anthropology, Psychology, Sociology.
Volume 6. Medicine, Technology.
Volume 7. Economics, Politics, Jurisprudence, Social Science.
Volume 8. Education, Religion.

The details and specifications of the volumes were prepared for competitive bids and submitted to twelve of the prominent publishers of the country. The most advantageous bid was received from Houghton, Mifflin & Company of Boston, Mass., and was accepted by the Exposition Company. The Administrative Board and the authorities of the Exposition feel deeply pleased at the result, inasmuch as the imprint of this firm guarantees a work in full accord with the high plane upon which the Congress has been conducted.

It was determined to print the entire proceedings in the English language, inasmuch as the Congress was held in an English-speaking country and the vast majority of the papers were read in that language. The consent of every foreign speaker was obtained for this procedure. It was found, after collecting, that the number of addresses to be translated was forty-four. The translators were selected by the editor upon the advice of the members of the Administrative Board and Organizing Committee, and great care was taken to find persons not only thoroughly trained in the two languages and possessing a good English style, but also persons who were thoroughly conversant with the subject on which the paper treated. Many of the translators were suggested by the foreign speakers themselves. As a result of this careful selection, the editor feels confident that the original value of the papers has been in no wise detracted from, and that both in form and content the translations are thoroughly satisfactory.

It will be found that some addresses are not closely related to the scheme of the Congress. Either through some misunderstanding of the exact purpose of the Congress, or through too close devotion to their own particular phase of investigation, some half-dozen speakers submitted papers dealing with special lines of work. These, while valuable and scholarly from their standpoint, do not accord with a series of papers prepared with a view to general relations and historical perspective. The exceptions are so few, however, as not seriously to interfere with the unity of the plan.

In the arrangement of the papers the order of the official programme is followed exactly, with the exception that, under Historical Science, Departments 3, 4, and 8, covering History of Politics, Law, and Religion, are combined in one volume; and Departments 5, 6, and 7, covering History of Language, Literature, and Art, are combined in the succeeding volume. In volume one, the first chapter is devoted to the history of the Congress, written by the editor, in which is set forth the plain narrative of the growth and development of the Congress, as much for the benefit of similar undertakings in the future as for the interest of those participating in this Congress. The second chapter contains the scientific introduction, written by Prof. Hugo Münsterberg of Harvard University, First Vice-President of the Congress and Member of the Organizing Committee. This is written for the purpose of giving in detail the principles upon which the classification was based, and the relations which the different sections and departments held to each other.

Each paper is prefaced by a very short biographical note in categorical form, for the purpose of insuring the identity of the speaker as long in the future as the volumes may exist. Appended to the addresses of each department is a short bibliography, which is essential for a general study of the subject in question. These are in no wise exhaustive or complete, but are rather designed to be a small, valuable, working reference library for students. The bibliographies have been prepared by eminent experts in the departments of the Congress, but are necessarily somewhat uneven, as some of the writers have gone into the subject more thoroughly than others. The general arrangement of the bibliographies is: 1. Historical books and standard works dealing with the subject. 2. General books for the whole department. 3. Books for sections of departments.

Appended also to the addresses of each department and sections are résumés of the ten-minute addresses delivered by invitation at the meeting of the department or section. Many of these papers are of high value; but inasmuch as very few of them were written in accord with the plan of the Congress, and with the main thought to be developed by the Congress, but deal rather with some interesting and detached phase of the subject, it has been deemed best not to print them in full, but to indicate in brief the subject and the treatment given it by the writer. Those which do accord with the plan of the Congress are given more extensive treatment.

CONCLUSION

What the results of the Congress will be; what influence it may have; was it worth the work and cost, are questions often fairly asked.

The lasting results and influences are of course problematical. They depend upon the character and soundness of the addresses, and whether the uniform strength of the publication will make the work as a whole, what it undoubtedly is in parts, a source-book for the future on the bases of scientific theory at the beginning of the twentieth century, and a reliable sketch of the growth of science during the nineteenth century. Critical study of the addresses will alone determine this, but from the favorable reception of those already published in reviews, and from editorial acquaintance with the others, it seems assured. That portion of the section addresses which deals with the inter-relations of science and demonstrates both its unity and variety of processes is new and authoritative thought, and will be the basis of much discussion and remodeling of theories in the future.

The immediate results of the Congress are highly satisfactory, and fully repay the work and the cost both from a scientific and an exposition standpoint. As an acknowledgment of the prominence of scientific methods, as a public recognition of the work of scientists, as the means of bringing to one place the most noted assemblage of thinkers the world has ever seen, as an opportunity for scholars to meet and know each other better, the Congress was an unqualified success and of enduring reputation. From the Exposition point of view, it was equally a success; not financially, nor was there ever a thought that it would be. Probably not more than seven thousand persons outside of St. Louis came primarily to attend the Congress, and their admission fees were a bagatelle; the revenue derived from the sale of the *Proceedings* will not meet the cost of printing. There has been no money value sought for in the Congress, — none received. Its value to the Exposition lies solely in the fact that it is the final argument to the world of the initial claims of the officials of the Exposition that its purpose was purely educational. Coördinate with the material exhibits, sought, classified, and installed on a rigidly scientific classification, the Congress, which relates, illumines, and defends the principles upon which the material portion was founded, has triumphantly vindicated the good faith, the wisdom, and the foresight of the Universal Exposition of 1904. This printed record of its proceedings will be a monument not only to the spirit of Science, but to the spirit of the Exposition, which will endure as long as the records of man are preserved.

In conclusion, the editor wishes to express his obligations to the many speakers and officers of the Congress, who have evinced great interest in the publication and assisted by valuable suggestions and advice. In particular, he acknowledges the help of President Butler of Columbia University, Professor Münsterberg of Harvard University, and Professor Small of the University of Chicago. Acknowledgments are with justice and pleasure made to the Committee on Congresses of the Exposition, and the able chairman, Hon. Frederick W. Lehmann, for their unwavering and prompt support on all matters of policy and detail, without which the full measure of success could not have been achieved. To the efficient secretary of the Department of Congresses, Mr. James Green Cotchett, an expression of obligation is due for his indefatigable labors during the Congress period, and for his able and painstaking work in compiling the detailed records of this publication.

At a meeting of the Executive Committee of the Exposition on January 3, 1905, there was unanimously voted the following resolution, recommended by the Administrative Board and approved by the Committee on Congresses: —

Moved: that a vote of thanks and an expression of deepest obligation be tendered to Dr. Simon Newcomb, President of the Congress, Prof. Hugo Münsterberg, vice-president of the Congress, and Prof. Albion W. Small, vice-president of the Congress, for their efficient, thorough, and comprehensive work in connection with the programme of the Congress, the selection and invitation of speakers, and the attention to detail in its execution. That, in view of the enormous amount of labor devolving upon these three gentlemen for the past eighteen months, to the exclusion of all opportunities for literary and other work outside their college departments, an honorarium of twenty-five hundred dollars be tendered to each of them.

At a subsequent meeting the following resolution was also passed:—

MOVED: that the Directors of the Louisiana Purchase Exposition Company place upon the record an expression of their appreciation of the invaluable aid so freely given by the Administrative Board of the Congress of Arts and Science. In organization, guidance, and results the Congress was the most notable of its kind in history. For the important part performed wisely and zealously by the Administrative Board the Exposition Management extends this acknowledgment.

SUMMARY OF EXPENSES OF THE CONGRESS

Office expenses	$7,025 82	
Travel	3,847 24	
Exploitation, Organising Committee abroad	8,663 16	
Traveling expenses, American Speakers	31,350	
Traveling expenses, Foreign Speakers	49,000	
Honorariums	7,500	
Banquet	3,500	
Expenses for editing proceedings	5,875	
Estimated cost of printing proceedings	22,000	$138,761 22

INTERNATIONAL

CONGRESS OF ARTS AND SCIENCE

UNIVERSAL EXPOSITION ST. LOUIS

SEPTEMBER 19–25 1904

PROGRAMME AND LIST OF SPEAKERS

PROGRAMME

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Organization of the Congress
Speakers and Chairmen
Chronological Order of Proceedings
Programme of Social Events
List of Ten-minute Speakers
List of Chairmen and Principal Speakers

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PURPOSE AND PLAN OF THE CONGRESS

The idea of the Congress grows out of the thought that the subdivision and multiplication of specialties in science has reached a stage at which investigators and scholars may derive both inspiration and profit from a general survey of the various fields of learning, planned with a view of bringing the scattered sciences into closer mutual relations. The central purpose is the unification of knowledge, an effort toward which seems appropriate on an occasion when the nations bring together an exhibit of their arts and industries. An assemblage is therefore to be convened at which leading representatives of theoretical and applied sciences shall set forth those general principles and fundamental conceptions which connect groups of sciences, review the historical development of special sciences, show their mutual relations and discuss their present problems.

The speakers to treat the various themes are selected in advance from the European and American continents. The discussions will be arranged on the following general plan: —

After the opening of the Congress on Monday afternoon, September 19, will follow, on Tuesday forenoon, addresses on main divisions of science and its applications, the general theme being the unification of each of the fields treated. These will be followed by two addresses on each of the twenty-four great departments of knowledge. The theme of one address in each case will be the Fundamental Conceptions and Methods, while the other will set forth the progress during the last century. The preceding addresses will be delivered by Americans, making the work of the first two days the contribution of American scholars.

On the third day, with the opening of the sections, the international work will begin. One hundred twenty-eight sectional meetings will be held on the four remaining days of the Congress, at each of which two papers will be read, the theme of one being suggested by the relations of the special branch treated to other branches; the other by its present problems. Three hours will be devoted to each sectional meeting, thus enabling each hearer to attend eight such meetings, if he so desires. The programme is so arranged that related subjects will be treated, as far as possible, at different times. The length of the principal addresses being limited to forty-five minutes each, there will remain at least one hour for five or six brief communications in each section. The addresses in each department will be collected and published in a special volume.

It is hoped that the living influence of this meeting will be yet more important than the formal addresses, and that the scholars whose names are announced in the following programme of speakers and chairmen will form only a nucleus for the gathering of thousands who feel in sympathy with the efforts to bring unity into the world of knowledge.

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HON. DAVID R. FRANCIS, A.M., LL.D.

DIRECTOR OF CONGRESSES,
HOWARD J. ROGERS, A.M., LL.D.
Universal Exposition, 1904.

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SPEAKERS AND CHAIRMEN

DIVISION A—NORMATIVE SCIENCE

Speaker: Professor Josiah Royce, Harvard University.
(Hall 6, September 20, 10 a. m.)

DEPARTMENT 1—PHILOSOPHY

(Hall 6, September 20, 11.15 a. m.)

Chairman: Professor Borden P. Bowne, Boston University.
Speakers: Professor George H. Howison, University of California.
Professor George T. Ladd, Yale University.

SECTION A. METAPHYSICS. *(Hall 6, September 21, 10 a. m.)*

Chairman: Professor A. C. Armstrong, Wesleyan University.
Speakers: Professor A. E. Taylor, McGill University, Montreal.
Professor Alexander T. Ormond, Princeton University.
Secretary: Professor A. O. Lovejoy, Washington University.

SECTION B. PHILOSOPHY OF RELIGION. *(Hall 1, September 21, 3 p. m.)*

Chairman: Professor Thomas C. Hall, Union Theological Seminary, N. Y.
Speakers: Professor Otto Pfleiderer, University of Berlin.
Professor Ernst Troeltsch, University of Heidelberg.
Secretary: Dr. W. P. Montague, Columbia University.

SECTION C. LOGIC. *(Hall 6, September 22, 10 a. m.)*

Chairman: Professor George M. Duncan, Yale University.
Speakers: Professor William A. Hammond, Cornell University.
Professor Frederick J. E. Woodbridge, Columbia University.
Secretary: Dr. W. H. Sheldon, Columbia University.

SECTION D. METHODOLOGY OF SCIENCE. *(Hall 6, September 22, 3 p. m.)*

Chairman: Professor James E. Creighton, Cornell University.
Speakers: Professor Wilhelm Ostwald, University of Leipzig.
Professor Benno Erdmann, University of Bonn.
Secretary: Dr. R. B. Perry, Harvard University.

SECTION E. ETHICS. *(Hall 6, September 23, 10 a. m.)*

Chairman: Professor George H. Palmer, Harvard University.
Speakers: Professor William R. Sorley, University of Cambridge.
Professor Paul Hensel, University of Erlangen.
Secretary: Professor F. C. Sharp, University of Wisconsin.

SECTION F. AESTHETICS. (*Hall 4, September 23, 3 p. m.*)

Chairman: Professor James H. Tufts, University of Chicago.
Speakers: Dr. Henry Rutgers Marshall, New York City.
Professor Max Dessoir, University of Berlin.
Secretary: Professor Max Meyer, University of Missouri.

DEPARTMENT 2 — MATHEMATICS

(*Hall 7, September 20, 11.15 a. m.*)

Chairman: Professor Henry S. White, Northwestern University.
Speakers: Professor Maxime Bôcher, Harvard University.
Professor James P. Pierpont, Yale University.

SECTION A. ALGEBRA AND ANALYSIS. (*Hall 9, September 22, 10 a. m.*)

Chairman: Professor E. H. Moore, University of Chicago.
Speakers: Professor Emile Picard, The Sorbonne; Member of the Institute of France.
Professor Heinrich Maschke, University of Chicago.
Secretary: Professor G. A. Bliss, University of Chicago.

SECTION B. GEOMETRY. (*Hall 9, September 24, 10 a. m.*)

Chairman: Professor M. W. Haskell, University of California.
Speakers: M. Gaston Darboux, Perpetual Secretary of the Academy of Sciences, Paris.
Dr. Edward Kasner, Columbia University.
Secretary: Professor Thomas J. Holgate, Northwestern University.

SECTION C. APPLIED MATHEMATICS. (*Hall 7, September 24, 3 p. m.*)

Chairman: Professor Arthur G. Webster, Clark University, Worcester, Mass.
Speakers: Professor Ludwig Boltzmann, University of Vienna.
Professor Henri Poincaré, The Sorbonne; Member of the Institute of France.
Secretary: Professor Henry T. Eddy, University of Minnesota.

DIVISION B — HISTORICAL SCIENCE

(*Hall 3, September 20, 10 a. m.*)

Speaker: President Woodrow Wilson, Princeton University.

DEPARTMENT 3 — POLITICAL AND ECONOMIC HISTORY

(*Hall 4, September 20, 11.15 a. m.*)

Chairman:
Speakers: Professor William M. Sloane, Columbia University.
Professor James H. Robinson, Columbia University.

SECTIONS A AND B. HISTORY OF GREECE, ROME, AND ASIA. (*Hall* 3, *September* 21, 10 *a. m.*)

Chairman: Professor Thomas D. Seymour, Yale University.
Speakers: Professor John P. Mahaffy, University of Dublin.
Professor Ettore Pais, University of Naples. Director of the National Museum of Antiquities, Naples.
Professor Henri Cordier, Ecole des Langues Vivantes Orientales, Paris.
Secretary: Professor Edward Capps, University of Chicago.

SECTION C. MEDIAEVAL HISTORY. (*Hall* 6, *September* 21, 3 *p. m.*)

Chairman: Professor Charles H. Haskins, Harvard University.
Speakers: Professor Karl Lamprecht, University of Leipzig.
Professor George B. Adams, Yale University.
Secretary: Professor Earle W. Dow, University of Michigan.

SECTION D. MODERN HISTORY OF EUROPE. (*Hall* 3, *September* 22, 10 *a. m.*)

Chairman: Honorable James B. Perkins, Rochester, N. Y.
Speakers: Professor J. B. Bury, University of Cambridge.
Professor Charles W. Colby, McGill University; Montreal.
Secretary: Professor Ferdinand Schwill, University of Chicago.

SECTION E. HISTORY OF AMERICA. (*Hall* 1, *September* 24, 10 *a. m.*)

Chairman: Dr. James Schouler, Boston.
Speakers: Professor Frederic J. Turner, University of Wisconsin.
Professor Edward G. Bourne, Yale University.
Secretary: Professor Evarts B. Greene, University of Illinois.

SECTION F. HISTORY OF ECONOMIC INSTITUTIONS. (*Hall* 2, *September* 23, 3 *p. m.*)

Chairman: Professor Frank A. Fetter, Cornell University.
Speakers: Professor J. E. Conrad, University of Halle.
Professor Simon N. Patten, University of Pennsylvania.
Secretary: Dr. J. Pease Norton, Yale University.

DEPARTMENT 4 — HISTORY OF LAW

(*Hall* 5, *September* 20, 11.15 *a. m.*)

Chairman: Honorable David J. Brewer, Associate Justice of the Supreme Court of the United States.
Speakers: Honorable Emlin McClain, Judge of the Supreme Court of Iowa, Iowa City.
Professor Nathan Abbott, Leland Stanford Jr. University.

SECTION A. HISTORY OF ROMAN LAW. (*Hall* 11, *September* 21, 3 *p. m.*)

Chairman:
Speakers: Mr. W. H. Buckler, Baltimore, Md.
Professor Munroe Smith, Columbia University.

SECTION B. HISTORY OF COMMON LAW. (*Hall* 11, *September* 21, 10 *a. m.*)

Chairman: Professor John D. Lawson, University of Missouri.
Speakers: Honorable Simeon E. Baldwin, Judge of the Supreme Court of Errors, New Haven, Conn.
Professor John H. Wigmore, Northwestern University.
Secretary: Professor C. H. Huberich, University of Texas.

SECTION C. COMPARATIVE LAW. (*Hall* 14, *September* 24, 3 *p. m.*)

Chairman: Honorable Jacob M. Dickinson, Chicago.
Speakers: Professor Nobushige Hozumi, University of Tokio.
Professor Alfred Nerincx, University of Louvain.
Secretary:

DEPARTMENT 5 — HISTORY OF LANGUAGE

(*Hall* 4, *September* 20, 2 *p. m.*)

Chairman: Professor George Hempl, University of Michigan.
Speakers: Professor T. R. Lounsbury, Yale University.
President Benjamin Ide Wheeler, University of California.

SECTION A. COMPARATIVE LANGUAGE. (*Hall* 4, *September* 21, 10 *a. m.*)

Chairman: Professor Francis A. March, Lafayette College.
Speakers: Professor Carl D. Buck, University of Chicago.
Professor Hans Oertel, Yale University.
Secretary: Professor E. W. Fay, University of Texas, Austin, Texas.

SECTION B. SEMITIC LANGUAGES. (*Hall* 4, *September* 21, 3 *p. m.*)

Chairman: Professor G. F. Moore, Harvard University.
Speakers: Professor James A. Craig, University of Michigan.
Professor Crawford H. Toy, Harvard University.
Secretary:

SECTION C. INDO-IRANIAN LANGUAGES. (*Hall* 8, *September* 22, 10 *a. m.*)

Chairman:
Speakers: Professor Sylvain Lévi, Collège de France, Paris.
Professor Arthur A. Macdonell, University of Oxford.
Secretary:

SECTION D. GREEK LANGUAGE. (*Hall* 3, *September* 22, 3 *p. m.*)

Chairman: Professor Martin L. D'Ooge, University of Michigan.
Speakers: Professor Herbert W. Smyth, Harvard University.
Professor Milton W. Humphreys, University of Virginia.
Secretary: Professor J. E. Harry, University of Cincinnati.

SECTION E. LATIN LANGUAGE. (*Hall* 9, *September* 23, 10 *a. m.*)

Chairman: Professor Maurice Hutton, University of Toronto.
Speakers: Professor E. A. Sonnenschein, University of Birmingham.
Professor William G. Hale, University of Chicago.
Secretary: Professor F. W. Shipley, Washington University.

SECTION F. ENGLISH LANGUAGE. (*Hall 3, September 23, 3 p. m.*)

Chairman: Professor Charles M. Gayley, University of California.
Speakers: Professor Otto Jespersen, University of Copenhagen.
Professor George L. Kittredge, Harvard University.
Secretary:

SECTION G. ROMANCE LANGUAGES. (*Hall 5, September 24, 10 a. m.*)

Chairman:
Speakers: Professor Paul Meyer, Collège de France, Paris.
Professor Henry A. Todd, Columbia University.
Secretary: Professor E. E. Brandon, Miami University.

SECTION H. GERMANIC LANGUAGES. (*Hall 3, September 24, 3 p. m.*)

Chairman: Professor Gustaf E. Karsten, Cornell University.
Speakers: Professor Eduard Sievers, University of Leipzig.
Professor Herman Collitz, Bryn Mawr College.
Secretary:

DEPARTMENT 6 — HISTORY OF LITERATURE

(*Hall 6, September 20, 4.15 p. m.*)

Chairman:
Speakers: Professor James A. Harrison, University of Virginia.
Professor Charles M. Gayley, University of California.

SECTION A. INDO-IRANIAN LITERATURE. (*Hall 8, September 24, 3 p. m.*)

Chairman: Professor Maurice Bloomfield, Johns Hopkins University.
Speaker: Professor A. V. W. Jackson, Columbia University.
Secretary:

SECTION B. CLASSICAL LITERATURE. (*Hall 3, September 21, 3 p. m.*)

Chairman: Professor Andrew F. West, Princeton University.
Speakers: Professor Paul Shorey, University of Chicago.
Professor John H. Wright, Harvard University.
Secretary: Professor F. G. Moore, Dartmouth College.

SECTION C. ENGLISH LITERATURE. (*Hall 1, September 22, 10 a. m.*)

Chairman:
Speakers: Professor Francis B. Gummere, Haverford College.
Professor John Hoops, University of Heidelberg.
Secretary:

SECTION D. ROMANCE LITERATURE. (*Hall 8, September 22, 3 p. m.*)

Chairman: Professor Adolphe Cohn, Columbia University.
Speakers: Professor Pio Rajna, Institute of Higher Studies, Florence, Italy.
Professor Alcée Fortier, Tulane University, New Orleans.
Secretary: Dr. Comfort, Haverford College.

SECTION E. GERMANIC LITERATURE. (*Hall 3, September 23, 10 a. m.*)

Chairman: Professor Kuno Francke, Harvard University.
Speakers: Professor August Sauer, University of Prague.
Professor J. Minor, University of Vienna.
Secretary: Professor D. K. Jessen, Bryn Mawr College.

SECTION F. SLAVIC LITERATURE. (*Hall 8, September 21, 10 a. m.*)

Chairman: Mr. Charles R. Crane, Chicago.
Speakers: Professor Leo Wiener, Harvard University.
Professor Paul Boyer, Ecole des Langues Vivantes Orientales, Paris.
Secretary: Mr. S. N. Harper, University of Chicago.

SECTION G. BELLES-LETTRES. (*Hall 3, September 24, 10 a. m.*)

Chairman: Professor Robert Herrick, University of Chicago.
Speakers: Professor Henry Schofield, Harvard University.
Professor Brander Matthews, Columbia University.
Secretary:

DEPARTMENT 7 — HISTORY OF ART

(*Hall 8, September 20, 11.15 a. m.*)

Chairman: Professor Halsey C. Ives, Washington University, St. Louis.
Speakers: Professor Rufus B. Richardson, New York, N. Y.
Professor John C. Van Dyke, Rutgers College.

SECTION A. CLASSICAL ART. (*Hall 12, September 22, 10 a. m.*)

Chairman: Professor Rufus B. Richardson, New York City.
Speakers: Professor Adolph Furtwängler, University of Munich.
Professor Frank B. Tarbell, University of Chicago.
Secretary: Dr. P. Baur, Yale University.

SECTION B. MODERN ARCHITECTURE. (*Hall 7, September 22, 3 p. m.*)

Chairman: Mr. Charles F. McKim, New York City.
Speakers: Professor C. Enlart, University of Paris.
Professor Alfred D. F. Hamlin, Columbia University.
Secretary: Mr. Guy Lowell, Boston, Mass.

SECTION C. MODERN PAINTING. (*Hall 4, September 24, 3 p. m.*)

Chairman:
Speakers: Professor Richard Muther, University of Breslau.
Mr. Okakura Kakuzo, Japan.
Secretary:

DEPARTMENT 8 — HISTORY OF RELIGION

(*Hall 5, September 20, 2 p. m.*)

Chairman: Rev. Wm. Eliot Griffis, Ithaca, N. Y.
Speakers: Professor George F. Moore, Harvard University.
Professor Nathaniel Schmidt, Cornell University.

SECTION A. BRAHMANISM AND BUDDHISM. (*Hall* 8, *September* 23, 10 *a. m.*)

Chairman:
Speakers: Professor Hermann Oldenberg, University of Kiel.
Professor Maurice Bloomfield, Johns Hopkins University.
Secretary: Dr. Reginald C. Robbins, Harvard University.

SECTION B. MOHAMMEDISM. (*Hall* 8, *September* 23, 3 *p. m.*)

Chairman: Professor James R. Jewett, University of Chicago.
Speakers: Professor Ignaz Goldziher, University of Budapest.
Professor Duncan B. Macdonald, Hartford Theological Seminary.
Secretary:

SECTION C. OLD TESTAMENT. (*Hall* 4, *September* 22, 10 *a. m.*)

Chairman: Professor A. S. Carrier, McCormick Theological Seminary.
Speakers: Professor James F. McCurdy, University College of Toronto.
Professor Karl Budde, University of Marburg.
Secretary: Professor James A. Kelso, Western Theological Seminary, Allegheny, Pa.

SECTION D. NEW TESTAMENT. (*Hall* 1, *September* 23, 10 *a. m.*)

Chairman: Professor Andrew C. Zenos, McCormick Theological Seminary.
Speakers: Professor Benjamin W. Bacon, Yale University.
Professor Ernest D. Burton, University of Chicago.
Secretary: Professor Clyde W. Votaw, University of Chicago.

SECTION E. HISTORY OF THE CHRISTIAN CHURCH. (*Hall* 2, *September* 24, 10 *a. m.*)

Chairman: Dr. Eri Baker Hulbert, University of Chicago.
Speakers: Professor Adolf Harnack, University of Berlin.
Professor Jean Réville, Faculty of Protestant Theology, Paris.
Secretary:

DIVISION C—PHYSICAL SCIENCE

(*Hall* 4, *September* 20, 10 *a. m.*)

Speaker: Professor Robert S. Woodward, Columbia University.

DEPARTMENT 9 — PHYSICS

(*Hall* 6, *September* 20, 2 *p. m.*)

Chairman: Professor Henry Crew, Northwestern University.
Speakers: Professor Edward L. Nichols, Cornell University.
Professor Carl Barus, Brown University.

SECTION A. PHYSICS OF MATTER. (*Hall* 11, *September* 23, 10 *a. m.*)

CHAIRMAN: PROFESSOR SAMUEL W. STRATTON, Director of the National Bureau of Standards, Washington.

SPEAKERS: PROFESSOR ARTHUR L. KIMBALL, Amherst College.
PROFESSOR FRANCIS E. NIPHER, Washington University.

SECRETARY: PROFESSOR R. A. MILLIKEN, University of Chicago.

SECTION B. PHYSICS OF ETHER. (*Hall* 11, *September* 23, 3 *p. m.*)

CHAIRMAN: PROFESSOR HENRY CREW, Northwestern University.

SPEAKER: PROFESSOR DEWITT B. BRACE, University of Nebraska.

SECRETARY: PROFESSOR AUGUSTUS TROWBRIDGE, University of Wisconsin.

SECTION C. PHYSICS OF THE ELECTRON. (*Hall* 5, *September* 22, 3 *p. m.*)

CHAIRMAN: PROFESSOR A. G. WEBSTER, Clark University.

SPEAKERS: PROFESSOR P. LANGEVIN, Collège de France.
PROFESSOR ERNEST RUTHERFURD, McGill University, Montreal.

SECRETARY: PROFESSOR W. J. HUMPHREYS, University of Virginia.

DEPARTMENT 10 — CHEMISTRY

(*Hall* 5, *September* 20, 4.15 *p. m.*)

CHAIRMAN: PROFESSOR JAMES M. CRAFTS, Massachusetts Institute of Technology.

SPEAKERS: PROFESSOR JOHN U. NEF, University of Chicago.
PROFESSOR FRANK W. CLARKE, Chief Chemist, U. S. Geological Survey.

SECTION A. INORGANIC CHEMISTRY. (*Hall* 16, *September* 21, 10 *a. m.*)

CHAIRMAN: PROFESSOR JOHN W. MALLET, University of Virginia.

SPEAKERS: PROFESSOR HENRI MOISSAN, The Sorbonne; Member of the Institute of France.
SIR WILLIAM RAMSAY, K.C.B., Royal Institution, London.

SECRETARY: PROFESSOR WILLIAM L. DUDLEY, Vanderbilt University.

SECTION B. ORGANIC CHEMISTRY. (*Hall* 16, *September* 21, 3 *p. m.*)

CHAIRMAN: PROFESSOR ALBERT B. PRESCOTT, University of Michigan.

SPEAKERS: PROFESSOR JULIUS STIEGLITZ, University of Chicago.
PROFESSOR WILLIAM A. NOYES, National Bureau of Standards.

SECRETARY:

SECTION C. PHYSICAL CHEMISTRY. (*Hall* 16, *September* 22, 10 *a. m.*)

CHAIRMAN: PROFESSOR WILDER D. BANCROFT, Cornell University.

SPEAKERS: PROFESSOR J. H. VAN T'HOFF, University of Berlin.
PROFESSOR ARTHUR A. NOYES, Massachusetts Institute of Technology.

SECRETARY: MR. W. R. WHITNEY, Schenectady, N. Y.

SECTION D. PHYSIOLOGICAL CHEMISTRY. (*Hall* 16, *September* 22, 3 *p. m.*)

CHAIRMAN: PROFESSOR WILBUR O. ATWATER, Wesleyan University.

SPEAKERS: PROFESSOR O. COHNHEIM, University of Heidelberg.
PROFESSOR RUSSELL H. CHITTENDEN, Yale University.

SECRETARY: DR. C. L. ALSBERG, Harvard University.

DEPARTMENT 11 — ASTRONOMY

(*Hall* 8, *September* 20, 4.15 *p. m.*)

CHAIRMAN: PROFESSOR GEORGE C. COMSTOCK, Director of the Observatory, Madison, Wisconsin.

SPEAKERS: PROFESSOR LEWIS BOSS, Director of Dudley Observatory.
PROFESSOR EDWARD C. PICKERING, Director of Harvard Observatory.

SECTION A. ASTROMETRY. (*Hall* 9, *September* 21, 10 *a. m.*)

CHAIRMAN: PROFESSOR ORMOND STONE, University of Virginia.

SPEAKERS: DR. OSKAR BACKLUND, Director of the Observatory, Pulkowa, Russia.
PROFESSOR JOHN C. KAPTEYN, University of Groningen, Holland.

SECRETARY: PROFESSOR W. S. EICHELBERGER, U. S. Naval Observatory.

SECTION B. ASTROPHYSICS. (*Hall* 9, *September* 21, 3 *p. m.*)

CHAIRMAN: PROFESSOR GEORGE E. HALE, Director of the Yerkes Observatory.

SPEAKERS: PROFESSOR HERBERT H. TURNER, F.R.S., University of Oxford.
PROFESSOR WILLIAM W. CAMPBELL, Director of the Lick Observatory, Mt. Hamilton, California.

SECRETARY: MR. W. S. ADAMS, Yerkes Observatory.

DEPARTMENT 12 — SCIENCES OF THE EARTH

(*Hall* 3, *September* 20, 11.15 *a. m.*)

CHAIRMAN: DR. G. K. GILBERT, U. S. Geological Survey.

SPEAKERS: PROFESSOR THOMAS C. CHAMBERLIN, University of Chicago.
PROFESSOR WILLIAM M. DAVIS, Harvard University.

SECTION A. GEOPHYSICS. (*Hall* 14, *September* 21, 10 *a. m.*)

CHAIRMAN: PROFESSOR CHRISTOPHER W. HALL, University of Minnesota.

SPEAKER: DR. GEORGE F. BECKER, Geologist, U. S. Geological Survey.

SECRETARY: PROFESSOR E. M. LEHNERTS, Minnesota State Normal School.

SECTION B. GEOLOGY. (*Hall* 14, *September* 21, 3 *p. m.*)

Chairman: Professor T. C. Chamberlin, University of Chicago.
Speakers: President Charles R. Van Hise, University of Wisconsin.

Secretary: Professor R. D. Salisbury, University of Chicago.

SECTION C. PALAEONTOLOGY. (*Hall* 11, *September* 22, 10 *a. m.*)

Chairman: Professor William B. Scott, Princeton University.
Speakers: Dr. A. S. Woodward, F.R.S., British Museum of Natural History, London.
Professor Henry F. Osborn, Columbia University.
Secretary: Dr. John M. Clarke, Albany, N. Y.

SECTION D. PETROLOGY AND MINERALOGY. (*Hall* 9, *September* 22, 3 *p. m.*)

Chairman: Dr. Oliver C. Farrington, Field Columbian Museum, Chicago.
Speaker: Professor F. Zirkel, University of Leipzig.
Secretary:

SECTION E. PHYSIOGRAPHY. (*Hall* 12, *September* 21, 10 *a. m.*)

Chairman: Mr. Henry Gannett, United States Geological Survey.
Speakers: Professor Albrecht Penck, University of Vienna.
Professor Israel C. Russell, University of Michigan.
Secretary: Dr. John M. Clarke, Albany, N. Y.

SECTION F. GEOGRAPHY. (*Hall* 11, *September* 22, 3 *p. m.*)

Chairman: Professor Israel C. Russell, University of Michigan.
Speakers: Dr. Hugh R. Mill, Director British Rainfall Organization, London.
Professor H. Yule Oldham, Cambridge, England.
Secretary: Professor R. D. Salisbury, University of Chicago.

SECTION G. OCEANOGRAPHY. (*Hall* 8, *September* 21, 3 *p. m.*)

Chairman: Rear-Admiral John R. Bartlett, United States Navy.
Speakers: Sir John Murray, K.C.B., F.R.S., Edinburgh.
Professor K. Mitsukuri, University of Tokio.
Secretary:

SECTION H. COSMICAL PHYSICS. (*Hall* 10, *September* 22, 10 *a. m.*)

Chairman: Professor Francis E. Nipher, Washington University.
Speakers: Professor Svante Arrhenius, University of Stockholm, Stockholm.
Dr. Abbott L. Rotch, Blue Hill Observatory.
Dr. L. A. Bauer, Washington, D. C.
Secretary:

DEPARTMENT 13 — BIOLOGY

(*Hall* 2, *September* 20, 11.15 *a. m.*)

Chairman: Professor William G. Farlow, Harvard University.
Speakers: Professor John M. Coulter, University of Chicago.
Professor Jacques Loeb, University of California.

SECTION A. PHYLOGENY. (*Hall* 2, *September* 21, 3 *p. m.*)

Chairman: Professor T. H. Morgan, Columbia University.
Speakers: Professor Hugo de Vries, University of Amsterdam.
Professor Charles O. Whitman, University of Chicago.
Secretary:

SECTION B. PLANT MORPHOLOGY. (*Hall* 2, *September* 22, 10 *a. m.*)

Chairman: Professor William Trelease, Washington University, St. Louis.
Speakers: Professor Frederick O. Bower, University of Glasgow.
Professor Karl F. Goebel, University of Munich.
Secretary: Professor F. E. Lloyd, Columbia University.

SECTION C. PLANT PHYSIOLOGY. (*Hall* 4, *September* 22, 3 *p. m.*)

Chairman: Professor Charles R. Barnes, University of Chicago.
Speakers: Professor Julius Wiesner, University of Vienna.
Professor Benjamin M. Duggar, University of Missouri.
Secretary: Professor F. C. Newcomb, University of Michigan.

SECTION D. PLANT PATHOLOGY. (*Hall* 7, *September* 23, 10 *a. m.*)

Chairman: Professor Chas. E. Bessey, University of Nebraska.
Speakers: Professor Joseph C. Arthur, Purdue University.
Merton B. Waite, U. S. Department of Agriculture.
Secretary: Dr. C. S. Shear, U. S. Department of Agriculture.

SECTION E. ECOLOGY. (*Hall* 7, *September* 23, 3 *p. m.*)

Chairman:
Speakers: Professor Oskar Drude, Kön. Technische Hochschule, Dresden.
Professor Benjamin Robinson, Harvard University.
Secretary: Professor F. E. Clements, University of Nebraska.

SECTION F. BACTERIOLOGY. (*Hall* 15, *September* 24, 10 *a. m.*)

Chairman: Professor Harold C. Ernst, Harvard University.
Speakers: Professor Edwin O. Jordan, University of Chicago.
Professor Theobald Smith, Harvard University.
Secretary: Dr. P. H. Hiss, Jr., Columbia University.

SECTION G. ANIMAL MORPHOLOGY. (*Hall* 2, *September* 21, 10 *a. m.*)

Chairman: Dr. Leland O. Howard, Department of Agriculture, Washington, D. C.
Speakers: Professor Charles B. Davenport, University of Chicago.
Professor Alfred Giard, The Sorbonne; Member of the Institute of France.
Secretary: Professor C. H. Herrick, Dennison University.

SECTION H. EMBRYOLOGY. (*Hall 9, September 23, 3 p. m.*)

Chairman: Professor Simon H. Gage, Cornell University.
Speakers: Professor Oskar Hertwig, University of Berlin.
Professor William K. Brooks, Johns Hopkins University.
Secretary: Professor T. G. Lee, University of Minnesota.

SECTION I. COMPARATIVE ANATOMY. (*Hall 2, September 24, 3 p. m.*)

Chairman: Professor James P. McMurrich, University of Michigan.
Speakers: Professor William E. Ritter, University of California.
Professor Yves Delage, The Sorbonne; Member of the Institute of France.
Secretary: Professor Henry B. Ward, University of Nebraska.

SECTION J. HUMAN ANATOMY. (*Hall 2, September 22, 3 p. m.*)

Chairman: Professor George A. Piersol, University of Pennsylvania.
Speakers: Professor Wilhelm Waldeyer, University of Berlin.
Professor H. H. Donaldson, University of Chicago.
Secretary: Dr. R. J. Terry, Washington University.

SECTION K. PHYSIOLOGY. (*Hall 4, September 23, 10 a. m.*)

Chairman: Dr. S. J. Meltzer, New York.
Speakers: Professor Max Verworn, University of Göttingen.
Professor William H. Howell, Johns Hopkins University.
Secretary: Dr. Reid Hunt, Washington.

DEPARTMENT 14 — ANTHROPOLOGY

(*Hall 8, September 20, 2 p. m.*)

Chairman: Professor Frederic W. Putnam, Harvard University.
Speakers: Dr. W J McGee, President American Anthropological Association, Washington, D. C.
Professor Franz Boas, Columbia University.

SECTION A. SOMATOLOGY. (*Hall 16, September 23, 3 p. m.*)

Chairman: Dr. Edward C. Spitzka, New York City.
Speakers: Professor L. Manouvrier, School of Anthropology, Paris.
Dr. George A. Dorsey, Field Columbian Museum, Chicago.
Secretary: Dr. E. A. Spitzka, New York City.

SECTION B. ARCHAEOLOGY. (*Hall 16, September 24, 10 a. m.*)

Chairman: Mr. M. H. Saville, American Museum of Natural History, New York.
Speakers: Señor Alfredo Chavero, Inspector of the National Museum, Mexico.
Professor Edouard Seler, University of Berlin.
Secretary: Professor William C. Mills, Ohio State University.

SECTION C. ETHNOLOGY. (*Hall* 16, *September* 24, 3 *p. m.*)

Chairman: Miss Alice C. Fletcher, President of the Washington Anthropological Society.
Speakers: Professor Frederick Starr, University of Chicago.
Professor A. C. Haddon, University of Cambridge.
Secretary: Professor F. W. Shipley, Washington University.

DIVISION D. — MENTAL SCIENCE

(*Hall* 7, *September* 20, 10 *a. m.*)

Speaker: President G. Stanley Hall, Clark University, Worcester, Mass.

DEPARTMENT 15 — PSYCHOLOGY

(*Hall* 7, *September* 20, 2 *p. m.*)

Chairman:
Speakers: Professor James McK. Cattell, Columbia University.
Professor J. Mark Baldwin, Johns Hopkins University.

SECTION A. GENERAL PSYCHOLOGY. (*Hall* 6, *September* 23, 3 *p. m.*)

Chairman: Professor Jos. Royce, Harvard University.
Speakers: Professor Harald Hoeffding, University of Copenhagen.
Professor James Ward, University of Cambridge, England.
Secretary: Dr. W. H. Davis, Lehigh University.

SECTION B. EXPERIMENTAL PSYCHOLOGY. (*Hall* 2, *September* 23, 10 *a. m.*)

Chairman: Professor Edward A. Pace, Catholic University of America.
Speakers: Professor Robert MacDougal, New York University.
Professor Edward B. Titchener, Cornell University.
Secretary: Dr. R. S. Woodworth, Columbia University.

SECTION C. COMPARATIVE AND GENETIC PSYCHOLOGY. (*Hall* 6, *September* 24, 10 *a. m.*)

Chairman: Professor Edmund C. Sanford, Clark University, Worcester, Mass.
Speakers: Principal C. Lloyd Morgan, University College, Bristol.
Professor Mary W. Calkins, Wellesley College.
Secretary: Dr. R. M. Yerkes, Harvard University.

SECTION D. ABNORMAL PSYCHOLOGY. (*Hall* 6, *September* 24, 3 *p. m.*)

Chairman: Dr. Edward Cowles, Waverley, Mass.
Speakers: Dr. Pierre Janet, College de France, Paris.
Dr. Morton Prince, Boston.
Secretary: Dr. Adolph Meyer, New York City.

DEPARTMENT 16 — SOCIOLOGY

(*Hall 7, September 20, 4.15 p. m.*)

CHAIRMAN: PROFESSOR FRANK W. BLACKMAR, University of Kansas.
SPEAKERS: PROFESSOR FRANKLIN H. GIDDINGS, Columbia University.
PROFESSOR GEORGE E. VINCENT, University of Chicago.

SECTION A. SOCIAL STRUCTURE. (*Hall 15, September 21, 10 a. m.*)

CHAIRMAN: PROFESSOR FREDERICK W. MOORE, Vanderbilt University.
SPEAKERS: FIELD MARSHAL GUSTAV RATZENHOFER, Vienna.
PROFESSOR F. TOENNIES, University of Kiel.
PROFESSOR LESTER F. WARD, U. S. National Museum.
SECRETARY: PROFESSOR JEROME DOWD, University of Wisconsin.

SECTION B. SOCIAL PSYCHOLOGY. (*Hall 15, September 23, 10 a. m.*)

CHAIRMAN: PROFESSOR CHARLES A. ELLWOOD, University of Missouri.
SPEAKERS: PROFESSOR WM. I. THOMAS, University of Chicago.
PROFESSOR EDWARD A. ROSS, University of Nebraska.

SECRETARY: PROFESSOR E. C. HAYES, Miami University.

DIVISION E — UTILITARIAN SCIENCES

(*Hall 1, September 20, 10 a. m.*)

SPEAKER: PRESIDENT DAVID STARR JORDAN, Leland Stanford Jr. University.

DEPARTMENT 17 — MEDICINE

(*Hall 1, September 20, 4.15 p. m.*)

CHAIRMAN: DR. WILLIAM OSLER, Johns Hopkins University.
SPEAKERS: DR. WILLIAM T. COUNCILMAN, Harvard University.
DR. FRANK BILLINGS, University of Chicago.

SECTION A. PUBLIC HEALTH. (*Hall 13, September 21, 10 a. m.*)

CHAIRMAN: DR. WALTER WYMAN, Surgeon-General of the U. S. Marine Hospital Service.
SPEAKERS: PROFESSOR WILLIAM T. SEDGWICK, Massachusetts Institute of Technology.
DR. ERNST J. LEDERLE, Former Commissioner of Health, New York City.
SECRETARY: DR. H. .M BRACKEN, St. Paul, Minn.

SECTION B. PREVENTIVE MEDICINE. (*Hall* 13, *September* 21, 3 *p. m.*)

Chairman: Dr. Joseph M. Mathews, President of the State Board of Health, Louisville, Ky.
Speaker: Professor Ronald Ross, F.R.S., School of Tropical Medicine, University College, Liverpool.
Secretary: Dr. J. N. Hurty, Indianapolis, Ind.

SECTION C. PATHOLOGY. (*Hall* 13, *September* 22, 10 *a. m.*)

Chairman: Professor Simon Flexner, Director of the Rockefeller Institute.
Speakers: Professor Ludwig Hektoen, University of Chicago.
Professor Johannes Orth, University of Berlin.
Professor Shibasaburo Kitasato, University of Tokio.
Secretary: Dr. W. McN. Miller, University of Missouri.

SECTION D. THERAPEUTICS AND PHARMACOLOGY. (*Hall* 13, *September* 24, 3 *p. m.*)

Chairman: Dr. Hobart A. Hare, Jefferson Medical College.
Speakers: Professor Oscar Liebreich, University of Berlin.
Sir Lauder Brunton, F.R.S., London.
Secretary: Dr. H. B. Favill, Chicago, Ill.

SECTION E. INTERNAL MEDICINE. (*Hall* 13, *September* 23, 3 *p. m.*)

Chairman: Professor Frederick C. Shattuck, Harvard University.
Speakers: Professor T. Clifford Allbutt, F.R.S., University of Cambridge.
Professor William S. Thayer, Johns Hopkins University.
Secretary: Dr. R. C. Cabot, Boston, Mass.

SECTION F. NEUROLOGY. (*Hall* 13, *September* 22, 3 *p. m.*)

Chairman: Professor Lewellyn F. Barker, University of Chicago.
Speaker: Professor James J. Putnam, Harvard University.
Secretary:

SECTION G. PSYCHIATRY. (*Hall* 7, *September* 22, 10 *a. m.*)

Chairman:
Speakers: Dr. Charles L. Dana, Cornell University, New York.
Dr. Edward Cowles, Boston.
Secretary: Dr. C. G. Chaddock, St. Louis, Mo.

SECTION H. SURGERY. (*Hall* 13, *September* 23, 10 *a. m.*)

Chairman: Professor Carl Beck, Post-Graduate Medical School, New York.
Speakers: Dr. Frederic S. Dennis, F.R.C.S., Cornell Medical College, New York City.
Professor Johannes Orth, University of Berlin.
Secretary: Dr. J. F. Binnie, Kansas City, Mo.

SECTION I. GYNECOLOGY. (*Hall 13, September 24, 10 a. m.*)

Chairman: Professor Howard A. Kelly, Johns Hopkins University.
Speaker: Professor J. Clarence Webster, Rush Medical College, Chicago.
Secretary: Dr. G. H. Noble, Atlanta, Ga.

SECTION J. OPHTHALMOLOGY. (*Hall 7, September 24, 10 a. m.*)

Chairman: Dr. George C. Harlan, Philadelphia, Pa.
Speakers: Dr. Edward Jackson, Denver, Col.
Dr. George M. Gould, Philadelphia, Pa.
Secretary: Dr. Wm. M. Sweet, Jefferson Medical College, Philadelphia, Pa.

SECTION K. OTOLOGY AND LARYNGOLOGY. (*Hall 7, September 21, 10 a. m.*)

Chairman: Professor William C. Glasgow, Washington University, St. Louis.
Speaker: Sir Felix Semon, C.V.O., Physician Extraordinary to His Majesty, the King, London.
Secretary: Dr. S. Spencer, Allenhurst, N. J.

SECTION L. PEDIATRICS. (*Hall 7, September 21, 3 p. m.*)

Chairman: Professor Thomas M. Rotch, Harvard University.
Speakers: Professor Theodore Escherich, University of Vienna.
Professor Abraham Jacobi, Columbia University.
Secretary: Dr. Samuel S. Adams, Washington, D. C.

DEPARTMENT 18 — TECHNOLOGY.

(*Hall 3, September 20, 2 p. m.*)

Chairman: Chancellor Winfield S. Chaplin, Washington University, St. Louis.
Speaker: Professor Henry T. Bovey, F.R.S., McGill University, Montreal.

SECTION A. CIVIL ENGINEERING. (*Hall 10, September 21, 10 a. m.*)

Chairman: Professor William H. Burr, Columbia University.
Speakers: Dr. J. A. L. Waddell, Consulting Engineer, Kansas City.
Mr. Lewis M. Haupt, Consulting Engineer, Philadelphia.
Secretary:

SECTION B. MECHANICAL ENGINEERING. (*Hall 10, September 23, 3 p. m.*)

Chairman: Professor James E. Denton, Stevens Institute of Technology.
Speaker: Professor Albert W. Smith, Leland Stanford Jr. University.
Secretary: Mr. George Dinkel, Jr., Jersey City.

SECTION C. ELECTRICAL ENGINEERING. (*Hall* 10, *September* 22, 3 *p. m.*)

CHAIRMAN:
SPEAKERS: PROFESSOR ARTHUR E. KENNELLY, Harvard University.
PROFESSOR MICHAEL I. PUPIN, Columbia University.
SECRETARY: MR. CARL HERING, Philadelphia, Pa.

SECTION D. MINING ENGINEERING. (*Hall* 11, *September* 24, 10 *a. m.*)

CHAIRMAN: MR. JOHN HAYS HAMMOND, New York City.
SPEAKERS: PROFESSOR ROBERT H. RICHARDS, Massachusetts Institute of Technology.
PROFESSOR SAMUEL B. CHRISTY, University of California.
SECRETARY: DR. JOSEPH STRUTHERS, New York City.

SECTION E. TECHNICAL CHEMISTRY. (*Hall* 16, *September* 23, 10 *a. m.*)

CHAIRMAN: DR. H. W. WILEY, Department of Agriculture.
SPEAKERS: PROFESSOR CHARLES E. MUNROE, George Washington University.
PROFESSOR WILLIAM H. WALKER, Massachusetts Institute of Technology.
SECRETARY: DR. MARCUS BENJAMIN, U. S. National Museum.

SECTION F. AGRICULTURE. (*Hall* 10, *September* 24, 3 *p. m.*)

CHAIRMAN: PROFESSOR H. J. WHEELER, Kingston, R. I.
SPEAKERS: PROFESSOR CHARLES W. DABNEY, JR., University of Cincinnati.
PROFESSOR LIBERTY H. BAILEY, Cornell University.
SECRETARY: PROFESSOR WILLIAM HILL, University of Chicago.

DEPARTMENT 19 — ECONOMICS

(*Hall* 1, *September* 20, 11.15 *a. m.*)

CHAIRMAN: PROFESSOR EMORY R. JOHNSON, University of Pennsylvania.
SPEAKERS: PROFESSOR FRANK A. FETTER, Cornell University.
PROFESSOR ADOLPH C. MILLER, University of California.

SECTION A. ECONOMIC THEORY. (*Hall* 15, *September* 22, 10 *a. m.*)

CHAIRMAN:
SPEAKERS: PROFESSOR JOHN B. CLARK, Columbia University.
PROFESSOR JACOB H. HOLLANDER, Johns Hopkins University.
SECRETARY: PROFESSOR JESSE E. POPE, University of Missouri.

SECTION B. TRANSPORTATION. (*Hall* 10, *September* 23, 10 *a. m.*)

CHAIRMAN: PROFESSOR J. LAWRENCE LAUGHLIN, University of Chicago.
SPEAKERS: PROFESSOR EUGENE VON PHILIPPOVICH, University of Vienna.
PROFESSOR WILLIAM Z. RIPLEY, Harvard University.
SECRETARY: MR. GEORGE G. TUNELL, Chicago.

SECTION C. COMMERCE AND EXCHANGE. (*Hall* 10, *September* 24, 10 *a. m.*)

Chairman:
Speakers: Professor E. D. Jones, University of Michigan.
Professor Carl Plehn, University of California.
Secretary:

SECTION D. MONEY AND CREDIT. (*Hall* 5, *September* 24, 3 *p. m.*)

Chairman: Mr. B. E. Walker, Canadian Bank of Commerce, Toronto.
Speakers: Mr. Horace White, New York City.
Professor J. Lawrence Laughlin, University of Chicago.
Secretary: Professor John Cummings, University of Chicago.

SECTION E. PUBLIC FINANCE. (*Hall* 1, *September* 21, 10 *a. m.*)

Chairman:
Speakers: Professor Henry C. Adams, University of Michigan.
Professor Edwin R. A. Seligman, Columbia University.
Secretary:

SECTION F. INSURANCE. (*Hall* 10, *September* 21, 3 *p. m.*)

Chairman: Dr. Emory McClintock, Actuary, Mutual Life Insurance Company, New York.
Speakers: Mr. Frederick L. Hoffman, Statistician, Prudential Insurance Company, Newark.
Professor Balthasar H. Meyer, University of Wisconsin.
Secretary:

DIVISION F — SOCIAL REGULATION

(*Hall* 2, *September* 20, 10 *a. m.*)

Speaker: Professor Abbott L. Lowell, Harvard University.

DEPARTMENT 20 — POLITICS

(*Hall* 2, *September* 20, 2 *p. m.*)

Chairman:
Speakers: Professor William A. Dunning, Columbia University.
Chancellor E. Benjamin Andrews, University of Nebraska.

SECTIONS A AND C. POLITICAL THEORY AND NATIONAL ADMINISTRATION. (*Hall* 15, *September* 22, 3 *p. m.*)

Chairman:
Speakers: Professor W. W. Willoughby, Johns Hopkins University.

Professor George G. Wilson, Brown University.
Right Hon. James Bryce, London, England.
Secretary: Dr. Charles E. Merriam, University of Chicago.

SECTION B. DIPLOMACY. (*Hall* 1, *September* 23, 3 *p. m.*)

Chairman:
Speakers: Honorable John W. Foster, Ex-Secretary of State.
Honorable David Jayne Hill, Minister of the United States to Switzerland.
Secretary:

SECTION D. COLONIAL ADMINISTRATION. (*Hall* 4, *September* 24, 10 *a. m.*)

Chairman: Professor Harry P. Judson, University of Chicago.
Speakers: Professor Bernard J. Moses, University of California.
Professor Paul S. Reinsch, University of Wisconsin.
Secretary:

SECTION E. MUNICIPAL ADMINISTRATION. (*Hall* 15, *September* 24, 3 *p. m.*)

Chairman:
Speakers: Mr. Albert Shaw, Editor American Monthly Review of Reviews.
Miss Jane Addams, Hull House, Chicago.
Secretary: Professor John A. Fairlie, University of Michigan.

DEPARTMENT 21 — JURISPRUDENCE

(*Hall* 3, *September* 20, 4.15 *p. m.*)

Chairman: Professor George W. Kirchwey, Columbia University.
Speakers: President Charles W. Needham, Columbian University, Washington.
Professor Joseph H. Beale, Harvard University.

SECTION A. INTERNATIONAL LAW. (*Hall* 14, *September* 22, 10 *a. m.*)

Chairman: Professor James B. Scott, Columbia University.
Speakers: Professor H. LaFontaine, Member of the Senate, Brussels, Belgium.
Professor Charles Noble Gregory, University of Iowa.
Count Albert Apponyi, Hungary.
Secretary: Dr. W. C. Dennis, Leland Stanford Jr. University.

SECTION B. CONSTITUTIONAL LAW. (*Hall* 14, *September* 24, 10 *a. m.*)

Chairman: Professor Henry St. George Tucker, George Washington University, Washington.
Speakers: Signor Attilio Brunialti, Councilor of State, Rome.
Professor John W. Burgess, Columbia University.
Professor Ferdinand Larnaude, University of Paris.
Secretary:

SECTION C. PRIVATE LAW. (*Hall 14, September 23, 3 p. m.*)

CHAIRMAN: PROFESSOR JAMES B. AMES, Dean, Harvard Law School.
SPEAKERS: PROFESSOR ERNST FREUND, University of Chicago.
HONORABLE EDWARD B. WHITNEY, New York.
SECRETARY: DEAN WILLIAM DRAPER LEWIS, University of Pennsylvania.

DEPARTMENT 22 — SOCIAL SCIENCE

(*Hall 1, September 20, 2 p. m.*)

CHAIRMAN: MR. WALTER L. SHELDON, Ethical Society, St. Louis.
SPEAKERS: PROFESSOR FELIX ADLER, Columbia University.
PROFESSOR GRAHAM TAYLOR, Chicago Theological Seminary.

SECTION A. THE FAMILY. (*Hall 5, September 21, 10 a. m.*)

CHAIRMAN: PROFESSOR SAMUEL G. SMITH, University of Minnesota.
SPEAKERS: DR. SAMUEL W. DIKE, Auburndale, Mass.
PROFESSOR GEORGE ELLIOTT HOWARD, University of Nebraska.
SECRETARY:

SECTION B. THE RURAL COMMUNITY. (*Hall 5, September 21, 3 p. m.*)

CHAIRMAN: HON. AARON JONES, Master of National Grange, South Bend, Ind.
SPEAKERS: PROFESSOR MAX WEBER, University of Heidelberg.
PRESIDENT KENYON L. BUTTERFIELD, Rhode Island State Agricultural College.
SECRETARY: PROFESSOR WILLIAM HILL, University of Chicago.

SECTION C. THE URBAN COMMUNITY. (*Hall 5, September 22, 10 a. m.*)
CHAIRMAN:
SPEAKERS: PROFESSOR T. JASTROW, University of Berlin.
PROFESSOR LOUIS WUARIN, University of Geneva.
SECRETARY:

SECTION D. THE INDUSTRIAL GROUP. (*Hall 14, September 22, 3 p. m.*)
CHAIRMAN:
SPEAKERS: PROFESSOR WERNER SOMBART, University of Breslau.
PROFESSOR RICHARD T. ELY, University of Wisconsin.
SECRETARY: PROFESSOR THOMAS S. ADAMS, Madison, Wis.

SECTION E. THE DEPENDENT GROUP. (*Hall 5, September 23, 10 a. m.*)

CHAIRMAN: MR. ROBERT W. DEFOREST, New York City.
SPEAKERS: PROFESSOR CHARLES R. HENDERSON, University of Chicago.
DR. EMIL MÜNSTERBERG, President City Charities, Berlin.
SECRETARY:

SECTION F. THE CRIMINAL GROUP. (*Hall 5, September 23, 3 p. m.*)

CHAIRMAN:
SPEAKER: MR. FREDERICK H. Wines, Secretary State Charities Aid Association, Upper Montclair, N. J.
SECRETARY:

DIVISION G — SOCIAL CULTURE

(*Hall* 5, *September* 20, 10 *a. m.*)

Speaker: Honorable William T. Harris, United States Commissioner of Education.

DEPARTMENT 23 — EDUCATION

(*Hall* 2, *September* 20, 4.15 *p. m.*)

Chairman:
Speakers: President Arthur T. Hadley, Yale University.
The Right Rev. John L. Spalding, Bishop of Peoria.

SECTION A. EDUCATIONAL THEORY. (*Hall* 12, *September* 24, 3 *p. m.*)

Chairman: Professor Charles DeGarmo, Cornell University.
Speakers: Professor Wilhelm Rein, University of Jena.
Professor Elmer E. Brown, University of California.
Secretary: Dr. G. M. Whittle, Cornell University.

SECTION B. THE SCHOOL. (*Hall* 12, *September* 23, 10 *a. m.*)

Chairman: Dr. F. Louis Soldan, Superintendent Public Schools, St. Louis.
Speakers: Dr. Michael E. Sadler, University of Manchester.
Dr. William H. Maxwell, Superintendent Public Schools, New York City.
Secretary: Professor A. S. Langsdorf, Washington University.

SECTION C. THE COLLEGE. (*Hall* 12, *September* 23, 3 *p. m.*)

Chairman: President W. S. Chaplin, Washington University.
Speakers: President William DeWitt Hyde, Bowdoin College.
President M. Carey Thomas, Bryn Mawr College.
Secretary: Professor H. H. Horne, Dartmouth College.

SECTION D. THE UNIVERSITY. (*Hall* 12, *September* 24, 10 *a. m.*)

Chairman:
Speakers: Professor C. Chabot, University of Lyons.
Professor Edward Delavan Perry, Columbia University.
Secretary:

SECTION E. THE LIBRARY. (*Hall* 12, *September* 22, 3 *p. m.*)

Chairman: Mr. Frederick M. Crunden, Librarian St. Louis Public Library.
Speakers: Mr. William A. E. Axon, Manchester, England.
Professor Guido Biagi, Royal Librarian, Florence.
Secretary: Mr. C. P. Pettus, Washington University.

DEPARTMENT 24 — RELIGION

(*Hall* 4, *September* 20, 4.15 *p. m.*)

CHAIRMAN: BISHOP JOHN H. VINCENT, Chautauqua, N. Y.
SPEAKERS: PRESIDENT HENRY C. KING, Oberlin College.
PROFESSOR FRANCIS G. PEABODY, Harvard University.

SECTION A. GENERAL RELIGIOUS EDUCATION. (*Hall* 11, *September* 24, 3 *p. m.*)

CHAIRMAN: PROFESSOR EDWIN D. STARBUCK, Earlham College, Richmond, Ind.
SPEAKERS: PROFESSOR GEORGE A. COE, Northwestern University.
DR. WALTER L. HERVEY, Examiner Board of Education, New York City.
SECRETARY:

SECTION B. PROFESSIONAL RELIGIOUS EDUCATION. (*Hall* 1, *September* 22, 3 *p. m.*)

CHAIRMAN:
SPEAKERS: PRESIDENT CHARLES CUTHBERT HALL, Union Theological Seminary.
PROFESSOR FRANK K. SANDERS, Yale University.
SECRETARY: PROFESSOR HERBERT L. WILLETT, Disciples Divinity House, Chicago, Ill.

SECTION C. RELIGIOUS AGENCIES. (*Hall* 15, *September* 23, 3 *p. m.*)

CHAIRMAN: PRESIDENT EDGAR G. MULLINS, Southern Baptist Theological Seminary, Louisville, Ky.
SPEAKERS: REV. WASHINGTON GLADDEN, Columbus, Ohio.
REV. JAMES M. BUCKLEY, Editor *The Christian Advocate*, New York.
SECRETARY: DR. IRA LANDRITH, General Secretary Religious Education Association, Chicago, Ill.

SECTION D. RELIGIOUS WORK. (*Hall* 1, *September* 24, 3 *p. m.*)

CHAIRMAN: RT. REV. THOMAS F. GAILOR, Memphis.
SPEAKERS: REV. FLOYD W. TOMKINS, Church of the Holy Trinity, Philadelphia.
REV. HENRY C. MABIE, Corresponding Secretary American Baptist Missionary Union.
SECRETARY:

SECTION E. RELIGIOUS INFLUENCE: PERSONAL. (*Festival Hall*, *September* 25, 10 *a. m.*)

CHAIRMAN: CHANCELLOR J. H. KIRKLAND, Vanderbilt University.
SPEAKERS: REV. HUGH BLACK, Edinburgh, Scotland.
PROFESSOR JOHN E. McFADYEN, Knox College.
REV. SAMUEL ELIOT, Boston, Mass.
REV. EDWARD B. POLLARD, Georgetown, Ky.
SECRETARY: PROFESSOR CLYDE W. VOTAW, University of Chicago.

SECTION F. RELIGIOUS INFLUENCE: SOCIAL. (*Festival Hall, September 25, 3 p. m.*)

CHAIRMAN:	DR. J. H. GARRISON, St. Louis.
SPEAKERS:	PRESIDENT JOSEPH SWAIN, Swarthmore College.
	DR. EMIL G. HIRSCH, Chicago, Ill.
	PROFESSOR EDWARD C. MOORE, Harvard University.
	DR. JOSIAH STRONG, League for Social Service, New York.
SECRETARY:	PROFESSOR CLYDE W. VOTAW, University of Chicago.

CHRONOLOGICAL ORDER OF PROCEEDINGS

MONDAY, SEPTEMBER 19.

3 P. M. Opening exercises of the Congress. Festival Hall (Hall 17).

The Congress will be called to order by the Director of Congresses, who will introduce the President of the Exposition.

Welcoming addresses will be delivered by the President of the Exposition and other officials.

A reply to these addresses of welcome will be made on behalf of the Congress by the Honorary Vice-President for Great Britain.

The Chairman of the Administrative Board will give an account of the origin and purpose of the Congress.

The President of the Congress will then be introduced and will deliver an introductory address, after which adjournment will follow.

TUESDAY, SEPTEMBER 20.

10.00 A. M. Meetings of the seven Divisions. The Divisional addresses will be given as follows: —

Hall 1, Utilitarian Sciences.
Hall 2, Social Regulation.
Hall 3, Historical Science.
Hall 4, Physical Science.
Hall 5, Social Culture.
Hall 6, Normative Science.
Hall 7, Mental Science.

11.15 to 6.00 P. M. Meetings of the Departments, with addresses: —

Meeting at 11.15 A. M.	Meeting at 2 P. M.
DEPARTMENTS.	DEPARTMENTS.
Hall 1, Economics.	Hall 1, Social Science.
Hall 2, Biology.	Hall 2, Politics.
Hall 3, Sciences of the Earth.	Hall 3, Technology.
Hall 4, Political History.	Hall 4, History of Language.
Hall 5, History of Law.	Hall 5, History of Religion.
Hall 6, Philosophy.	Hall 6, Physics.
Hall 7, Mathematics.	Hall 7, Psychology.
Hall 8, History of Art.	Hall 8, Anthropology.
Adjournment at 1 P. M.	Adjournment at 3.45 P. M.

Meeting at 4.15 P. M.

DEPARTMENTS.

Hall 1, Medicine.
Hall 2, Education.
Hall 3, Jurisprudence.
Hall 4, Religion.
Hall 5, Chemistry.
Hall 6, History of Literature.
Hall 7, Sociology.
Hall 8, Astronomy.

Adjournment at 6. P. M.

On the four days following, the Sectional meetings will be held. The duration of each session will be three hours. The morning sessions will extend from 10 A. M. until 1 P. M.; the afternoon sessions from 3 P. M. to 6 P. M.

The meetings of some of the religious sections will be held on Sunday, September 25, in Festival Hall. Further announcements concerning these Sunday Meetings will be made in Registration Hall, in the daily press of St. Louis, and in the World's Fair Official Programme.

WEDNESDAY, SEPTEMBER 21.

Meeting at 10 A. M.

Hall 1, Public Finance.
Hall 2, Animal Morphology.
Hall 3, History of Greece, Rome, and Asia.
Hall 4, Comparative Language.
Hall 5, The Family.
Hall 6, Metaphysics.
Hall 7, Otology and Laryngology.
Hall 8, Slavic Literature.
Hall 9, Astrometry.
Hall 10, Civil Engineering.
Hall 11, History of Common Law.
Hall 12, Physiography.
Hall 13, Public Health.
Hall 14, Geophysics.
Hall 15, Social Structure.
Hall 16, Inorganic Chemistry.

Adjournment at 1 P. M.

Meeting at 3 P. M.

Hall 1, Philosophy of Religion.
Hall 2, Phylogeny.
Hall 3, Classical Literature.
Hall 4, Semitic Languages.
Hall 5, The Rural Community.
Hall 6, Medieval History.
Hall 7, Pediatrics.
Hall 8, Oceanography.
Hall 9, Astrophysics.
Hall 10, Insurance.
Hall 11, History of Roman Law.
Hall 13, Preventive Medicine.
Hall 14, Geology.
Hall 16, Organic Chemistry.

Adjournment at 6 P. M.

Immediately following the Section of Geophysics in the morning, and the Section of Geology in the afternoon, in Room 14, the Eighth International Geographic Congress will hold sessions in the same room, Hall 14, Mines and Metallurgy Building.

THURSDAY, SEPTEMBER 22.

Meeting at 10 A. M.

Hall 1, English Literature.
Hall 2, Plant Morphology.
Hall 3, Modern History of Europe.
Hall 4, Old Testament.
Hall 5, The Urban Community.
Hall 6, Logic.
Hall 7, Psychiatry.
Hall 8, Indo-Iranian Languages.
Hall 9, Algebra and Analysis.
Hall 10, Cosmical Physics.
Hall 11, Palæontology.
Hall 12, Classical Art.
Hall 13, Pathology.
Hall 14, International Law.
Hall 15, Economic Theory.
Hall 16, Physical Chemistry.

Adjournment at 1 P. M.

Meeting at 3 P. M.

Hall 1, Professional Religious Education.
Hall 2, Human Anatomy.
Hall 3, Greek Language.
Hall 4, Plant Physiology.
Hall 5, Physics of the Electron.
Hall 6, Methodology of Science.
Hall 7, Modern Architecture.
Hall 8, Romance Literature.
Hall 9, Petrology and Mineralogy.
Hall 10, Electrical Engineering.
Hall 11, Geography.
Hall 12, The Library.
Hall 13, Neurology.
Hall 14, The Industrial Group.
Hall 15, Political Theory and National Administration.
Hall 16, Physiological Chemistry.

Adjournment at 6 P. M.

FRIDAY, SEPTEMBER 23.

Meeting at 10 A. M.

Hall 1, New Testament.
Hall 2, Experimental Psychology.
Hall 3, Germanic Literature.
Hall 4, Physiology.
Hall 5, The Dependent Group.
Hall 6, Ethics.
Hall 7, Plant Pathology.
Hall 8, Brahmanism and Buddhism.
Hall 9, Latin Language.
Hall 10, Transportation.
Hall 11, Physics of Matter.
Hall 12, The School.
Hall 13, Surgery.
Hall 15, Social Psychology.
Hall 16, Technical Chemistry.

Adjournment at 1 P. M.

Meeting at 3 P. M.

Hall 1, Diplomacy.
Hall 2, History of Economic Institutions.
Hall 3, English Language.
Hall 4, Æsthetics.
Hall 5, The Criminal Group.
Hall 6, General Psychology.
Hall 7, Ecology.
Hall 8, Mohammedism.
Hall 9, Embryology.
Hall 10, Mechanical Engineering.
Hall 11, Physics of Ether.
Hall 12, The College.
Hall 13, Internal Medicine.
Hall 14, Private Law.
Hall 15, Religious Agencies.
Hall 16, Somatology.

Adjournment at 6 P. M.

SATURDAY, SEPTEMBER 24.

Meeting at 10 A. M.

Hall 1, History of America.
Hall 2, History of the Christian Church.
Hall 3, Belles-Lettres.
Hall 4, Colonial Administration.
Hall 5, Romance Languages.
Hall 6, Comparative and Genetic Psychology.
Hall 7, Ophthalmology.
Hall 8, History of Asia.
Hall 9, Geometry.
Hall 10, Commerce and Exchange.
Hall 11, Mining Engineering.
Hall 12, The University.
Hall 13, Gynecology.
Hall 14, Constitutional Law.
Hall 15, Bacteriology.
Hall 16, Archæology.

Adjournment at 1 P. M.

Meeting at 3 P. M.

Hall 1, Religious Work.
Hall 2, Comparative Anatomy.
Hall 3, Germanic Languages.
Hall 4, Modern Painting.
Hall 5, Money and Credit.
Hall 6, Abnormal Psychology.
Hall 7, Applied Mathematics.
Hall 8, Indo-Iranian Literature.
Hall 10, Agriculture.
Hall 11, ...
Hall 12, Educational Theory.
Hall 13, Therapeutics and Pharmacology.
Hall 14, Comparative Law.
Hall 15, Municipal Administration.
Hall 16, Ethnology.

Adjournment at 6 P. M.

SUNDAY, SEPTEMBER 25.

Festival Hall.

Meeting at 10 A. M.
Religious Influence: Personal.

Meeting at 3 P. M.
Religious Influence: Social.

PROGRAMME OF SOCIAL EVENTS

MONDAY EVENING, SEPTEMBER 19. — Grand Fête night in honor of the Congress of Arts and Science. Special illuminations about the Grand Basin. Lagoon fête.

Banquet by the St. Louis Chemical Society, at the Southern Hotel, to the members of the Chemical Sections.

TUESDAY EVENING, SEPTEMBER 20. — General Reception by Board of Lady Managers to the officers and speakers of the Congress and officials of the Exposition.

WEDNESDAY AFTERNOON, SEPTEMBER 21. — Garden fête to be given to the members of the Congress of Arts and Science, at the French Pavilion, by the Commissioner-General from France.

WEDNESDAY EVENING, SEPTEMBER 21. — General reception by the German Imperial Commissioner-General to the members of the Congress of Arts and Science, at the German State House.

THURSDAY EVENING. — Shaw banquet at the Buckingham Club to the foreign delegates.

FRIDAY EVENING, SEPTEMBER 23. — General banquet to the speakers and officials of the Congress of Arts and Science in the banquet-hall of the Tyrolean Alps. 8 P. M.

SATURDAY EVENING, SEPTEMBER 24. — Banquet at St. Louis Club by Round Table of St. Louis, to the foreign members of the Congress.

Banquet given by Imperial Commissioner-General from Japan to the Japanese delegation to the Congress and Exposition officials.

Dinner given by Commissioner-General from Great Britain to the English members of the Congress.

ALPHABETICAL LIST OF MEMBERS WHO MADE 10-MINUTE ADDRESSES

The following list differs from the original programme, in that it contains the names only of those who actually read addresses. It was planned that each Section should meet for three hours. When authors of ten-minute papers were not present, and where not enough of these shorter papers were offered to fill out the time, the Chairmen invited discussions from the floor until the time was filled.

Professor R. G. Aitken	Lick Observatory	Astronomy
James W. Alexander, Esq.	New York City	Insurance
Frederick Almy	Buffalo, N. Y.	Social Science
Professor S. G. Ashmore	Union College	Latin Language
Professor L. A. Bauer	Carnegie Institute	Cosmical Physics
Dr. Marcus Benjamin	National Museum	Technical Chemistry
Professor H. T. Blickfeldt	Leland Stanford Univ.	Geometry
Professor Ernest W. Brown	Haverford College	Lunar Theory
Dr. Henry Dickson Bruns	New Orleans	Municipal Administration
Dr. F. K. Cameron	Dep't of Agriculture	Physical Chemistry
Rear-Admiral C. M. Chester, U. S. N.	United States Naval Observatory	Astronomy
H. H. Clayton, Esq.	Blue Hill Observatory	Cosmical Physics
Professor Charles A. Coffin	New York City	Modern Painting
Dr. George Coronilas	Athens, Greece	Tuberculosis
Professor J. E. Denton	Stevens Institute	Mechanical Engineering
Professor L. W. Dowling	Univ. of Wisconsin	Geometry
Professor H. C. Elmer	Cornell Univ.	Latin Language
Professor A. Emch	Univ. of Colorado	Geometry
Professor H. R. Fanclough	Leland Stanford Univ.	Classical Literature
Professor W. S. Ferguson	Univ. of California	History of Greece, Rome, and Asia
Dr. Carlos Finley	Havana	Pathology
Dr. C. E. Fisk	Centralia, Ill.	History of America
Homer Folks, Esq.	New York City	Social Science
Professor F. C. French	Univ. of Nebraska	Philosophy of Religion
H. L. Gannt, Esq.	Schenectady, N. Y.	Mechanical Engineering
Dr. F. P. Gorham	Brown Univ.	Bacteriology
Professor Evarts B. Greene	Univ. of Illinois	History of America
Stansbury Hagar, Esq.	Brooklyn, N. Y.	Ethnology
J. D. Hague, Esq.	New York City	Mining Engineering

Professor G. B. Halstead	Kenyon College	Geometry
Professor A. D. F. Hamlin	Columbia Univ.	Æsthetics
Professor H. Hancock	Univ. of Cincinnati	Geometry
Professor J. A. Harris	St. Louis, Mo.	Plant Morphology
Professor M. W. Haskell	Univ. of California	Algebra and Analysis
Professor J. T. Hatfield	Northwestern Univ.	Germanic Language
Professor E. C. Hayes	Miami Univ.	Social Psychology
Professor W. E. Heidel	Iowa College	Greek Language
Dr. C. L. Herrick	Granville, Ohio	Neurology
Dr. C. Judson Herrick	Granville, Ohio	Animal Morphology
Professor W. H. Hobbs	Univ. of Wisconsin	Petrology and Mineralogy
Professor A. R. Hohlfeld	Univ. of Wisconsin	Germanic Literature
Professor H. H. Horne	Dartmouth College	Educational Theory
Dr. E. V. Huntington	Harvard Univ.	Algebra and Analysis
Dr. Reid Hunt	U. S. Marine Hospital	Alcohol, etc.
Dr. J. N. Hurty	Indianapolis, Ind.	Public Health
Professor J. J. Hutchinson	Cornell Univ.	Algebra and Analysis
Rev. Thomas E. Judge	Catholic Review of Reviews	General Religious Education
Professor L. Kahlenburg	Univ. of Wisconsin	Physical Chemistry
Professor Albert G. Keller	Yale University	Municipal Administration
Professor George Lefevre	Univ. of Missouri	Comparative Anatomy
President Henry C. King	Oberlin College	Education, The College
Dr. Ira Landrith	Belmont College	Religious Agencies
Professor M. D. Learned	Univ. of Pennsylvania	Germanic Literature
Professor A. O. Leuschner	Univ. of California	Astronomy
Dr. E. P. Lyon	St. Louis Univ.	Physiology
Dr. Duncan B. Macdonald	Hartford Theological Seminary	Semitic Languages
Professor A. MacFarlane	Chatham, Ontario	Applied Mathematics
Professor James McMahon	Cornell Univ.	Applied Mathematics
Mr. Edward Mallinckrodt	St. Louis, Mo.	Chemistry
Professor H. P. Manning	Brown Univ.	Geometry
Professor G. A. Miller	Leland Stanford Univ.	Algebra and Analysis.
Dr. W. C. Mills	Ohio State Univ.	Archæology
Professor W. S. Milner	Univ. of Toronto	Classical Literature
Professor F. G. Moore	Dartmouth College	Classical Literature
Dr. W. P. Montague	Columbia Univ.	Metaphysics
Clarence B. Moore, Esq.	Philadelphia	Archæology.
Professor F. R. Moulton	Univ. of Chicago	Astronomy.
Dr. J. G. Needham	Lake Forest Univ.	Animal Morphology
Professor Alex. T. Ormond	Princeton Univ.	Philosophy of Religion
Professor Frederic L. Paxton	Univ. of Colorado	History of America
Dr. Carl Pfister	St. Mark's Hospital, New York City	Surgery
Professor M. B. Porter	Univ. of Texas	Algebra and Analysis
Dr. A. J. Reynolds	Chicago	Public Health
Professor S. P. Sadtler	Philadelphia College of Pharmacy	Technical Chemistry
Dr. John A. Sampson	Albany, N. Y.	Gynæcology
Oswald Schreiner, Esq.	U. S. Dep't of Agriculture	Chemistry

Rev. Frank Sewall	Washington, D. C.	Social Science, The Family
Professor H. C. Sheldon	Boston Univ.	History of the Christian Church
Professor Frank C. Sharp	Univ. of Wisconsin	Ethics
Professor J. B. Shaw	Milliken Univ.	Algebra and Analysis
Professor W. B. Smith	Tulane Univ.	New Testament
Professor Marshall S. Snow	Washington Univ.	History of America
Professor Henry Snyder	Univ. of Minnesota	Social Science
Professor Edwain D. Starbuck	Earlham College	General Religious Education
Professor George B. Stewart	Auburn Theological Seminary	Professional Religious Education
John M. Stahl	Quincy, Ill.	The Rural Community
Professor J. Stieglitz	Univ. of Chicago	Chemistry
Professor Robert Stein	U. S. Geological Survey	Comparative Language
Mr. Teitaro Suzuki	La Salle, Ill.	Brahmanism and Buddhism
Col. T. W. Symonds, U. S. A.	Washington, D. C.	Civil Engineering
Professor Teissier	Lyons, France	Pathology
Judge W. H. Thomas	Montgomery, Ala.	Private Law
Professor O. H. Tittmann	U. S. C. and G. Survey	Astronomy
Professor Alfred M. Tozzer	Peabody Museum	Anthropology
Dr. Benjamin F. Trueblood	Univ. of Missouri	Medieval History
Professor Clyde W. Votaw	Univ. of Chicago	New Testament
Professor John B. Watson	Univ. of Chicago	Psychology
Professor H. L. Willett	Disciples Divinity House, Chicago	Professional Religious Education
President Mary E. Woolley	Mt. Holyoke College	Education, The College
H. Zwaarddemaker	Utrecht	Otology and Laryngology.

THE SCIENTIFIC PLAN OF THE CONGRESS

BY PROF. HUGO MÜNSTERBERG

I

THE PURPOSE OF THE CONGRESS

1. *The Centralization of the Congress*

THE history of the Congress has been told. It remains to set forth the principles which controlled the work of the Congress week, and thus scientifically to introduce the scholarly undertaking, the results of which are to speak for themselves in the eight volumes of this publication. Yet in a certain way this scientific introduction has once more to use the language of history. It does not deal with the external development of the Congress, and the story which it has to tell is thus not one of dates and names and events. But the principles which shaped the whole undertaking have themselves a claim to historical treatment; they do not lie before us simply as the subject for a logical disputation or as a plea for a future work. That was the situation of three years ago. At that time various ideas and opposing principles entered into the arena of discussion; but now, since the work is completed, the question can be only of what principles, right or wrong, have really determined the programme. We have thus to interpret that state of mind out of which the purposes and the scientific arrangement of the Congress resulted; and no after-thought of to-day would be a desirable addition. Whatever possible improvements of the plan may suggest themselves in the retrospect can be given only a closing word. It was certainly easy to learn from experience, but first the experience had to be passed through. We have here to interpret the view from that standpoint from which the experience of the Congress was still a matter of the future, and of an uncertain future indeed, full of doubts and fears, and yet full of hopes and possibilities.

The St. Louis World's Fair promised, through the vast extent of its grounds, through the beautiful plans of the buildings, through the eagerness of the United States, through the participation of all countries on earth, and through the gigantic outlines of the internal plans, to become the most monumental expression of the energies with which the twentieth century entered on its course. Commerce and industry, art and social work, politics and education, war and peace, country and city, Orient and Occident, were all to be focussed for a few summer months in the ivory city of the Mississippi Valley. It seemed most natural that science and productive scholarship should also find its characteristic place among the factors of our modern civilization. Of course the scientist had his word to say on almost every square foot of the Exposition. Whether the building was devoted to electricity or to chemistry, to anthropology or to metallurgy, to civic administration or to medicine, to transportation or to industrial arts, it was everywhere the work of the scientist which was to win the triumph; and the Palace of Education, the first in any universal exposition, was to combine under its roof not only the school work of all countries, but the visible record of the world's universities and technical schools as well. And yet it seemed not enough to gather the products and records of science and to make science serve with its tools and inventions. Modern art, too, was to reign over every hall and to beautify every palace, and yet demanded its own unfolding in the gallery of paintings and sculptures. In the same way it was not enough for science to penetrate a hundred exhibitions and turn the wheels in every hall, but it must also seek to concentrate all its energies in one spot and show the cross-section of human knowledge in our time, and, above all, its own methods.

An exhibition of scholarship cannot be arranged for the eyes. The great work which grows day by day in quiet libraries and laboratories, and on a thousand university platforms, can express itself only through words. Yet heaped up printed volumes would be dead to a World's Fair spectator; how to make such words living was the problem. Above all, scholarship does not really exhibit its methods, if it does not show itself in production. It is no longer scholarship which speaks of a truth-seeking that has been performed instead of going on with the search for further truth. If the world's science was to be exhibited, a form had to be sought in which the scholarly work on the spot would serve the ideals of knowledge, would add to the storehouse of truth, and would thus work in the service of human progress at the same moment in which it contributed to the completeness of the exhibition.

The effort was not without precedent. Scholarly production had been connected with earlier expositions, and the large gatherings of scholars at the Paris Exposition were still in vivid memory. A large number of scientific congresses of specialists had been held there, and many hundred scholarly papers had been read. Yet the results hardly suggested the repetition of such an experiment. Every one felt too strongly that the outcome of such disconnected congresses of specialists is hardly comparable with the glorious showing which the arts and industries have made and were to make again. In every other department of the World's Fair the most careful preparation secured an harmonious effect. The scholarly meetings alone failed even to aim at harmony and unity. Not only did the congresses themselves stand apart without any inner relation, grouped together by calendar dates or by their alphabetical order from Anthropology to Zoölogy; but in every congress, again, the papers read and the manuscripts presented were disconnected pieces without any programme or correlation. Worse than that, they could not even be expected in their isolatedness to add anything which would not have been worked out and communicated to the world just as well without any congress. The speaker at such a meeting is asked to contribute anything he has at hand, and he accepts the invitation because he has by chance a completed paper or a research ready for publication. In the best case it would have appeared in the next number of the specialistic magazine, in not unfrequent cases it has appeared already in the last number. Such a congress is then only an accident and does not itself serve the progress of knowledge.

Even that would be acceptable if at least the best scholars would come out with their latest investigations, or, still more delightful, if they would enter into an important discussion. But experience has too often shown that the conditions are most favorable for the opposite outcome. The leading scholars stay away partly to give beginners the chance to be heard, partly not to be grouped with those who habitually have the floor at such gatherings. These are either the men whose day has gone by or those whose day has not yet come; and both groups tyrannize alike an unwilling audience. Yet it may be said that in scientific meetings of specialists the reading of papers is nonessential and no harm is done even if they do not contribute anything to the status of scholarship; their great value lies in the personal contact of fellow workers and in the discussions and informal exchange of opinions. All that is true, and completely justifies the yearly meetings of scholarly associations. But these advantages are much diminished whenever such gatherings take on an international character, and thus introduce the confusion of tongues. And hardly any one can doubt that the turmoil of a world's fair is about the worst possible background for such exchange of thought, which demands repose and quietude. Yet even with the certainty of all these disadvantages the city of Paris, with its large body of scholars, with its venerable scholarly traditions, and with its incomparable attractions, could overcome every resistance, and its convenient location made it natural that in vacation time, in an exposition summer, the scholars should gather there, not on account of, but in spite of, the hundred congresses. With this the city of St. Louis could make no claim to rivalry. Its recent growth, its minimum of scholarly tradition, its great distance from the old centres of knowledge even in the New World, the apathy of the East and the climatic fears of Europe, all together made it clear that a mere repetition of unrelated congresses would be not only useless, but a disastrous failure. These very fears, however, themselves suggested the remedy.

If the scholarly work of our time was to be represented at St. Louis, something had to be attempted which should be not simply an imitation of the branch-congresses which every scientific specialty in every country is calling every year. Scholarship was to be asked to show itself really in process, and to produce for the World's Fair meeting something which without it would remain undone. To invite the scholars of the world for their leisurely enjoyment and reposeful discussion of work done elsewhere is one thing; to call them together for work which they would not do otherwise, and which ought to be done, is a very different thing. The first had in St. Louis all odds against it; it seemed worth while to try the second. And it seemed not only worth while in the interest of scholarship, it seemed, above all, the only way to give to the scholarship of our time a chance for the complete demonstration of its productive energies.

The plan of unrelated congresses, with chance combinations of papers prepared at random, was therefore definitively replaced by the plan of only one representative gathering, bound together by one underlying thought, given thus the unity of one scholarly aim, whose fulfillment is demanded by the scientific needs of our time, and is hardly to be reached by other methods. Every arbitrary and individual choice was then to be eliminated and every effort was to be controlled by the one central purpose; the work thus to be organized and prepared with the same carefulness of adjustment and elaboration which was doubtless to be applied in the admirable exhibitions of the United States Government or in the art exhibition. The open question was, of course, what topic could fulfill these various demands most completely; wherein lay the greatest scholarly need of our time; what task could be least realized by the casual efforts of scholarship at random; where was the unity of a world organization most needed?

One thought was very naturally suggested by the external circumstances. St. Louis had asked the nations of the world to a celebration of the Louisiana Purchase. Historical thoughts thus gave meaning and importance to the whole undertaking. The pride of one century's development had stimulated the gigantic work from its inception. An immense territory had been transformed from a half wilderness into a land with a rich civilization, and with a central city in which eight thousand factories are at work. No thought lay nearer than to ask how far this century was of similar importance for the changes in the world of thought. How have the sciences developed themselves since the days of the Louisiana Purchase? That is a topic which with complete uniformity might be asked from every special science, and which might thus offer a certain unity of aim to scholars of all scientific denominations. There was indeed no doubt that such an historical question would have to be raised if we were to live up to the commemorative idea of the whole Fair. And yet it seemed still more certain that the retrospective problem did not justify itself as a central topic for a World's Congress. There were sciences for which the story of the last hundred years was merely the last chapter of a history of three thousand years and other sciences whose life history did not begin until one or two decades ago. It would thus be a very external uniformity; the question would have a very different meaning for the various branches of knowledge, and the treatment would be of very unequal interest and importance. More than that, it would not abolish the unrelated character of the endeavors; while the same topic might be given everywhere, yet every science would remain isolated; there would be no internal unity, and thus no inner reason for bringing together the best workers of all spheres. And finally the mere retrospective attitude brings with it the depressing mood of perfunctory activity. Certainly to look back on the advance of a century can be most suggestive for a better understanding of the way which lies before us; and we felt indeed that the occasion for such a backward glance ought not to be missed. Yet there would be something lifeless if the whole meeting were devoted to the consideration of work that had been completed; a kind of necrological sentiment would pervade the whole ceremony, while our chief aim was to serve the progress of knowledge and thus to stimulate living interests.

This language of life spoke indeed in the programme of another plan which seemed also to be suggested by the character of the Exposition. The St. Louis Fair desired not merely to look backward and to revive the historical interest in the Louisiana Purchase, but its first aim seemed to be to bring into sharp relief the factors which serve to-day the practical welfare and the achievements of human society. If all the scholars of all sciences were to convene under one flag, would it not thus seem most harmonious with the occasion, if, as the one controlling topic, the question were proposed, "What does your science contribute to the practical progress of mankind?" No one can deny that such a formulation would fit in well with the lingering thoughts of every World's Fair visitor. Whoever wanders through the aisles of exhibition palaces and sees amassed the marvelous achievements of industry and commerce, and the thousand practical arts of modern society, may indeed turn most naturally to a gathering of scholars with the question, "What have you to offer of similar import?" All your thinking and speaking and writing, are they merely words on words, or do you also turn the wheels of this gigantic civilization?

Such a question would give a noble opening indeed to almost every science. Who would say that the opportunity is confined to the man of technical science? Does not the biologist also prepare the achievements of modern medicine, does not the mathematician play his most important rôle in our mastery over stubborn nature, do we not need language for our social intercourse, and law and religion for our practical social improvement? Yes, is there any science which has not directly or indirectly something to contribute to the practical development of the modern man and his civilization? All this is true, and yet the perspective of this truth, too, appears at once utterly distorted if we take the standpoint of science itself. The one end of knowledge is to reach the truth. The belief in the absolute value of truth gives to it meaning and significance. This value remains the controlling influence even where the problem to be solved is itself a practical one, and the spirit of science remains thus essentially theoretical even in the so-called applied sciences. But incomparably more intense in that respect is the spirit of all theoretical disciplines. Philosophy and mathematics, history and philology, chemistry and biology, astronomy and geology, may be and ought to be helpful to practical civilization everywhere; and every step forward which they take will be an advance for man's practical life too. And yet their real meaning never lies in their technical by-product. It is not the scholar who peers in the direction of practical use who is most loyal to the deepest demand of scholarship, and every relation to practical achievement is more or less accidental or even artificial for the real life interests of productive scholarship.

But if the contrast between his real intention and his social technical successes may not appear striking to the physicist or chemist, it would appear at least embarrassing to the scholars in many other departments and directly bewildering to not a few. Perhaps two thirds of the sciences to which the best thinkers of our time are faithfully devoted would then be grouped together and relegated to a distant corner, their only practical technical function would be to contribute material to the education of the cultured man. For what else do we study Sanscrit or medieval history or epistemology? And finally even the uniform topic of practical use would not have brought the different sciences nearer to each other; the Congress would still have remained a budget of disconnected records of scholarship. If the practical side of the Exposition was to suggest anything, it should then not be more than an appeal not to overlook the importance of the applied sciences which too often play the rôle of a mere appendix to the system of knowledge. The logical one-sidedness which considers practical needs as below the dignity of pure science was indeed to be excluded, but to choose practical service as the one controlling topic would be far more anti-scientific.

2. *The Unity of Knowledge*

There was another side of the Exposition plan which suggested a stronger topic. The World's Fair was not only an historical memorial work, and was not only a show of the practical tools of technical civilization; its deepest aim was after all the effort to bring the energies of our time into inner relation. The peoples of the whole globe, separated by oceans and mountains, by language and custom, by politics and prejudice, were here to come in contact and to be brought into correlation by better mutual understanding of the best features of their respective cultures. The various industries and arts, the most antagonistic efforts of commerce and production, separated by the rivalry of the market and by the diversity of economic interests were here to be brought together in harmony, were to be correlated for the eye of the spectator. It was a near-lying thought to choose correlation as the controlling thought of a scientific World's Congress too. That was the topic which was finally agreed upon: the inner relation of the sciences of our day.

The fitness and the external advantages of such a scheme are evident. First of all, the danger of disconnectedness now disappears completely. If the sciences are to examine what binds them together, their usual isolation must be given up for the time being and a concerted effort must control the day. The bringing together of scholars of all scientific specialties is then no longer a doubtful accidental feature, but becomes a condition of the whole undertaking. More than that, such a topic, with all that it involves, makes it a matter of course that the call goes out to the really leading scholars of the time. To aim at a correlation of sciences means to seek for the fundamental principles in each territory of knowledge and to look with far-seeing eye beyond the limits of its field; but just this excludes from the outset those who like to be the self-appointed speakers in routine gatherings. It excludes from the first the narrow specialist who does not care for anything but for his latest research, and ought to exclude not less the vague spirits who generalize about facts of which they have no concrete substantial knowledge, as their suggestions towards correlation would lack inner productiveness and outer authority. Such a plan has room only for those men who stand high enough to see the whole field and who have yet the full authority of the specialistic investigator; they must combine the concentration on specialized productive work with the inspiration that comes from looking over vast regions. With such a topic the usual question does not come up whether one or another strong man would feel attracted to take part in the gathering, but it would be justified and necessary to confine the active participation from the outset to those who are leaders, and thus to guarantee from the beginning a representation of science equal in dignity to the best efforts of the exhibiting countries in all other departments. In this way such a plan had the advantage of justifying through its topic the administrative desire to bring all sciences to the same spot, and at the same time of excluding all participants but the best scholars: with isolated gatherings or with second-rate men, this subject would have been simply impossible.

Yet all these halfway external advantages count little compared with the significance and importance of the topic for the inner life of scientific thought of our time. We all felt it was the one topic which the beginning of the twentieth century demanded and which could not be dealt with otherwise than by the combined labors of all nations and of all sciences. The World's Fair was the one great opportunity to make a first effort in this direction; we had no right to miss this opportunity. Thus it was decided to have a congress with the definite purpose of working towards the unity of human knowledge, and with the one mission, in this time of scattered specializing work, of bringing to the consciousness of the world the too-much neglected idea of the unity of truth. To quote from our first tentative programme: "Let the rush of the world's work stop for one moment for us to consider what are the underlying principles, what are their relations to one another and to the whole, what are their values and purposes; in short, let us for once give to the world's sciences a holiday. The workaday functions are much better fulfilled in separation, when each scholar works in his own laboratory or in his library; but this holiday task of bringing out the underlying unity, this synthetic work, this demands really the coöperation of all, this demands that once at least all sciences come together in one place at one time."

Yet if our work stands for the unity of knowledge, aims to consider the fundamental conceptions which bind together all the specialistic results, and seeks to inquire into the methods which are common to various fields, all this is after all merely a symptom of the whole spirit of our times. A reaction against the narrowness of mere fact-diggers has set in. A mere heaping up of disconnected, unshaped facts begins to disappoint the world; it is felt too vividly that a mere dictionary of phenomena, of events and laws, makes our knowledge larger but not deeper, makes our life more complex but not more valuable, makes our science more difficult but not more harmonious. Our time longs for a new synthesis and looks towards science no longer merely with a desire for technical prescriptions and new inventions in the interest of comfort and exchange. It waits for knowledge to fulfill its higher mission, it waits for science to satisfy our higher needs for a view of the world which shall give unity to our scattered experience. The indications of this change are visible to every one who observes the gradual turning to philosophical discussion in the most different fields of scientific life.

When after the first third of the nineteenth century the great philosophic movement which found its climax in Hegelianism came to disaster in consequence of its absurd neglect of hard solid facts, the era of naturalism began its triumph with contempt for all philosophy and for all deeper unity. Idealism and philosophy were stigmatized as the enemies of true science and natural science had its great day. The rapid progress of physics and chemistry fascinated the world and produced modern technique; the sciences of life, physiology, biology, medicine, followed; and the scientific method was carried over from body to mind, and gave us at the end of the nineteenth century modern psychology and sociology. The lifeless and the living, the physical and the mental, the individual and the social, all had been conquered by analytical methods. But just when the climax was reached and all had been analyzed and explained, the time was ripe for disillusion, and the lack of deeper unity began to be felt with alarm in every quarter. For seventy years there had been nowhere so much philosophizing going on as suddenly sprung up among the scientists of the last decade. The physicists and the mathematicians, the chemists and the biologists, the geologists and the astronomers, and, on the other side, the historians and the economists, the psychologists and the sociologists, the jurists and the theologians — all suddenly found themselves again in the midst of discussions on fundamental principles and methods, on general categories and conditions of knowledge, in short, in the midst of the despised philosophy. And with those discussions has come the demand for correlation. Everywhere have arisen leaders who have brought unconnected sciences together and emphasized the unity of large divisions. The time seems to have come again when the wave of naturalism and realism is ebbing, and a new idealistic philosophical tide is swelling, just as they have always alternated in the civilization of two thousand years.

No one dreams, of course, that the great synthetic apperception, for which our modern time seems ripe, will come through the delivery of some hundred addresses, or the discussions of some hundred audiences. An ultimate unity demands the gigantic thought of a single genius, and the work of the many can, after all, be merely the preparation for the final work of the one. And yet history shows that the one will never come if the many have not done their share. What is needed is to fill the sciences of our time with the growing consciousness of belonging together, with the longing for fundamental principles, with the conviction that the desire for correlation is not the fancy of dreamers, but the immediate need of the leaders of thought. And in this preparatory work the St. Louis Congress of Arts and Science seemed indeed called for an important part when it was committed to this topic of correlation.

To call the scholars of the world together for concerted action towards the correlation of knowledge meant, of course, first of all, to work out a detailed programme, and to select the best authorities for every special part of the whole scheme. Nothing could be left to chance methods and to casual contributions. The preparation needed the same administrative strictness which would be demanded for an encyclopedia, and the same scholarly thoroughness which would be demanded for the most scientific research. A plan was to be devised in which every possible striving for truth would find its place, and in which every section would have its definite position in the system. And such a ground-plan given, topics were to be assigned to every department and sub-department, the treatment of which would bring out the fundamental principles and the inner relations in such a way that the papers would finally form a close-woven intellectual fabric. There would be plenty of room for a retrospective glance at the historical development of the sciences and plenty of room for emphasis on their practical achievements; but the central place would always belong to the effort towards unity and internal harmonization.

We thus divided human knowledge into large parts, and the parts into divisions, and the divisions into departments, and the departments into sections. As the topic of the general divisions — we proposed seven of them — it was decided to discuss the Unity of the whole field. As topic for the departments — we had twenty-four of them — the addresses were to discuss the fundamental Conceptions and Methods and the Progress during the last century; and in the sections, finally — our plan provided for one hundred and twenty-eight of them — the topics were in every one the Relation of the special branch to other branches, and those most important Present Problems which are essential for the deeper principles of the special field. In this way the ground-plan itself suggested the unity of the practically separated sciences; and, moreover, our plan provided from the first that this logical relation should express itself externally in the time order of the work. We were to begin with the meetings of the large divisions, the meetings of the departments were to follow, and the meetings of the sections and their ramifications would follow the departmental gatherings.

3. *The Objections to the Plan*

It was evident that even the most modest success of that gigantic undertaking depended upon the right choice of speakers, upon the value of the ground-plan, and upon many external conditions; thus no one was in doubt as to the difficulty in realizing such a scheme. Yet there were from the scholarly side itself objections to the principles involved, objections which might hold even if those other conditions were successfully met. The most immediate reason for reluctance lies in the specializing tendencies of our time. Those who devote all their working energy as loyal sons of our analyzing period of science to the minute detail of research come easily into the habit of a nervous fear with regard to any wider general outlook. The man of research sees too often how ignorance hides itself behind generalities. He knows too well how much easier it is to formulate vague generalities than to contribute a new fact to human knowledge, and how often untrained youngsters succeed with popular text-books which are rightly forgotten the next day. Methodical science must thus almost encourage this aversion to any deviation from the path of painstaking specialistic labor. Then, of course, it seems almost a scientific duty to declare war against an undertaking which explicitly asks everywhere for the wide perspectives and the last principles, and does not aim at adding at this moment to the mere treasury of information.

But such a view is utterly one-sided, and to fight against such one-sidedness and to overcome the specializing narrowness of the scattered sciences was the one central idea of the plan. If there existed no scholars who despise the philosophizing connection, there would have hardly been any need for this whole undertaking; but to yield to such philosophy-phobia means to declare the analytic movement of science permanent, and to postpone a synthetic movement indefinitely. Our time has just to emphasize, and the leaders of thought daily emphasize it more, that a mere heaping up of information can be merely a preparation for knowledge, and that the final aim is a *Weltanschauung*, a unified view of the whole of reality. All that our Congress had to secure was thus merely that the generalizing discussion of principles should not be left to men who generalized because they lacked the substantial knowledge which is necessary to specialize. The thinkers we needed were those who through specialistic work were themselves led to a point where the discussion of general principles becomes unavoidable. Our plan was by no means antagonistic to the patient labors of analysis; the aim was merely to overcome its one-sidedness and to stimulate the synthesis as a necessary supplement.

But the objections against a generalizing plan were not confined to the mistaken fear that we sought to antagonize the productive work of the specialist. They not seldom took the form of a general aversion to the logical side of the ground-plan. It was often said that such a scheme has after all interest only for the logician, for whom science as such is an object of study, and who must thus indeed classify the sciences and determine their logical relation. The real scientist, it was said, does not care for such methodological operations, and should be suspicious from the first of such philosophical high-handedness. The scientist cannot forget how often in the history of civilization science was the loser when it trusted its problems to the metaphysical thinker who substituted his lofty speculations for the hard work of the investigator. The true scholar will thus not only object to generalizing "commonplaces" as against solid information, but he will object as well to logical demarcation lines and systematization as against the practical scientific work which does not want to be hampered by such philosophical subtleties. Yet all these fears and suspicions were still more mistaken.

Nothing was further from our intentions than a substitution of metaphysics for concrete science. It was not by chance that we took such pains to find the best specialists for every section. No one was invited to enter into logical discussions and to consider the relations of science merely from a dialectic point of view. The topic was everywhere the whole living manifoldness of actual relations, and the logician had nothing else to do than to prepare the programme. The outlines of the programme demanded, of course, a certain logical scheme. If hundreds of sciences are to take part, they have to be grouped somehow, if a merely alphabetical order is not adopted; and even if we were to proceed alphabetically, we should have to decide beforehand what part of knowledge is to be recognized as a special science. But the logical order of the ground-plan refers, of course, merely to the simple relation of coördination, subordination, and superordination, and the logician is satisfied with such a classification. But the endless variety of internal relations is no longer to be dealt with from the point of view of mere logic. We may work out the ground-plan in such a way that we understand that logically zoölogy is coördinated to botany and subordinated to mechanics and superordinated to ichthyology; but this minimum of determination gives, of course, not even a hint of that world of relations which exists from the standpoint of the biologist between the science of zoölogy and the science of botany, or between the biological and the mechanical studies. To discuss these relations of real scientific life is the work of the biologist and not at all of the logician.

The foregoing answers also at once an objection which might seem more justified at the first glance. It has been said that we were undertaking the work of bringing about a synthesis of scientific endeavors, and that we yet had that synthesis already completed in the programme on which the work was to be based. The scholars to be invited would be bound by the programme, and would therefore have no other possibility than to say with more words what the programme had settled beforehand. The whole effort would then seem determined from the start by the arbitrariness of the proposed ground-plan. Now it cannot be denied indeed that a certain factor of arbitrariness has to enter into a programme. We have already referred to the fact that some one must decide beforehand what fraction of science is to be acknowledged as a self-dependent discipline. If a biologist were to work out the scheme, he might decide that the whole of philosophy was just one science; while the philosopher might claim a large number of sections for logic and ethics and philosophy of religion, and so on. And the philosopher, on the other hand, might treat the whole of medicine as one part in itself, while the physician might hold that even otology has to be separated from rhinology. A certain subjectivity of standpoint is unavoidable, and we know very well that instead of the one hundred and twenty-eight sections of our programme we might have been satisfied with half that number or might have indulged in double that number. And yet there was no possible plan which would have allowed us to invite the speakers without defining beforehand the sectional field which each was to represent. A certain courage of opinion was then necessary, and sometimes also a certain adjustment to external conditions.

Quite similar was the question of classification. Just as we had to take the responsibility for the staking-out of every section, we had also to decide in favor of a certain grouping, if we desired to organize the Congress and not simply to bring out haphazard results. The principles which are sufficient for a mere directory would never allow the shaping of a programme which can be the basis for synthetic work. Even a university catalogue begins with a certain classification, and yet no one fancies that such catalogue grouping inhibits the freedom of the university lecturer. It is easy to say, as has been said, that the essential trait of the scientific life of to-day is its live-and-let-live character. Certainly it is. In the regular work in our libraries and laboratories the year round, everything depends upon this democratic freedom in which every one goes his own way, hardly asking what his neighbor is doing. It is that which has made the specialistic sciences of our day as strong as they are. But it has brought about at the same time this extreme tendency to unrelated specialization with its discouraging lack of unity; this heaping up of information without an outer harmonious view of the world; and if we were really at least once to satisfy the desire for unity, then we had not the right to yield fully to this live-and-let-live tendency. Therefore some principle of grouping had to be accepted, and whatever principle had been chosen, it would certainly have been open to the criticism that it was a product of arbitrary decision, inasmuch as other principles might have been possible.

A classification which in itself expresses all the practical relations in which sciences stand to each other is, of course, absolutely impossible. A programme which should try to arrange the place of a special discipline in such a way that it would become the neighbor of all those other sciences with which it has internal relation is unthinkable. On the other hand, only if we had tried to construct a scheme of such exaggerated ambitions should we have been really guilty of anticipating a part of that which the specialistic scholars were to tell us. The Congress had to leave it to the invited participants to discuss the totality of relations which practically exist between their fields and others, and the organizers confined themselves to that minimum of classification which just indicates the pure logical relations, a minimum which every editor of encyclopedic work would be asked to initiate without awakening suspicions of interference with the ideas of his contributors.

The only justified demand which could be met was that a system of division and classification should be proposed which should give fair play to every existing scientific tendency. The minimum of classification was to be combined with the maximum of freedom, and to secure that a careful consideration of principles was indeed necessary. To bring logical order into the sciences which stand out clearly with traditional rights is not difficult; but the chances are too great that certain tendencies of thought might fail to find recognition or might be suppressed by scientific prejudice. Any serious omission would indeed have necessarily inhibited the freedom of expression. To secure thus the greatest inner fullness of the programme, seemed indeed the most important task in the elaboration of the ground-plan. The fears that we might offer empty generalization instead of scholarly facts, or that we might simply heap up encyclopedic information instead of gaining wide perspectives, or that we might interfere with the living connections of sciences by the logical demarcation lines, or that we might disturb the scholar in his freedom by determining beforehand his place in the classification, — all these fears and objections, which were repeatedly raised when the plan was first proposed, seemed indeed unimportant compared with the fear that the programme might be unable to include all scientific tendencies of the time.

That would have been, indeed, the one fundamental mistake, as the whole Congress work was planned in the service of the great synthetic movement which pervades the intellectual life of to-day. The undertaking would be useless and even hindering if it were not just the newer and deeper tendencies that came to most complete expression in it. Everything depended, therefore, upon the fullest possible representation of scientific endeavors in the plan. But no one can become aware of this manifoldness and of the logical relations who does not go back to the ultimate principles of the human search for truth. We have, therefore, to enter now into a full discussion of the principles which have controlled the classification and subdivision of the whole work. The discussion is necessarily in its essence a philosophical one, as it was earlier made plain that philosophy must lay out the plan, while in the realization of the plan through concrete work the scientist alone, and not the logician, has to speak. Yet here again it may be said that while our discussion of principles in its essence is logical, in another respect it is a merely historical account. The question is not what principles of classification are to be acknowledged as valuable now that the work of the Congress lies behind us, but what principles were accepted and really led to the organization of the work in that form in which it presents itself in the records of the following volumes.

II

THE CLASSIFICATION OF THE SCIENCES

1. *The Development of Classification*

The problem of dividing and subdividing the whole of human knowledge and of thus bringing order into the manifoldness of scientific efforts has fascinated the leading thinkers of all ages. It may often be difficult to say how far the new principles of classification themselves open the way for new scientific progress and how far the great forward movements of thought in the special sciences have in turn influenced the principles of classification. In any case every productive age has demanded the expression of its deepest energy in a new ordering of human science. The history of these efforts leads from Plato and Aristotle to Bacon and Locke, to Bentham and Ampère, to Kant and Hegel, to Comte and Spencer, to Wundt and Windelband. And yet we can hardly speak of a real historical continuity. In a certain way every period took up the problem anew, and the new aspects resulted not only from the development of the sciences themselves which were to be classified, but still more from the differences of logical interest. Sometimes the classification referred to the material, sometimes to the method of treatment, sometimes to the mental energies involved, and sometimes to the ends to be reached. The reference to the mental faculties was certainly the earliest method of bringing order into human knowledge, for the distinction of the Platonic philosophy between dialectics, physics, and ethics pointed to the threefold character of the mind, to reason, perception, and desire; and it was on the threshold of the modern time, again, when Bacon divided the intellectual globe into three large parts according to three fundamental psychical faculties: memory, imagination, and reason. The memory gives us history; the imagination, poetry; the reason, philosophy, or the sciences. History was further divided into natural and civil history; natural history into normal, abnormal, and artificial phenomena; civil history into political, literary, and ecclesiastical history. The field of reason was subdivided into man, nature, and God; the domain of man gives, first, civil philosophy, parted off into intercourse, business, and government, and secondly, the philosophy of humanity, divided into that of body and of soul, wherein medicine and athletics belong to the body, logic and ethics to the soul. Nature, on the other hand, was divided into speculative and applied science, — the speculative containing both physics and metaphysics; the applied, mechanics and magic. All this was full of artificial constructions, and yet still more marked by deep insight into the needs of Bacon's time, and not every modification of later classifiers was logically a step forward.

Yet modern efforts had to seek quite different methods, and the energies which have been most effective for the ordering of knowledge in the last decades spring unquestionably from the system of Comte and his successors. He did not aim at a system of ramifications; his problem was to show how the fundamental sciences depend on each other. A series was to be constructed in which each member should presuppose the foregoing. The result was a simplicity which is certainly tempting, but this simplicity was reached only by an artificial emphasis which corresponded completely to the one-sidedness of naturalistic thought. It was a philosophy of positivism, the background for the gigantic work of natural science and technique in the last two thirds of the nineteenth century. Comte's fundamental thought is that the science of Morals, in which we study human nature for the government of human life, is dependent on sociology. Sociology, however, depends on biology; this on chemistry; this on physics; this on astronomy; and this finally on mathematics. In this way, all mental and moral sciences, history and philology, jurisprudence and theology, economics and politics, are considered as sociological phenomena, as dealing with functions of the human being. But as man is a living organism, and thus certainly falls under biology, all the branches of knowledge from history to ethics, from jurisprudence to æsthetics, can be nothing but subdivisions of biology. The living organism, on the other hand, is merely one type of the physical bodies on earth, and biology is thus itself merely a department of physics. But as the earthly bodies are merely a part of the cosmic totality, physics is thus a part of astronomy; and as the whole universe is controlled by mathematical laws, mathematics must be superordinated to all sciences.

But there followed a time which overcame this thinly disguised example of materialism. It was a time when the categories of the physiologist lost slightly in credit and the categories of the psychologist won repute. This newer movement held that it is artificial to consider ethical and logical life, historic and legal action, literary and religious emotions, merely as physiological functions of the living organism. The mental life, however necessarily connected with brain processes, has a positive reality of its own. The psychical facts represent a world of phenomena which in its nature is absolutely different from that of material phenomena, and, while it is true that every ethical action and every logical thought can, from the standpoint of the biologist, be considered as a property of matter, it is not less true that the sciences of mental phenomena, considered impartially, form a sphere of knowledge closed in itself, and must thus be coördinated, not subordinated, to the knowledge of the physical world. We should say thus: all knowledge falls into two classes, the physical sciences and the mental sciences. In the circle of physical sciences we have the general sciences, like physics and chemistry, the particular sciences of special objects, like astronomy, geology, mineralogy, biology, and the formal sciences, like mathematics. In the circle of mental sciences we have correspondingly, as a general science, psychology, and as the particular sciences all those special mental and moral sciences which deal with man's inner life, like history or jurisprudence, logic or ethics, and all the rest. Such a classification, which had its philosophical defenders about twenty years ago, penetrated the popular thought as fully as the positivism of the foregoing generation, and was certainly superior to its materialistic forerunner.

Of course it was not the first time in the history of civilization that materialism was replaced by dualism, that biologism was replaced by psychologism; and it was also not the first time that the development of civilization led again beyond this point: that is, led beyond the psychologizing period. There is no doubt that our time presses on, with all its powerful internal energies, away from this *Weltanschauung* of yesterday. The materialism was anti-philosophic, the psychological dualism was unphilosophic. To-day the philosophical movement has set in. The one-sidedness of the nineteenth century creed is felt in the deeper thought all over the world: popular movements and scholarly efforts alike show the signs of a coming idealism, which has something better and deeper to say than merely that our life is a series of causal phenomena. Our time longs for a new interpretation of reality; from the depths of every science wherein for decades philosophizing was despised, the best scholars turn again to a discussion of fundamental conceptions and general principles. Historical thinking begins again to take the leadership which for half a century belonged to naturalistic thinking; specialistic research demands increasingly from day to day the readjustment toward higher unities, and the technical progress which charmed the world becomes more and more simply a factor in an ideal progress. The appearance of this unifying congress itself is merely one of a thousand symptoms of this change appearing in our public life, and if the scientific philosophy is producing to-day book upon book to prove that the world of phenomena must be supplemented by the world of values, that description must yield to interpretation, and that explanation must be harmonized with appreciation: it is but echoing in technical terms the one great emotion of our time.

This certainly does not mean that any step of the gigantic materialistic, technical, and psychological development will be reversed, or that progress in any one of these directions ought to cease. On the contrary, no time was ever more ready to put its immense energies into the service of naturalistic work; but it does mean that our time recognizes the one-sidedness of these movements, recognizes that they belong only to one aspect of reality, and that another aspect is possible; yes, that the other aspect is that of our immediate life, with its purposes and its ideals, its historical relations and its logical aims. The claim of materialism, that all psychical facts are merely functions of the organism, was no argument against psychology, because, though the biological view was possible, yet the other aspect is certainly a necessary supplement. In the same way it is no argument against the newer view that all purposes and ideals, all historical actions and logical thoughts, can be considered as psychological phenomena. Of course we can consider them as such, and we must go on doing so in the service of the psychological and sociological sciences; but we ought not to imagine that we have expressed and understood the real character of our historical or moral, our logical or religious life when we have described and explained it as a series of phenomena. Its immediate reality expresses itself above all in the fact that it has a meaning, that it is a purpose which we want to understand, not by considering its causes and effects, but by interpreting its aims and appreciating its ideals.

We should say, therefore, to-day that it is most interesting and important for the scientist to consider human life with all its strivings and creations from a biological, psychological, sociological point of view; that is, to consider it as a system of causal phenomena; and many problems worthy of the highest energies have still to be solved in these sciences. But that which the jurist or the theologian, the student of art or of history, of literature or of politics, of education or of morality, is dealing with, refers to the other aspect in which inner life is not a phenomenon but a system of purposes, not to be explained but to be interpreted, to be approached not by causal but by teleological methods. In this case the historical sciences are no longer sub-sections of psychological or of sociological sciences; the conception of science is no longer identical with the conception of the science of phenomena. There exist sciences which do not deal with the description or explanation of phenomena at all, but with the internal relation and connection, the interpretation and appreciation of purpose. In this way modern thought demands that sciences of purpose be coördinated with sciences of phenomena. Only if all these tendencies of our time are fully acknowledged can the outer framework of our classification offer a fair field to every scientific thought, while a positivistic system would cripple the most promising tendencies of the twentieth century.

2. *The Four Theoretical Divisions*

We have first to determine the underlying structure of the classification, that is, we have to seek the chief Divisions, of which our plan shows seven; four theoretical and three practical ones. It will be a secondary task to subdivide them later into the 24 Departments and 128 Sections. We desire to divide the whole of knowledge in a fundamental way, and we must therefore start with the question of principle: — what is knowledge? This question belongs to epistemology, and thus falls, indeed, into the domain of philosophy. The positivist is easily inclined to substitute for the philosophical problem the empirical question: how did that which we call knowledge grow and develop itself in our individual mind, or in the mind of the nations? The question becomes, then, of course, one which must be answered by psychology, by sociology, and perhaps by biology. Such genetic inquiries are certainly very important, and the problem of how the processes of judging and conceiving and thinking are produced in the individual or social consciousness, and how they are to be explained through physical and psychical causes, deserves fullest attention. But its solution cannot even help us as regards the fundamental problem, what we mean by knowledge, and what the ultimate value of knowledge may be, and why we seek it. This deeper logical inquiry must be answered somehow before those genetic studies of the psychological and the sociological positivists can claim any truth at all, and thus any value, for their outcome. To explain our present knowledge genetically from its foregoing causes means merely to connect the present experience, which we know, with a past experience, which we remember, or with earlier phenomena which we construct on the basis of theories and hypotheses; but in any case with facts which we value as parts of our knowledge and which thus presuppose the acknowledgment of the value of knowledge. We cannot determine by linking one part of knowledge with another part of knowledge whether we have a right to speak of knowledge at all and to rely on it.

We can thus not start from the childhood of man, or from the beginning of humanity, or from any other object of knowledge, but we must begin with the state which logically precedes all knowledge; that is, with our immediate experience of real life. Here, in the naïve experience in which we do not know ourselves as objects which we perceive, but where we feel ourselves in our subjective attitudes as agents of will, as personalities, here we find the original reality not yet shaped and remoulded by scientific conceptions and by the demands of knowledge. And from this basis of primary, naïve reality we must ask ourselves what we mean by seeking knowledge, and how this demand of ours is different from the other activities in which we work out the meaning and the ideals of our life.

One thing is certain, we cannot go back to the old dogmatic standpoint, whether rationalistic or sensualistic. In both cases dogmatism took for granted that there is a real world of things which exist in themselves independent of our subjective attitudes, and that our knowledge has to give us a mirror picture of that self-dependent world. Sensualism averred that we get this knowledge through our perceptions; rationalism, that we get it by reasoning. The one asserted that experience gives us the data which mere abstract reasoning can never supply; the other asserted that our knowledge speaks of necessity which no mere perception can find out. Our modern time has gone through the school of philosophical criticism, and the dogmatic ideas have lost for us their meaning. We know that the world which we think as independent cannot be independent of the forms of our thinking, and that no science has reference to any other world than the world which is determined by the categories of our apperception. There cannot be anything more real than the immediate pure experience, and if we seek the truth of knowledge, we do not set out to discover something which is hidden behind our experience, but we set out simply to make something out of our experience which satisfies certain demands. Our immediate experience does not contain an objective thing and a subjective picture of it, but they are completely one and the same piece of experience. We have the object of our immediate knowledge not in the double form of an outer object independent of ourselves and an idea in us, but we have it as our object there in the practical world before science for its special purposes has broken up that bit of reality into the physical material thing and the psychical content of consciousness. And if this doubleness does not hold for the immediate reality of pure experience, it cannot enter through that reshaping and reconstructing and connecting and interpreting of pure experience which we call our knowledge. All that science gives to us is just such an endlessly enlarged experience, of which every particle remains objective and independent, inasmuch as it is not in us as psychical individuals, while yet completely dependent upon the forms of our subjective experience. The ideal of truth is thus not to gain by reason or by observation ideas in ourselves which correspond as well as possible to absolute things, but to reconstruct the given experience in the service of certain purposes. Everything which completely fulfills the purposes of this intentional reconstruction is true.

What are these purposes? One thing is clear from the first: There cannot be a purpose where there is not a will. If we come from pure experience to knowledge by a purposive transformation, we must acknowledge the reality of will in ourselves, or rather, we must find ourselves as will in the midst of pure experience before we reach any knowledge. And so it is indeed. We can abstract from all those reconstructions which the sciences suggest to us and go back to the most immediate naïve experience; but we can never reach an experience which does not contain the doubleness of subject and object, of will and world. That doubleness has nothing whatever to do with the difference of physical and psychical; both the physical thing and the psychical idea are objects. The antithesis is not that between two kinds of objects, since we have seen that in the immediate experience the objects are not at all split up into the two groups of material and mental things; it is rather the antithesis between the object in its undifferentiated state on the one side and the subject in its will-attitude on the other side. Yes, even if we speak of the subject which stands as a unity behind the will-attitudes, we are already reconstructing the real experience in the interest of the purposes of knowledge. In the immediate experience, we have the will-attitudes themselves, and not a subject which wills them.

If we ask ourselves finally what is then the ultimate difference between those two elements of our pure experience, between the object and the will-attitude, we stand before the ultimate data: we call that element which exists merely through a reference to its opposite, the object, and we call that element of our experience which is complete in itself, the attitude of the will. If we experienced liking or disliking, affirming or denying, approving or disapproving in the same way in which we experience the red and the green, the sweet and the sour, the rock and the tree and the moon, we should know objects only. But we do experience them in quite a different way. The rock and the tree do not point to anything else, but the approval has no reality if it does not point to its opposition in disapproval, and the denial has no meaning if it is not meant in relation to the affirmative. This doubleness of our primary experience, this having of objects and of antagonistic attitudes must be acknowledged wherever we speak of experience at all. We know no object without attitude, and no attitude without object. The two are one state; object and attitude form a unity which we resolve by the different way in which we experience these two features of the one state: we find the object and we live through the attitude. It is a different kind of awareness, the having of the object and the taking of the attitude. In real life our will is never an object which we simply perceive. The psychologist may treat the will as such, but in the immediate experience of real life, we are certain of our action by doing it and not by perceiving our doing; and this our performing and rejecting is really our self which we posit as absolute reality, not by knowing it, but by willing it. This cornerstone of the Fichtean philosophy was forgotten throughout the uncritical and unphilosophical decades of a mere naturalistic age. But our time has finally come to give attention to it again.

Our pure experience thus contains will-attitudes and objects of will, and the different attitudes of the will give the fundamental classes of human activity. We can easily recognize four different types of will-relation towards the world. Our will submits itself to the world; our will approves the world as it is; our will approves the changes in the world; our will transcends the world. Yet we must make at once one more most important discrimination. We have up to this point simplified our pure experience too much. It is not true that we experience only objects and our own will-attitudes. Our will reaches out not only to objects, but also to other subjects. In our most immediate experience, not reshaped at all by theoretical science, our will is in agreement or disagreement with other wills; tries to influence them, and receives influences and suggestions from them. The pseudo-philosophy of naturalism must say of course that the will does not stand in any direct relation to another will, but that the other persons are for us simply material objects which we perceive, like other objects, and into which we project mental phenomena like those which we find in ourselves by the mere conclusion of analogy. But the complex reconstructions of physiological psychology are therein substituted for the primary experience. If we have to express the agreement or disagreement of wills in the terms of causal science, we may indeed be obliged to transform the real experience into such artificial constructions; but in our immediate consciousness, and thus at the starting-point of our theory of knowledge, we have certainly to acknowledge that we understand the other person, accept or do not accept his suggestion, agree or disagree with him, before we know anything of a difference between physical and mental objects.

We cannot agree with an object. We agree directly with a will, which does not come to us as a foreign phenomenon, but as a proposition which we accept or decline. In our immediate experience will thus reaches will, and we are aware of the difference between our will-attitude as merely individual and our will-attitude as act of agreement with the will-attitude of other individuals. We can go still further. The circle of other individuals whose will we express in our own will-act may be narrow or wide, may be our friends or the nation, and this relation clearly constitutes the historical significance of our attitude. In the one case our act is a merely personal choice for personal purposes without any general meaning; in the other case it is the expression of general tendencies and historical movements. Yet our will-decisions can have connections still wider than those with our social community or our nation, or even with all living men of to-day.

It can seek a relation to the totality of those whom we aim to acknowledge as real subjects. It thus becomes independent of the chance experience of this or that man, or this or that movement, which appeals to us, but involves in an independent way the reference to every one who is to be acknowledged as a subject at all. Such reference, which is no longer bound to any special group of historical individuals, thus becomes strictly over-individual. We can then discriminate three stages: our merely individual will; secondly, our will as bound by other historical individuals; and thirdly, our over-individual will, which is not influenced by any special individual, but by the general demands for the idea of a personality.

Each of those four great types of will-attitude which we insisted on — that is, of submitting, of approving the given, of approving change, and of transcending — can be carried out on these three stages, that is, as individual act, as historical act, and as over-individual act. And we may say at once that only if we submit and approve and change and transcend in an over-individual act, do we have Truth and Beauty and Morality and Conviction. If we approve, for instance, a given experience in an individual will-act, we have simply personal enjoyment and its object is simply agreeable; if we approve it in harmony with other individuals, we reach a higher attitude, yet one which cannot claim absolute value, as it is dependent on historical considerations and on the tastes and desires of a special group or a school or a nation or an age. But if we approve the given object just as it is in an over-individual will-act, then we have before us a thing of beauty, whose value is not dependent upon our personal enjoyment as individuals, but is demanded as a joy forever, by every one whom we acknowledge at all as a complete subject. In exactly the same way, we may approve a change in the world from any individual point of view: we have then to do with technical, practical achievements; or we may approve it in agreement with others: we then enter into the historical interests of our time. Or we may approve it, finally, in an over-individual way, without any reference to any special personality: then only is it valuable for all time, then only is it morally good. And if our will is transcending experience in an individual way, it can again claim no more than a subjective satisfaction furnished by any superstition or hope. But if the transcending will is over-individual, it reaches the absolute values of religion and metaphysics.

Exactly the same differences, finally, must occur when our will submits itself to experience. This submission may be, again, an individual decision for individual purposes; no absolute value belongs to it. Or it may be again a yielding to the suggestions of other individuals; or it may, finally, again be an over-individual submission, which seeks no longer a personal interest. This submission is not to the authority of others, and is without reference to any individual; we assume that every one who is to share with us our world of experience has to share this submission too. That alone is a submission to truth, and experience, considered in so far as we submit ourselves to it over-individually, constitutes our knowledge.

The system of knowledge is thus the system of experience with all that is involved in it in so far as it demands submission from our over-individual will, and the classification which we are seeking must be thus a division and subdivision of our over-individual submissions. But the submission itself can be of very different characters and these various types must give the deepest logical principles of scientific classification. To point at once to the fundamental differences: our will acknowledges the demands of other wills and of objects. We cannot live our life — and this is not meant in a biological sense, but, first of all, in a teleological sense — our life becomes meaningless, if our will does not respect the reality of will-demands and of objects of will. Now we have seen that the will which demands our decision may be either the individual will of other subjects or the over-individual will, which belongs to every subject as such and is independent of any individuality. We can say at once that in the same way we are led to acknowledge that the object has partly an over-individual character, that is, necessarily belongs to the world of objects of every possible subject, and partly an individual character, as our personal object. We have thus four large groups of experiences to which we submit ourselves: over-individual will-acts, individual will-acts, over-individual objects, individual objects. They constitute the first four large divisions of our system.

The over-individual will-acts, which are as such teleologically binding for every subject and therefore norms for his will, give us the Normative Sciences. The individual will-acts in the world of historical manifoldness give us the Historical Sciences. The objects, in so far as they belong to every individual, make up the physical world, and thus give us the Physical Sciences; and finally the objects, in so far as they belong to the individual, are the contents of consciousness, and thus give us the Mental Sciences. We have then the demarcation lines of our first four large divisions: the Normative, the Historical, the Physical, and the Mental Sciences. Yet their meaning and method and difference must be characterized more fully. We must understand why we have here to deal with four absolutely different types of scientific systems, why the over-individual objects lead us to general laws and to the determination of the future, while the study of the individual will-acts, for instance, gives us the system of history, which turns merely to the past and does not seek natural laws; and why the study of the norms gives us another kind of system in which neither a causal nor an historical, but a purely logical connection prevails. Yet all these methodological differences result necessarily from the material with which these four different groups of sciences are working.

Let us start again from the consideration of our original logical purpose. We feel ourselves bound and limited in our will by physical things, by psychical contents, by the demands of other subjects, and by norms. The purpose of all our knowledge is to develop completely all that is involved in this bondage. We want to develop in an over-individual way all the obligations for our submission which are necessarily included in the given objects and the given demands of subjects. We start of course everywhere and in every direction from the actual experience, but we expand the experience by seeking those objects and those demands to which, as necessarily following from the immediately given experience, we must also submit. And in thus developing the whole system of submissions, the interpretation of the experience itself becomes transformed: the physicist may perhaps substitute imperceptible atoms for the physical object and the psychologist may substitute sensations for the real idea, and the historian may substitute combinations of influences for the real personality, and the student of norms may substitute combinations of conflicting demands for the one complete duty; yet in every case the substitution is logically necessary and furnishes us what we call truth inasmuch as it is needed to develop the concrete system of our submissions and thus to express our confidence in the order-lines of reality. And each of these substitutions and supplementations becomes, as material of knowledge, itself a part of the world of experience.

3. *The Physical and the Mental Sciences*

The physicist, we said, speaks of the world of objects in so far as they belong to every possible subject, and are material for a merely passive spectator. Of course the pure experience does not offer us anything of that kind. We insisted that the objects of our real life are objects of our will and of our attitudes, and are at the same time undifferentiated into the physical things outside of us and the psychical ideas in us. To reach the abstraction of the physicist, we have thus to cut loose the objects from our will and to separate the over-individual elements from the individual elements. Both transformations are clearly demanded by our logical aims. As to the cutting loose from our will, it means considering the object as if it existed for itself, as if it were a mere passively given material and not a material of our personal interests. But just that is needed. We want to find out how far we have to submit ourselves to the object. If we want to live our life, we must adjust our attitudes to things, and, as we know our will, we must seek to understand the other factor in the complex experience, the object of our will, and we must find out what it involves in itself. But we do not understand the object and the submission which it demands if we do not completely understand its relation to our desires. Our total submission to the thing thus involves our acknowledgment of all that we have to expect from it. And although the real experience is a unity of will and thing, we have thus the most immediate interest in considering what we have to expect from the thing in itself, without reference to our will. That means finding out the effects of the given object with a subject as the passive spectator. We eliminate artificially, therefore, the activity of the subject and construct as presupposition for this circle of knowledge a nowhere existing subject without activity, for which the thing exists merely as a cause of the effects which it produces.

The first step towards natural science is, therefore, to dissolve the real experience into thing and personality; that is, into object and active subject, and to eliminate in an artificial abstraction the activity of the subject, making the object material of merely passive awareness, and related no longer to the will but merely to other objects. It may be more difficult to understand the second step which naturalism has to take before a natural science is possible. It must dissolve the object of will into an over-individual and an individual part and must eliminate the individual. That part of my objects which belongs to me alone is their psychical side; that which belongs to all of us and is the object of ever new experience is the physical object. As a physicist, in the widest sense of the word, I have to ignore the objects in so far as they are my ideas and have to consider the stones and the stars, the inorganic and the organic objects, as they are outside of me, material for every one. The logical purpose of this second abstraction may be perhaps formulated in the following way.

We have seen that the purpose of the study of the objects is to find out what we have to expect from them; that is, to what effects of the given thing we have to submit ourselves in anticipation. The ideal aim is thus to understand completely how present objects and future objects — that is, how causes and effects — are connected. The first stage in such knowledge of causal connections is, of course, the observation of empirical consequences. Our feeling of expectation grows with the regularity of observed succession; yet the ideal aim can never be fulfilled in that way. The mere observation of regularities can help us to reduce a particular case to a frequently observed type, but what we seek to understand is the necessity of the process. Of course we have to formulate laws, and as soon as we acknowledge a special law to be expressive of a necessity, the subsumption of the particular case under the law will satisfy us even if the necessity of the connection is not recognized in the particular case. We are satisfied because the acknowledgment of the law involved all possible cases. But we do not at all feel that we have furnished a real explanation if the law means to us merely a generalization of routine experiences, and if thus no absolute validity is attached to the law. This necessity between cause and effect must thus have its ultimate reason in our own understanding. We must be logically obliged to connect the objects in such a way, and wherever observation seems to contradict that which is logically necessary, we must reshape our idea of the object till the demands of reason are fulfilled. That is, we must substitute for the given object an abstraction which serves the purpose of a logically necessary connection. That demand is clearly not satisfied if we simply group the totality of such causal judgments under the single name, Causality, and designate thus all these judgments as results of a special disposition of the understanding. We never understand why just this cause demands just this effect so long as we rely on such vague and mystical power of our reason to link the world by causality.

But the situation changes at once if we go still further back in the categories of our understanding. While a mere demand for causality never explains what cause is to be linked with what effect, the vagueness disappears when we understand this demand for causality itself as the product of a more fundamental demand for identity. That an object remains identical with itself does not need for us any further interpretation. That is the ultimate presupposition of our thought, and where a complete identity is found nothing demands further explanation. All scientific effort aims at so rethinking different experiences that they can be regarded as partially identical, and every discovery of necessary connection is ultimately a demonstration of identity. If we seek connections with the final aim to understand them as necessary, we must conceive the world of our objects in such a way that it is possible to consider the successive experiences as parts of a self-identical world; that is, as parts of a world in which no substance and no energy can disappear or appear anew. To reach this end it is obviously needed that we eliminate from the world of objects all that cannot be conceived as identically returning in a new experience; that is, all that belongs to the present experience only. We do eliminate this by taking it up conceptually into the subject and calling it psychical, and thus leaving to the object merely that which is conceived as belonging to the world of everybody's experience, that is, of over-individual experience. The whole history of natural science is first of all the gigantic development of this transformation, resolution, and reconstruction. The objects of experience are re-thought till everything is eliminated which cannot be conceived as identical with itself in the experiences of all individuals and thus as belonging to the over-individual world. All the substitutions of atoms for the real thing, and of energies for the real changes, are merely conceptional schemes to satisfy this demand.

The logically primary step is thus not the separation of the physical and the psychical things plus the secondary demand to connect the physical things causally; the order is exactly opposite. The primary desire is to connect the real objects and to understand them as causes and effects. This understanding demands not only empirical observation, but insight into the necessary connection. Necessary connection, on the other hand, exists merely for identical objects and identical qualities. But in the various experiences only that is identical which is independent of the momentary individual experiences, and therefore we need as the ultimate aim a reconstruction of the object into the two parts, the one perceptional, which refers to our individual experience; and the other conceptional, which expresses that which can be conceived as identical in every new experience. The ideal of this constructed world is the mechanical universe in which every atom moves by causal necessity because there is nothing in that universe, no element of substance and no element of energy, which will not remain identical in all changes of the universe which are possibly to be expected. It becomes completely determinable by anticipation and the system of our submissions to the object can be completely constructed. The totality of intellectual efforts to reconstruct such a causally connected over-individual world of objects clearly represents a unity of its own. It is the system of physical sciences.

The physical universe is thus not the totality of our objects. It is a substitution for our real objects, constructed by eliminating the individual parts of our objects of experience. These individual parts are the psychical aspects of our objective experience, and they clearly awake our scientific interest too. The physical sciences need thus as counterpart a division of mental sciences. Their aim must be the same. We want to foresee the psychical results and to understand causally the psychical experience. Yet it is clear that the plan of the mental sciences must be quite different in principle from that of the sciences of nature. The causal connection of the physical universe was ultimately anchored in the identity of the object through various experiences; while the object of experience was psychical for us just in so far as it could never be conceived as identical in different phases of reality. The psychical object is an ever new creation; my idea can never be your idea. Their meaning may be identical, but the psychical stuff, the content of my consciousness, can never be object for any one else, and even in myself the idea of to-day is never the idea of yesterday or to-morrow. But if there cannot be identity in different psychical experiences, it is logically impossible to connect them directly by necessity. If we yet want to master their successive appearance, we must substitute an indirect connection for the direct one, and must describe and explain the psychical phenomena through reference to the physical world. It is in this way that modern psychology has substituted elementary sensations for the real contents of consciousness and has constructed relations between these elementary mental states on the basis of processes in the organism, especially brain processes. Here, again, reality is left behind and a mere conceptional construction is put in its place. But this construction fulfills its purpose and thus gives us truth; and if the basis is once given, the psychological sciences can build up a causal system of the conscious processes in the individual man and in society.

4. *The Historical and the Normative Sciences*

The two divisions of the physical and mental sciences represent our systematized submission to objects. But we saw from the first that it is an artificial abstraction to consider in our real experience the object alone. We saw clearly that we, as acting personalities, in our will and in our attitudes, do not feel ourselves in relation to objects, merely, but to will-acts; and that these will-acts were the individual ones of other subjects or the over-individual ones which come to us in our consciousness of norms. The sciences which deal with our submissions to the individual will-acts of others are the Historical Sciences. Their starting-point is the same as that of the object sciences, the immediate experience. But the other subjects reach our individuality from the start in a different way from the objects. The wills of other subjects come to us as propositions with which we have to agree or disagree; as suggestions, which we are to imitate or to resist; and they carry in themselves that reference to an opposite which, as we saw, characterizes all will-activity. The rock or the tree in our surroundings may stimulate our reactions, but does not claim to be in itself a decision with an alternative. But the political or legal or artistic or social or religious will of my neighbors not only demands my agreement or disagreement, but presents itself to me in its own meaning as a free decision which rejects the opposite, and its whole meaning is destroyed if I consider it like the tree or the rock as a mere phenomenon, as an object in the world of objects. Whoever has clearly understood that politics and religion and knowledge and art and law come to me from the first quite differently from objects, can never doubt that their systematic connection must be most sharply separated from all the sciences which connect impressions of objects, and is falsified if the historical disciplines are treated simply as parts of the sciences of phenomena — for instance, as parts of sociology, the science of society as a psycho-physical object.

Just as natural science transcends the immediately experienced object and works out the whole system of our necessary submissions to the world of objects, so the historical sciences transcend the social will-acts which approach us in our immediate experience, and again seek to find what we are really submitting to if we accept the suggestions of our social surroundings. And yet this similar demand has most dissimilar consequences. We submit to an object and want to find out what we are really submitting to. That cannot mean anything else, as we have seen, than to seek the effects of the object and thus to look forward to what we have to expect from the object. On the other hand, if we want to find out what we are really submitting to if we agree with the decision of our neighbor, the only meaning of the question can be to ask what our neighbor really is deciding on, what is contained in his decision; and as his decision must mean an agreement or disagreement with the will-act of another subject, we cannot understand the suggestion which comes to us without understanding in respect to what propositions of others it takes a stand. Our interest is in this case thus led from those subjects of will which enter into our immediate experience to other subjects whose purposes stand in the relation of suggestion and demand to the present ones. And if we try to develop the system of these relations, we come to an endless chain of will-relations, in which one individual will always points back in its decisions to another individual will with which it agrees or disagrees, which it imitates or overcomes by a new attitude of will; and the whole network of these will-relations is the political or religious or artistic or social history of mankind. This system of history as a system of teleologically connected will-attitudes is elaborated from the will-propositions which reach us in immediate experience, with the same necessity with which the mechanical universe of natural science is worked out from the objects of our immediate experience.

The historical system of will-connections is similar to the system of object-connections, not only in its starting in the immediate experience, but further in its also seeking identities. Without this feature history would not offer to our understanding real connections. We must link the will-attitudes of men by showing the identity of the alternatives. Just as the physical thing is substituted by a large number of atoms which remain identical in the causal changes, in the same way the personality is substituted by an endless manifoldness of decisions and becomes linked with the historical community by the thought that each of these partial decisions refers to an alternative which is identical with that of other persons. And yet there remains a most essential difference between the historical and the causal connection. In a world of things the mere identical continuity is sufficient to determine the phenomena of any given moment. In a world of will the identity of alternatives cannot determine beforehand the actual decision; that belongs to the free activity of the subject. If this factor of freedom were left out, man would be made an object and history a mere appendix of natural science. The connection of the historian can therefore never be a necessary one, however much we may observe empirical regularities. If there were no identities, our reason could not find connection in history; but if the historical connections were necessary, like the causal ones, it would not be history. The historian is, therefore, unable and without the ambition to look into the future like the naturalist; his domain is the past.

Yet will-attitudes and will-acts can also be brought into necessary connection; that is, we can conceive will-acts as teleologically identical with each other and exempt from the freedom of the individual. That is clearly possible only if they are conceived as beyond the freedom of individual decision and related to the over-individual subject. The question is then no longer how this special man wills and decides, but how far a certain will-decision binds every possible individual who performs this act if he is to share our common world of will and meaning. Such an over-individual connection of will-acts is what we call the logical connection. It shares with all other connections the dependence upon the category of identity. The logical connection shows how far one act or combination of acts involves, and thus is partially identical with, a new combination. This logical connection has, in common with the causal connection, necessity; and in common with the historical connection, teleological character. Any individual will-act of historical life may be treated for certain purposes as such a starting-point of over-individual relations; it would then lead to that scientific treatment which gives us an interpretation, for instance, of law. Such interpretative sciences belong to the system of history in the widest sense of the word.

The chief interest, however, must belong to the logical connections of those will-acts which themselves have over-individual character. A merely individual proposition can lead to necessary logical connection, but cannot claim that scientific importance which belongs to the logical connection of those propositions which are necessary for the constitution of every real experience: the science of chess cannot stand on the same level with the science of geometry, the science of local legal statutes not on the same level with the system of ethics. The logical connections of the over-individual attitudes thus constitute the fourth large division besides the physical, the mental, and the historical sciences. It must thus comprise the systems of all those propositions which are presuppositions of our common reality, independent of the free individual decision. Here belong the acts of approval — the ethical approval of changes and achievements, as well as the æsthetic approval of the given world; the acts of conviction — the religious convictions of a superstructure of the world as well as the metaphysical convictions of a substructure; and above all, the acts of affirmation and submission, the logical as well as the mathematical. But to be consistent we must really demand that merely the over-individual logical connections are treated in this division. If we deal, for instance, with the æsthetical or ethical acts as psychological experiences, or as historical propositions, they belong to the psychical or historical division. Only the philosophical system of ethics or æsthetics finds its place in this division. It is difficult to find a suitable name for this whole system of logical connections of over-individual attitudes. Perhaps it would be most correct to call it the Sciences of Values, inasmuch as every one of these over-individual decisions constitutes a value in our world which our individual will finds as an absolute datum like the objects of experience. Seen from another point of view, these values appear as norms which bind our practical will inasmuch as these absolute values demand of our will to realize them, and it may thus be permitted to designate this whole group of sciences as a Division of Normative Sciences.

Our logical explanation of the meaning of these four divisions naturally began with the interpretation of that science which usually takes precedence in popular thought — with the science of nature, that is, and passed then to those groups whose methodological situation is seen rather vaguely by our positivistic age. But as soon as we have once defined and worked out the boundary lines of each of these four divisions, it would appear more logical to change their order and to begin with that division whose material is those over-individual will-acts on which all possible knowledge must depend, and then to turn to those individual will-acts which determine the formulation of our present-day knowledge, and then only to go to the objects of knowledge, the over-individual and the individual ones. In short, we must begin with the normative sciences, consider in the second place the historical sciences, in the third place the physical sciences, and in the fourth place the psychical sciences. There cannot be a scientific judgment which must not find its place somewhere in one of these four groups. And yet can we really say that these four great divisions complete the totality of scientific efforts? The plan of our Congress contains three important divisions besides these.

5. *The Three Divisions of Practical Sciences*

The three divisions which still lie before us represent Practical Knowledge. Have we a logical right to put them on an equal level with the four large divisions which we have considered so far? Might it not rather be said that all that is knowledge in those practical sciences must find its place somewhere in the theoretical field, and that everything outside of it is not knowledge, but art? It cannot be denied indeed that the logical position of the practical sciences presents serious problems. That the function of the engineer or of the physician, of the lawyer or of the minister, of the diplomat or of the teacher, contains elements of an art cannot be doubted. They all need not only knowledge, but a certain instinct and power and skill, and their schooling thus demands a training and discipline through imitation which cannot be substituted by mere learning. Yet when it comes to the classification of sciences, it seems very doubtful whether practical sciences have to be acknowledged as special divisions, inasmuch as the factor of art must have been eliminated at the moment they are presented as sciences. The auscultation of the physician certainly demands skill and training, yet this practical activity itself does not enter into the science of medicine as presented in medical writings. As soon as the physician begins to deal with it scientifically, he needs, as does any scholar, not the stethoscope, but the pen. He must formulate judgments; and as soon as he simply describes and analyzes and explains and interprets his stethoscopic experiences, his statements become a system of theoretical ideas.

We can say in general that the science of medicine or of engineering, of jurisprudence or of education, contains, as science, no element of art, but merely theoretical judgments which, as such, can find their place somewhere in the complete systems of the theoretical sciences. If the physician describes a disease, its symptoms, the means of examining them, the remedies, their therapeutical effects, and the prophylaxis, in short, everything which the physician needs for his art, he does not record anything which would not belong to an ideally complete description and explanation of the processes in the human body. In the same way it can be said that if the engineer characterizes the conditions under which an iron bridge will be safe, it is evident that he cannot introduce any facts which would not find their logical place in an ideally complete description of the properties of inorganic nature; and finally, the same is true for the statements of the politician, the jurist, the pedagogue, or the minister. Whatever is said about their art is a theoretical judgment which connects facts of the ideally complete system of theoretical science; in their case the facts of course belong in first line to the realm of the psychological, historical, and normative sciences. There never has been or can be practical advice in the form of words which is not in principle a statement of facts which belong to the absolute totality of theoretical knowledge. Seen from this point of view, it is evident that all our knowledge is fundamentally theoretical, and that the conception of practical knowledge is logically unprecise.

But the opposite point of view might also be taken. It might be said that after all every kind of knowledge is practical, and our own deduction of the meaning of science might be said to suggest such interpretation. We acknowledged at the outset that the so-called theoretical knowledge is by no means a passive mirror-picture of an independent outside world; but that in every judgment real experience is remoulded and reshaped in the service of certain purposes of will. Here lies the true core of that growing popular philosophy of to-day which, under the name of pragmatism, or under other titles, mingles the purposive character of our knowledge and the evolutionary theories of modern biology in the vague notion that men created knowledge because the biological struggle for existence led to such views of the world; and that we call true that correlation of our experiences which has approved itself through its harmony with the phylogenetic development. Certainly we must reject such circle philosophies. We must see clearly that the whole conception of a biological development and of a struggle of organisms is itself only a part of our construction of causal knowledge. We must have knowledge to conceive ourselves as products of a phylogenetic history, and thus cannot deduce from it the fact, and, still less, the justification of knowledge. Yet one element of this theory remains valuable: knowledge is indeed a purposive activity, a reconstruction of the world in the service of ideals of the will. We have thus from one side the suggestion that all knowledge is merely theoretical, from the other side the claim that all knowledge is practical activity. It seems as if both sides might agree that it is superfluous and unjustified to make a demarcation line through the field of knowledge and to separate two sorts of knowledge, theoretical and practical. For both theories demand that all knowledge be of one kind, and they disagree only as to whether we ought to call it all theoretical or all practical.

Yet the true situation is not characterized by such an antithesis. If we say that all knowledge is ultimately practical, we are speaking from an epistemological point of view, inasmuch as we take it then as a reconstruction of the world through the purposive activity of the over-individual subject. On the other hand it is an empirical point of view from which ultimately all knowledge, that of the physician and engineer and lawyer, as well as that of the astronomer, appears theoretical. But this antithesis can, therefore, not decide the further empirical question, whether or not in the midst of theoretical knowledge two kinds of sciences may be discriminated, of which the one refers to empirical practical purposes and the other not. Such an inquiry would have nothing to do with the epistemological problem of pragmatism; it would be strictly non-philosophical, just as the separation of chemistry into organic and inorganic chemistry. This empirical question is indeed to be answered in the affirmative. If we ask what causes bring about a certain effect, for the sake of a practical purpose of ours, — for instance, the curing a patient of a disease, — no one can state facts which are not in principle to be included in the complete system of physical causes and effects and thus in the system of physical sciences. And yet it may well be that the physical sciences, as such, have not the slightest reason to mention the effect of that special drug on that special pathological alteration of the tissues of the organism. The descriptions and explanations of science are not a mere heaping up of material, but a steady selection in the interest of the special aim of the science. No physical science describes every special pebble on the beach; no historical science deals with the chance happenings in the daily life of any member of the crowd. And we already well know the point of view from which the selection is to be performed. We want to know in the physical and psychical sciences whatever is involved in the object of our experience, and in the historical and normative sciences whatever is involved in the demands which reach our will. But whether we have to do with the objects or with the demands, in both cases we have systems before us which are determined only by the objects or demands themselves, without any relation to our individual will and our own practical activity. Theoretically, of course, our will, our activity, our organism, our personality is included in the complete system; and if we knew absolutely everything of the empirical effects of the object or of the consequences of these demands, we should find among them their relation to our individual interests; but that relation would be but one chance case among innumerable others, and the sciences would not have the slightest interest in giving any attention to that particular case. Thus if our knowledge of chemical substances were complete, we should certainly have to know theoretically that a few grains of antipyrine introduced into the organism have an influence on those brain centres which regulate the temperature of the human body. Yet if the chemist does not share the interest of the physician who wants to fight a fever, he would have hardly any reason for examining this particular relation, as it hardly throws light on the chemical constitution as such. In this way we might say in general that the relation of the world to us as acting individuals is in principle contained in the total system of the relations of our world of experience, but has a strictly accidental place there and can never be in itself a centre around which the scientific data are clustered, and science will hardly have an interest in giving any attention to its details.

This relation of the world, the physical, the psychical, the historical, and the normative world, to our individual, practical purposes can, however, indeed become the centre of scientific interest, and it is evident that the whole inquiry receives thereupon a perfectly new direction which demands not only a completely new grouping of facts and relations, but also a very different shading in elaboration. As long as the purpose was to understand the world without relation to our individual aims, science had to gather endless details which are for us now quite indifferent, as they do not touch our aims; and in other respects science was satisfied with broad generalizations and abstractions where we have now to examine the most minute details. In short, the shifting of the centre of gravity creates perfectly new sciences which must be distinguished; and if we call them again theoretical and practical sciences, it is clear that this difference has then no longer anything to do with the philosophical problems from which we started.

The term practical may be preferable to the other term which is sometimes used: Applied Science. If we construct the antithesis of theoretical and applied science, the underlying idea is clearly that we have to do on the practical side with a discipline which teaches how to apply a science which logically exists as such beforehand. Engineering, for instance, is an applied science because it applies the science of physics; but this is not really our deepest meaning here. Our practical sciences are not meant as mere applications of theoretical sciences. They are logically somewhat degraded if they are treated in such a way. Their real logical meaning comes out only if they are acknowledged as self-dependent sciences whose material is differentiated from that of the theoretical sciences by the different point of view and purpose. They are methodologically perfectly independent, and the fact that a large part or theoretically even everything of their teaching overlaps the teaching of certain theoretical sciences ought not to have any influence on their logical standing. The practical sciences could be conceived as completely self-dependent, without the existence of any so-called theoretical sciences; that is, the relations of the world of experience to our individual aims might be brought into complete systems without working out in principle the system of independent experience. We might have a science of engineering without acknowledging an independent science of theoretical physics besides it. To be sure, such a science of engineering would finally develop itself into a system which would contain very much that might just as well be called theoretical physics; yet all would be held together by the point of view of the engineer, and that part of theoretical physics which the engineer applies might just as well be considered as depracticalized engineering. If this logical self-dependence of the practical science holds true even for such technological disciplines, it is still more evident that it would cripple the meaning and independent character of jurisprudence and social science, or of pedagogy and theology, to treat them simply as applied sciences, that is, as applications of theoretical science.

This point of view determines, also, of course, the classification of the Practical Sciences. If they were really merely applied sciences it would be most natural to group them according to the classification of the theoretical sciences which are to be applied. We should then have applied physical sciences, applied psychological sciences, applied historical sciences, and applied normative sciences. Yet even from the standpoint of practice, we should come at once into difficulties, and indeed much of the superficiality of practical sciences to-day results from the hasty tendency to consider them as applied sciences only, and thus to be determined by the points of view of the theoretical discipline which is to be applied. Then, for instance, pedagogy becomes simply applied psychology, and the psychological point of view is substituted for the educational one. Pedagogy then becomes simply a selection of those chapters in psychology which deal with the mental functions of the child. Yet as soon as we really take the teachers' point of view, we understand at once that it is utterly artificial to substitute the categories of the psychologist for those of immediate practical will-relations and to consider the child in the class-room as a causal system of pyscho-physical elements instead of a personality which is teleologically to be interpreted, and whose aims are not to be connected with causal effects but with over-individual attitudes. In this way the historical relation and the normative relation have to play at least as important a rôle in the pedagogical system as the psycho-physical relation, and we might quite as well call education applied history and applied ethics.

Almost every practical science can be shown in this way to apply a number of theoretical sciences; it synthesizes them to a new unity. But better, we ought to say, that it is a unity in itself from the start, and that it only overlaps with a number of theoretical sciences. If we want to classify the practical sciences, we have thus only the one logical principle at our disposal: we must classify them in accordance with the group of human individual aims which control those different disciplines. If all practical sciences deal with the relation of the world of experience to our individual practical ends, the classes of those ends are the classes of our practical sciences, whatever combinations of applied theoretical sciences may enter into the group. Of course a special classification of these aims must remain somewhat arbitrary; yet it may seem most natural to separate three large divisions. We called them the Utilitarian Sciences, the Sciences of Social Regulation, and the Sciences of Social Culture. Utilitarian we may call those sciences in which our practical aim refers to the world of things; it may be the technical mastery of nature or the treatment of the body, or the production, distribution, and consumption of the means of support. The second division contains everything in which our aim does not refer to the thing, but to the other subjects; here naturally belong the sciences which deal with the political, legal, and social purposes. And finally the sciences of culture refer to those aims in which not the individual relations to things or to other subjects are in the foreground, but the purposes of the teleological development of the subject himself; education, art, and religion here find their place. It is, of course, evident that the material of these sciences frequently allows the emphasis of different aspects. For instance, education, which aims primarily at self-development, might quite well be considered also from the point of view of social regulation; and still more naturally could the utilitarian sciences of the economic distribution of the means of support be considered from this point of view. Yet a classification of sciences nowhere suggests by its boundary lines that there are no relations and connections between the different parts; on the contrary, it is just the manifoldness of these given connections which makes it so desirable to become conscious of the principles involved, and thus to emphasize logical demarcation lines, which of course must be obliterated as soon as any material is to be treated from every possible point of view. It may thus well be that, for instance, a certain industrial problem could be treated in the Normative Sciences from the point of view of ethics; in the Historical Sciences, from the point of view of the history of economic institutions; in the Physical Sciences, from the point of view of physics or chemistry; in the Mental Sciences, from the point of view of sociology; in the Utilitarian Sciences, from the point of view of medicine or of engineering, or of commerce and transportation; and finally in the Regulative Sciences, from the point of view of political administration, or in the Social Sciences, from the standpoint of the urban community, and so on. The more complex the relations are, the more necessary is it to make clean distinctions between the different logical purposes with which the scientific inquiries start. Practical life may demand a combination of historical, sociological, psychological, economical, social, and ethical considerations; but not one of these sciences can contribute its best if the consciousness of these differences is lost and the deliberate combination is replaced by a vague mixture of the problems.

6. *The Subdivisions*

We have now before us the ground-plan of the scheme, the four theoretical divisions, and the three practical divisions; every additional comment on the classification must be of secondary importance, as it has to refer to the smaller subdivisions, which cannot change the principles of the plan, and which have not seldom, indeed, been a result of practical considerations. If, for instance, our Division of Cultural Sciences shows in the final plan merely the departments of Education and of Religion, while the originally planned Department of Art is left out, there was no logical reason for it, but merely the practical ground that it seemed difficult to bring such a practical art section to a desirable scientific level; we confine art, therefore, to the normative æsthetic and historical points of view. Or, to choose another illustration, if it happened that the normative sciences were finally organized without a section for the philosophy of law, this resulted from the fact that the American jurists, in contrast with their Continental European colleagues, showed a general lack of appreciation for such a section. A few sections had to be left out even for the chance reason that the leading speakers were obliged to withdraw at a time when it was too late to ask substitutes to work up addresses. And almost everywhere there had to be something arbitrary in the limitation of the special sections. Though Otology and Laryngology were brought together into one section, they might just as well have been placed in two; and Rhinology, which was left out, might have been added as a third in that company. As to this subtler ramification, the plan has been changed several times during the period of the practical preparation of the plan, and much is the result of adjustment to questions of personalities. No one claims, thus, any special logical value for the final formulation of the sectional details, for which our chief aim was not to go beyond eight times sixteen, that is 128, sections, inasmuch as it was planned to have the meetings at eight different time-periods in sixteen different halls. If we had fulfilled all the wishes which were expressed by specialists, the number would have been quickly doubled.

Yet a few remarks may be devoted to the branching off within the seven divisions, as a short discussion of some of these details may throw additional light on the general principles of the whole plan. If we thus begin with the Normative Sciences, we stand at once before one feature of the plan which has been in an especially high degree a matter of both approval and criticism: the fact that Mathematics is grouped with Philosophy. The Division was to contain, as we have seen, the systems of logically connected will-acts of the over-individual subject. That Ethics or Logic or Æsthetics or Philosophy of Religion deals with such over-individual attitudes cannot be doubted; but have we a right to coördinate the mathematical sciences with these philosophical sciences? Has Mathematics not a more natural place among the physical sciences coördinated with and introductory to Mechanics, Physics, and Astronomy? The mathematicians themselves would often be inclined to accept without hesitation this neighborhood of the physical sciences. They would say that the mathematical objects are independent realities whose properties we study like those of nature, whose relations we "observe," whose existence we "discover," and in which we are interested because they belong to the real world. All this is true, and yet the objects of the mathematician are objects made by the logical will only, and thus different from all phenomena into which sensation enters. The mathematician, of course, does not reflect on the purely logical origin of the objects which he studies, but the system of knowledge must give to the study of the mathematical objects its place in the group where the functions and products of the over-individual attitudes are classified. The mathematical object is a free creation, and a creation not only as to the combination of elements — that would be the case with many laboratory substances of the chemist too — but a creation as to the elements themselves, and the value of that creation, its "mathematical interest," is to be judged by ideals of thought; that is, by logical purposes. No doubt this logical purpose is its application in the world of objects and the mathematical concepts must thus fit the objective world so absolutely that mathematics can be conceived as a description of the world after abstracting not only from the will-relations, as physics does, but also from the content. Mathematics would, then, be the phenomenalistic science of the form and order of the world. In this way, mathematics has indeed a claim to places in both divisions: among the physical sciences if we emphasize its applicability to the world, and among the teleological sciences if we emphasize the free creation of the objects by the logical will. But if we really go back to epistemological principles, our system has to prefer the latter emphasis; that is, we must coördinate mathematics with logic and not with physics.

As to the subdivision of philosophy, it is most essential for us to point to the negative fact that of course psychology cannot have a place in the philosophical department, as part of the Normative Division. There is perhaps no science whose position in the system of knowledge offers so many methodological difficulties as psychology. Historical tradition of course links it with philosophy; throughout a great part of its present endeavors it is, on the other hand, linked with physiology. Thus we find it sometimes coördinated with logic and ethics, and sometimes, especially in the classical positivistic systems, coördinated with the sciences of the organic functions. We have seen why a really logical treatment has to disregard those historical and practical relations and has to separate the psychological sciences from the philosophical and the biological sciences. Yet even this does not complete the list of problems which must be settled, inasmuch as modern thinkers have frequently insisted that psychology itself allows a twofold aspect. We can have a psychology which describes and explains the mental life by analyzing it into its elements and by connecting these elements through causality. But there may be another psychology which treats inner life in that immediate unity in which we experience it and seeks to interpret it as the free function of personality. This latter kind of psychology has been called voluntaristic psychology as against the phenomenalistic psychology which seeks description and explanation. Such voluntaristic psychology would clearly belong again to a different division. It would be a theory of individual life as a function of will, and would thus be introductory to the historical sciences and to the normative sciences too. Yet we left out this teleological psychology from our programme, as such a science is as yet a programme only. Wherever an effort is made to realize it, it becomes an odd mixture of an inconsistent phenomenalistic psychology on the one side, and philosophy of history, logic, ethics, and æsthetics on the other side. The only science which really has a right to call itself psychology is the one which seeks to describe and to explain inner life and treats it therefore as a system of psychical objects, that is, as contents of consciousness, that is, as phenomena. Psychology belongs, then, in the general division of psychical sciences as over against physical sciences, and both deal with objects as over against philosophy and history, which deal with subjects of will.

The subdivision of the Historical Sciences offers no methodological difficulty as soon as those epistemological arguments are acknowledged by which we sharply distinguish history from the Physical and Mental Sciences. If history is a system of will-relations which is in teleological connection with the will-demands that surround us, then political history loses its predominant rôle, and the history of law and of literature, of language and of economy, of art and religion, become coördinated with political development, while the mere anthropological aspect of man is relegated to the physical sciences. The more complete original scheme was here again finally condensed for practical reasons; for instance, the planned departments on the History of Education, on the History of Science, and on the History of Philosophy were sacrificed, and the department of Economic History was joined to that of Political History. In the same way we felt obliged to omit in the end many important sections in the departments; we had, for instance, in the History of Language at first a section on Slavic Languages; yet the number of scholars interested was too small to justify its existence beside a section on Slavic Literature. Also the History of Music was omitted from the History of Art; and the History of Law was planned at first with a fuller ramification.

The division of Physical Sciences naturally suggested that kind of subdivision which the positivistic classification presents as a complete system of sciences. Considering physics and chemistry as the two fundamental sciences of general laws, we turn first to astronomy, then from the science of the whole universe to the one planet, to the sciences of the earth; thence to the living organisms on the earth; and from biology to the still narrower circle of anthropology. The special classification of physics offers a certain difficulty. To divide it in textbook fashion into sound, light, electricity, etc., seems hardly in harmony with the effort to seek logical principles in the other parts of the classification. The three groups which we finally formed, Physics of Matter, Physics of Ether, and Physics of Electron, may appear somewhat too much influenced by the latest theories of to-day, yet it seemed preferable to other principles. In the biological department, criticism seems justified in view of the fact that we constructed a special section, Human Anatomy. A strictly logical scheme might have acknowledged that human anatomy is to-day not a separate science, and that it has resolved itself into comparative anatomy. Sections of Invertebrate and Vertebrate Anatomy might have been more satisfactory. The final arrangement was a concession to the practical interests of the physicians, who have naturally to emphasize the anatomy of the human organism.

In the division of Mental Sciences, we have the Department of Sociology. We were, of course, aware that the sociological interest includes not only the psychological, but also the physiological life of society, and that it thus has relations to the physical sciences too. Yet these relations are logically not more fundamental than those of the individual mental life to the functions of the individual organism. Much of the physiological side was further to be handed over to the Department of Anthropology, and thus we felt justified in grouping sociology with psychology under the Mental Sciences, as the psychology of the social organism. Here, too, a larger number of sections was intended and only the two most essential ones, Social Structure and Social Psychology, were finally admitted.

The ramifications of the practical sciences had to follow the general principle that their character is determined by purpose and not by material. The difficulty was here merely in the extreme specialization of the practical disciplines, which suggests on the whole the forming of very small units, while our plan was to provide for fifty practical sections only. It seemed, therefore, incongruous to have the whole of Internal Medicine or the whole of Private Law condensed into one section. Yet as the purpose of the scheme was a theoretical and not a practical one, even where the theory of practical sciences was in question, we felt justified in constructing coördinated sections, even where the practical importance was very unequal. On the other hand, some glaring defects just here are due merely to chance circumstances. That there were, for instance, no sections on Criminal Law or Ecclesiastical Law in the Department of Jurisprudence, nor on Legal Procedure, resulted from the unfortunate accident that in these cases the speakers who were to come from Europe were withheld by illness or public duties. The absence of the Department of Art in the Division of Social Culture, and thus of the Sections on the theory and practice of the different arts, has been explained before. It is evident that also in the Economical Department the practical development has interfered with the original symmetrical arrangement of the sections. This is not true of the Religious Department, whose six sections express the tendencies of the original plan. The frequently expressed criticism that the different religions and their denominations ought to have found place there shows a misconception of our purpose; a Parliament of Religion did not belong to this plan.

III

THE RESULTS OF THE CONGRESS

The programme of the Congress, as outlined in the previous pages, was in this case somewhat more than a mere programme. It not only invited to do a piece of work, but it sought to contribute to the work itself. Yet the chief work had to be done by others, and their part needed careful preparation. Yet very little of the preparation showed itself to the eyes of the larger public, and few were fully aware what a complex organization was growing up and how many persons of mark were coöperating.

It was essential to find for every address the best man. Specialists only could suggest to the committees where to find him. It has been told before how our invitations were brought to the foreigners first till the desired number of foreign participants was secured, and how the Americans followed. As could not be otherwise expected, interferences of all kinds disturbed the ideal configuration of the first list of acceptances; substitutes had sometimes to be relied on; and yet, when on the nineteenth of September President Francis welcomed the Congress of Arts and Science in the gigantic Festival Hall of the St. Louis Exposition, the Committee knew that almost four hundred speakers had completed their manuscripts, and that it was a galaxy which far surpassed in importance that of any previous international congress. And the list of those who stood for the success of the work was not confined to the official speakers. Each Department and each Section had its own honorary President, who was also chosen by the consent of leading specialists and whose introductory remarks were to give additional importance to the gathering. At their side stood the hundred and thirty Secretaries, carefully chosen from among the productive scholars of the younger generation. And a large number of informal, yet officially invited contributors, had announced valuable discussions and addresses for almost every Section. Invitations to membership finally had been sent to the universities and scholarly societies of all countries.

That the turmoil of a world's fair is out of harmony with the scholar's longing for repose and quietude is a natural presupposition, which has not been disproved by the experience of St. Louis. When Professor Newcomb, our President, spoke to the opening assembly on the dignity of scholarship, the scholar's peaceful address was accentuated by the thunder of the cannons with which Boer and British forces were playing at war near by. The roaring of the Pike overpowered many a quiet session, and the patient speaker had not seldom to fight heroically with a brass band on the next lawn. The trains were delayed, trunks were mixed up, and the sultry St. Louis weather stirred much secret longing for the seashore and the mountains, which most had to leave too early for that pilgrimage to the Mississippi Valley. Yet all this could have been easily foreseen, and every one knew that all this would soon be forgotten. These slight discomforts were many times made up for by the overwhelming beauty of that ivory city in which the civilization of the world was focused by the united energy of the nations, and it seemed well worth while to cross the ocean for the delight of that enchantment which came with every evening's myriad illumination. And every day brought interesting festivities. No one will forget the receptions of the foreign commissioners, or the charming hospitality of the leading citizens of St. Louis, or the enthusiastic banquet which brought one thousand speakers and presidents and official members of the Congress together as guests of the master mind of the Exposition, President Francis.

While the discomfort of external shortcomings was thus easily balanced, it is more doubtful whether the internal shortcomings of the work can be considered as fully compensated for. It would be impossible to overlook these defects in the realization of our plans, even if it may be acknowledged that they were unavoidable under the given conditions. The principal difficulty has been that many speakers have not really treated the topic for the discussion of which they were invited. This deviation from the plan took various forms. There was in some cases a fundamental attitude taken which did not harmonize with those logical principles which had led to the classification; for instance, we had sharply separated, for reasons fully stated above, the Division of History from the Division of Mental Sciences, including sociology; yet some papers for the Division of History clearly indicated sympathy with the traditional positivistic view, according to which history becomes simply a part of sociology. And similar variations of the general plan occur in almost every division. But there cannot be any objection to this secondary variety as long as the whole framework gives the primary uniformity. Certainly no one of the contributors is to be blamed for it; no one was pledged to the philosophy of the general plan, and probably few would have agreed if any one had had the idea of demanding from every contributor an identical background of general convictions. Such monotony would have been even harmful, as the work would have become inexpressive of the richness of tendencies in the scholarly life of our time. This was not an occasion where educated clerks were to work up in a second-hand way a report whose general trend was determined beforehand;

the work demanded original thinkers, with whom every word grows out of a rich individual view of the totality. If every paper had been meant merely as a detailed amplification of the logical principles on which the whole plan was based, it would have been wiser to set young Doctor candidates to work, who might have elaborated the hint of the general scheme. To invite the leaders of knowledge meant to give them complete freedom and to confine the demands of the plan to a most general direction.

The same freedom, which every one was to have as to the general standpoint, was intended also for all with regard to the arrangement and limitation of the topic. All the sectional addresses were supposed to deal either with relations or with fundamental problems of to-day. It would have been absurd to demand that in every case the totality of relations or of problems should be covered or even touched. The result would have become perfunctory and insignificant. No one intended to produce a cyclopedia. It was essential everywhere to select that which was most characteristic of the tendencies of the age and most promising for the science of the twentieth century. Those problems were to be emphasized whose solution is most demanded for the immediate progress of knowledge, and those relations had to be selected through which new connections, new synthetic thoughts prepare themselves to-day. That this selection had to be left to the speaker was a matter of course.

Yet it may be said that in all these directions, with reference to the general standpoint and with reference to problems and relations, the Organizing Committee had somewhat prepared the choice through the selection of the speakers themselves. As the standpoints of the leading speakers were well known, it was not difficult to invite as far as possible for every place a scholar whose general views would be least out of harmony with the principles of the plan. For instance, when we had the task before us of selecting the divisional speakers for the Normative and for the Mental Sciences, it was only natural to invite for the first a philosopher of idealistic type and for the latter a philosopher of positivistic stamp, inasmuch as the whole scheme gave to the mental sciences the same place which they would have had in a positivistic scheme, while the normative sciences would have lost the meaning which they had in our plan if a positivist had simply psychologized them. In the same way we gave preference as far as possible, for the addresses on relations, to those scholars whose previous work was concerned with new synthetic movements, and as speakers on problems those were invited who were in any case engaged in the solution of those problems which seemed central in the present state of science. Thus it was that on the whole the expectation was justified that the most characteristic relations and the most characteristic problems would be selected if every invited speaker spoke essentially on those relations and on those problems with which his own special work was engaged.

Yet there is no doubt that this expectation was sometimes fulfilled beyond our anticipation, in an amount of specialization which was no longer entirely in harmony with the general character of the undertaking. The general problem has become sometimes only the starting-point or almost the pretext for speaking on some relation or problem so detailed that it can hardly stand as a representative symbol of the whole movement in that sectional field. Especially in the practical sciences more room was sometimes taken for particular hobbies and chance aspects than in the eyes of the originators the occasion may have called for. Yet on the whole this was the exception. The overwhelming majority of the addresses fulfilled nobly the high hopes of the Boards, and even in those exceptional cases where the speaker went his own way, it was usually such an original and stimulating expression of a strong personality that no one would care to miss this tone in the symphony of science.

Even now of course, though the Congress days have passed, and only typewritten manuscripts are left from all those September meetings, it would be easy to provide, by editorial efforts, for a greater uniformity and a smoother harmonization. Most of the authors would have been quite willing to retouch their addresses in the interest of greater objective uniformity and to accept the hint of an editorial committee in elaborating more fully some points and in condensing or eliminating others. Much was written in the desire to bring a certain thought for discussion before such an eminent audience, while the speaker would be ready to substitute other features of the subject for the permanent form of the printed volume. Yet such editorial supervision and transformation would be not only immodest but dangerous. We might risk gaining some external uniformity, but only to lose much of the freshness and immediacy and brilliancy of the first presentation. And who would dare to play the critical judge when the international contributors are the leaders of thought? There was therefore not the slightest effort made to suggest revision of the manuscripts, for which the whole responsibility must thus fall to the particular author. The reduction to a uniform language seemed, on the other hand, most natural, and those who had delivered their addresses in French, German, or Italian themselves welcomed the idea that their papers should be translated into English by competent specialists. The short bibliographies, selected mostly through the chairman of the departments, and the very full index with references may add to the general usefulness of the eight volumes in which the work is to be presented.

But the significance of the Congress of Arts and Science ought not to be measured and valued only by reference to this printed result.

Its less visible side-effects seem in no way less important for scholarship, and they are fourfold. There was, first, the personal contact between the scholarly public and the leaders of thought; there was, secondly, the first academic alliance between the United States and Europe; there was, thirdly, the first demonstration of a world congress crystallized about one problem; there was, fourthly, the unique accentuation of the thought of unity in all human science; and each of these four movements will be continued and reinforced by the publication of these proceedings.

The first of these four features, the contact of the scholarly public with the best thinkers of our time, had, to be sure, its limitations. It was not sought to create a really popular congress. Neither the level of the addresses, nor the size of the halls, nor the number of invitations sent out, nor the general conditions of a world's fair at which the expense of living is high and the distractions thousandfold, favored the attendance of crowds. It was planned from the first that on the whole scholars and specialists should attend and that the army should be made up essentially of officers. If in an astronomical section perhaps thirty men were present, among whom practically every one was among the best known directors of observatories or professors of mathematics, astronomy, or physics, from all countries of the globe, much more was gained than if three thousand had been in the audience, brought together by an interest of curiosity in moon and stars. For the most part there must have been between a hundred and two hundred in each of the 128 sectional meetings, and that was more than the organizers expected. This direct influence on the interested public is now to be expanded a thousandfold by the mission work of these volumes. The concentration of these hundreds of addresses into a few days made it in any case impossible to listen to more than to a small fraction; these volumes will bring at last all speakers to coördinated effectiveness; and while one hall suffered from bad acoustics, another from bad ventilation, and a third from the passing of the intermural trains, here at least is an audience in which nothing will disturb the sensitive nerves of the willing follower.

But much more emphasis is due to the second feature. The Congress was an epoch-making event for the international world of scholarship from the fact that it was the first great undertaking in which the Old and the New Worlds stood on equal levels and in which Europe really became acquainted with the scientific life of these United States. The contact of scholarship between America and Europe has, indeed, grown in importance through many decades. Many American students had studied in European and especially in German universities and had come back to fill the professorial chairs of the leading academic institutions. The spirit of the Graduate School and the work towards the Doctor's degree, yes, the whole productive scholarship of recent decades had been influenced by European ideals, and the results were no longer ignored at the seats of learning throughout the whole world. European scholars had here and there come as visiting lecturers or as assimilated instructors, and a few American scholars belonged to the leading European Academies. Yet, whoever knew the real development of American post-graduate university life, the rapid advance of genuine American scholarship, the incomparable progress of the scientific institutions of the New World, of their libraries and laboratories, museums and associations, was well aware that Europe had hardly noticed and certainly not fully understood the gigantic strides of the country which seemed a rival only on commercial and industrial ground. Europe was satisfied with the traditional ideas of America's scientific standing which reflected the situation of thirty years ago, and did not understand that the changes of a few lustres mean in the New World more than under the firmer traditions of Europe. American scientific literature was still neglected; American universities treated in a condescending and patronizing spirit and with hardly any awareness of the fundamental differences in the institutions of the two sides. Those European scholars who crossed the ocean did it with missionary, or perhaps with less unselfish, intentions, and the Americans who attended European congresses were mostly treated with the friendliness which the self-satisfied teacher shows to a promising pupil. The time had really come when the contrast between the real situation and the traditional construction became a danger for the scientific life of the time. Both sides had to suffer from it. The Americans felt that their serious and important achievements did not come to their fullest effectiveness through the insistent neglect of those who by the tradition of centuries had become the habitual guardians of scientific thought. A kind of feeling of dependency as it usually develops in weak colonies too often depressed the conscientious scholarship on American soil as the result of this undue condescension. Yet the greater harm was to the other side. Once before Europe had had the experience of surprise when American successes presented themselves where nothing of that kind was anticipated in the Old World. It was in the field of economic life that Europe looked down patronizingly on America's industrial efforts, and yet before she was fully aware how the change resulted, suddenly the warning signal of the "American danger" was heard everywhere. The surprise in the intellectual field will not be less. The unpreparedness was certainly the same. Of course, there cannot be any danger of rivalry in the scientific field, inasmuch as science knows no competition but only coöperation. And yet it cannot be without danger for European science if it willfully neglects and recklessly ignores this eager working of the modern America. For both sides a change in the situation was thus not only desirable, but necessary; and to prepare this change, to substitute knowledge for ignorance, nothing could have been more effective than this Congress of Arts and Science.

Even if we abstract from the not inconsiderable number of those European scholars who followed naturally in the path of the invited guests, and if we consider merely the function of these invited participants, the importance of the procedure is evident. More than a hundred leading scholars from all European countries came under conditions where academic fellowship on an equal footing was a necessary part of the work. There was not the slightest premium held out which might have attracted them had not real interacademic interest brought them over the ocean, and no missionary spirit was appealed to, as everything was equally divided between American and foreign contributors. It was a real feast of international scholarship, in which the importance and the number of foreigners stamped it as the first significant alliance of the spirit of learning in the New and the Old Worlds. And it was essentially for this purpose that the week of personal intermingling in St. Louis itself was preceded and followed by happy weeks of visits to leading universities. Almost every one of those one hundred European scholars visited Harvard and Yale, Chicago and Johns Hopkins, Columbia and Pennsylvania, saw the treasures of Washington and examined the exhibitions of American scholarship in the World's Fair itself. The change of opinion, the disappearance of prejudice, the growth of confidence, the personal intercollegiate ties which resulted from all that, have been evident since those days all over Europe. And it is not surprising that it is just the most famous and most important of the visitors, famous and important through their width and depth of view, whose expression of appreciation and admiration for the new achievements has been loudest.

We insisted that the effectiveness of the Congress showed itself in two other directions still: on the one side, there was at last a congress with a unified programme, a congress which stood for a definite thought, and which brought all its efforts to bear on the solution of one problem. There seemed a far-reaching agreement of opinion that this new principle of congress administration had successfully withstood the test of practical realization. Mere conglomerations of unconnected meetings with casual programmes and unrelated papers cannot claim any longer to represent the only possible form of international gatherings of scholars. More than that, their superfluous and disheartening character will be felt in future more strongly than before. No congress will appear fully justified whose printed proceedings do not show a real plan in its programme. And the consciousness of this mission of the Congress will certainly be again reinforced by the publication of these volumes, inasmuch as it is evident that they represent a substantial contribution to the knowledge of our time which would not have been made without the special stimulating occasion of the Congress.

And, finally, whether such a congress is held again or not, the impulse of this one cannot be lost on account of the special end to which all its efforts have been directed: the unity of scientific knowledge. We had emphasized from the first that here was the centre of our purposes in a time whose scientific specialization necessarily involves a scattering of scholarly work and which yet in its deepest meaning strives for a new synthesis, for a new unity, which is to give to all this scattered labor a real dignity and significance; truly nothing was more needed than an intense accentuation of the internal harmony of all human knowledge. But for that it is not enough that the masses feel instinctively the deep need of such unifying movements, nor is it enough that the philosophers point with logical arguments towards the new synthesis. The philosopher can only stand by and point the way; the specialists themselves must go the way. And here at last they have done so. Leaders of thought have interrupted their specialistic work and have left their detailed inquiries to seek the fundamental conceptions and methods and principles which bind all knowledge together, and thus to work towards that unity from which all special work derives its meaning. Whether or not their coöperation has produced anything which is final is a question almost insignificant compared with the fundamental fact that they coöperated at all for this ideal synthetic purpose. This fact can never lose its influence on the scholarly effort of our age, and will certainly find its strongest reinforcement in this unified publication. It has fulfilled its noblest purpose if it adds strength to the deepest movement of our time, the movement towards unity of meaning in the scattered manifoldness of scientific endeavor with which the twentieth century has opened.

PROCEEDINGS OF THE CONGRESS

INTRODUCTORY ADDRESS

DELIVERED AT THE OPENING EXERCISES AT FESTIVAL HALL BY
PROFESSOR SIMON NEWCOMB, PRESIDENT OF THE CONGRESS

THE EVOLUTION OF THE SCIENTIFIC INVESTIGATOR

As we look at the assemblage gathered in this hall, comprising so many names of widest renown in every branch of learning, — we might almost say in every field of human endeavor, — the first inquiry suggested must be after the object of our meeting. The answer is, that our purpose corresponds to the eminence of the assemblage. We aim at nothing less than a survey of the realm of knowledge, as comprehensive as is permitted by the limitations of time and space. The organizers of our Congress have honored me with the charge of presenting such preliminary view of its field as may make clear the spirit of our undertaking.

Certain tendencies characteristic of the science of our day clearly suggest the direction of our thoughts most appropriate to the occasion. Among the strongest of these is one toward laying greater stress on questions of the beginning of things, and regarding a knowledge of the laws of development of any object of study as necessary to the understanding of its present form. It may be conceded that the principle here involved is as applicable in the broad field before us as in a special research into the properties of the minutest organism. It therefore seems meet that we should begin by inquiring what agency has brought about the remarkable development of science to which the world of to-day bears witness. This view is recognized in the plan of our proceedings, by providing for each great department of knowledge a review of its progress during the century that has elapsed since the great event commemorated by the scenes outside this hall. But such reviews do not make up that general survey of science at large which is necessary to the development of our theme, and which must include the action of causes that had their origin long before our time. The movement which culminated in making the nineteenth century ever memorable in history is the outcome of a long series of causes, acting through many centuries, which are worthy of especial attention on such an occasion as this. In setting them forth we should avoid laying stress on those visible manifestations which, striking the eye of every beholder, are in no danger of being overlooked, and search rather for those agencies whose activities underlie the whole visible scene, but which are liable to be blotted out of sight by the very brilliancy of the results to which they have given rise. It is easy to draw attention to the wonderful qualities of the oak; but from that very fact, it may be needful to point out that the real wonder lies concealed in the acorn from which it grew.

Our inquiry into the logical order of the causes which have made our civilization what it is to-day will be facilitated by bringing to mind certain elementary considerations — ideas so familiar that setting them forth may seem like citing a body of truisms — and yet so frequently overlooked, not only individually, but in their relation to each other, that the conclusion to which they lead may be lost to sight. One of these propositions is that psychical rather than material causes are those which we should regard as fundamental in directing the development of the social organism. The human intellect is the really active agent in every branch of endeavor, — the *primum mobile* of civilization, — and all those material manifestations to which our attention is so often directed are to be regarded as secondary to this first agency. If it be true that "in the world is nothing great but man; in man is nothing great but mind," then should the keynote of our discourse be the recognition of this first and greatest of powers.

Another well-known fact is that those applications of the forces of nature to the promotion of human welfare which have made our age what it is, are of such comparatively recent origin that we need go back only a single century to antedate their most important features, and scarcely more than four centuries to find their beginning. It follows that the subject of our inquiry should be the commencement, not many centuries ago, of a certain new form of intellectual activity.

Having gained this point of view, our next inquiry will be into the nature of that activity, and its relation to the stages of progress which preceded and followed its beginning. The superficial observer, who sees the oak but forgets the acorn, might tell us that the special qualities which have brought out such great results are expert scientific knowledge and rare ingenuity, directed to the application of the powers of steam and electricity. From this point of view the great inventors and the great captains of industry were the first agents in bringing about the modern era. But the more careful inquirer will see that the work of these men was possible only through a knowledge of the laws of nature, which had been gained by men whose work took precedence of theirs in logical order, and that success in invention has been measured by completeness in such knowledge. While giving all due honor to the great inventors, let us remember that the first place is that of the great investigators, whose forceful intellects opened the way to secrets previously hidden from men. Let it be an honor and not a reproach to these men, that they were not actuated by the love of gain, and did not keep utilitarian ends in view in the pursuit of their researches. If it seems that in neglecting such ends they were leaving undone the most important part of their work, let us remember that nature turns a forbidding face to those who pay her court with the hope of gain, and is responsive only to those suitors whose love for her is pure and undefiled. Not only is the special genius required in the investigator not that generally best adapted to applying the discoveries which he makes, but the result of his having sordid ends in view would be to narrow the field of his efforts, and exercise a depressing effect upon his activities. The true man of science has no such expression in his vocabulary as "useful knowledge." His domain is as wide as nature itself, and he best fulfills his mission when he leaves to others the task of applying the knowledge he gives to the world.

We have here the explanation of the well-known fact that the functions of the investigator of the laws of nature, and of the inventor who applies these laws to utilitarian purposes, are rarely united in the same person. If the one conspicuous exception which the past century presents to this rule is not unique, we should probably have to go back to Watt to find another.

From this viewpoint it is clear that the primary agent in the movement which has elevated man to the masterful position he now occupies, is the scientific investigator. He it is whose work has deprived plague and pestilence of their terrors, alleviated human suffering, girdled the earth with the electric wire, bound the continent with the iron way, and made neighbors of the most distant nations. As the first agent which has made possible this meeting of his representatives, let his evolution be this day our worthy theme. As we follow the evolution of an organism by studying the stages of its growth, so we have to show how the work of the scientific investigator is related to the ineffectual efforts of his predecessors.

In our time we think of the process of development in nature as one going continuously forward through the combination of the opposite processes of evolution and dissolution. The tendency of our thought has been in the direction of banishing cataclysms to the theological limbo, and viewing nature as a sleepless plodder, endowed with infinite patience, waiting through long ages for results. I do not contest the truth of the principle of continuity on which this view is based. But it fails to make known to us the whole truth. The building of a ship from the time that her keel is laid until she is making her way across the ocean is a slow and gradual process; yet there is a cataclysmic epoch opening up a new era in her history. It is the moment when, after lying for months or years a dead, inert, immovable mass, she is suddenly endowed with the power of motion, and, as if imbued with life, glides into the stream, eager to begin the career for which she was designed.

I think it is thus in the development of humanity. Long ages may pass during which a race, to all external observation, appears to be making no real progress. Additions may be made to learning, and the records of history may constantly grow, but there is nothing in its sphere of thought, or in the features of its life, that can be called essentially new. Yet, nature may have been all along slowly working in a way which evades our scrutiny until the result of her operations suddenly appears in a new and revolutionary movement, carrying the race to a higher plane of civilization.

It is not difficult to point out such epochs in human progress. The greatest of all, because it was the first, is one of which we find no record either in written or geological history. It was the epoch when our progenitors first took conscious thought of the morrow, first used the crude weapons which nature had placed within their reach to kill their prey, first built a fire to warm their bodies and cook their food. I love to fancy that there was some one first man, the Adam of evolution, who did all this, and who used the power thus acquired to show his fellows how they might profit by his example. When the members of the tribe or community which he gathered around him began to conceive of life as a whole, — to include yesterday, to-day, and to-morrow in the same mental grasp — to think how they might apply the gifts of nature to their own uses, — a movement was begun which should ultimately lead to civilization.

Long indeed must have been the ages required for the development of this rudest primitive community into the civilization revealed to us by the most ancient tablets of Egypt and Assyria. After spoken language was developed, and after the rude representation of ideas by visible marks drawn to resemble them had long been practiced, some Cadmus must have invented an alphabet. When the use of written language was thus introduced, the word of command ceased to be confined to the range of the human voice, and it became possible for master minds to extend their influence as far as a written message could be carried. Then were communities gathered into provinces; provinces into kingdoms; kingdoms into the great empires of antiquity. Then arose a stage of civilization which we find pictured in the most ancient records, — a stage in which men were governed by laws that were perhaps as wisely adapted to their conditions as our laws are to ours, — in which the phenomena of nature were rudely observed, and striking occurrences in the earth or in the heavens recorded in the annals of the nation.

Vast was the progress of knowledge during the interval between these empires and the century in which modern science began. Yet, if I am right in making a distinction between the slow and regular steps of progress, each growing naturally out of that which preceded it, and the entrance of the mind at some fairly definite epoch into an entirely new sphere of activity, it would appear that there was only one such epoch during the entire interval. This was when abstract geometrical reasoning commenced, and astronomical observations aiming at precision were recorded, compared, and discussed. Closely associated with it must have been the construction of the forms of logic. The radical difference between the demonstration of a theorem of geometry and the reasoning of every-day life which the masses of men must have practiced from the beginning, and which few even to-day ever get beyond, is so evident at a glance that I need not dwell upon it. The principal feature of this advance is that, by one of those antinomies of the human intellect of which examples are not wanting even in our own time, the development of abstract ideas preceded the concrete knowledge of natural phenomena. When we reflect that in the geometry of Euclid the science of space was brought to such logical perfection that even to-day its teachers are not agreed as to the practicability of any great improvement upon it, we cannot avoid the feeling that a very slight change in the direction of the intellectual activity of the Greeks would have led to the beginning of natural science. But it would seem that the very purity and perfection which was aimed at in their system of geometry stood in the way of any extension or application of its methods and spirit to the field of nature. One example of this is worthy of attention. In modern teaching the idea of magnitude as generated by motion is freely introduced. A line is described by a moving point; a plane by a moving line; a solid by a moving plane. It may, at first sight, seem singular that this conception finds no place in the Euclidian system. But we may regard the omission as a mark of logical purity and rigor. Had the real or supposed advantages of introducing motion into geometrical conceptions been suggested to Euclid, we may suppose him to have replied that the theorems of space are independent of time; that the idea of motion necessarily implies time, and that, in consequence, to avail ourselves of it would be to introduce an extraneous element into geometry.

It is quite possible that the contempt of the ancient philosophers for the practical application of their science, which has continued in some form to our own time, and which is not altogether unwholesome, was a powerful factor in the same direction. The result was that, in keeping geometry pure from ideas which did not belong to it, it failed to form what might otherwise have been the basis of physical science. Its founders missed the discovery that methods similar to those of geometric demonstration could be extended into other and wider fields than that of space. Thus not only the development of applied geometry, but the reduction of other conceptions to a rigorous mathematical form was indefinitely postponed.

Astronomy is necessarily a science of observation pure and simple, in which experiment can have no place except as an auxiliary. The vague accounts of striking celestial phenomena handed down by the priests and astrologers of antiquity were followed in the time of the Greeks by observations having, in form at least, a rude approach to precision, though nothing like the degree of precision that the astronomer of to-day would reach with the naked eye, aided by such instruments as he could fashion from the tools at the command of the ancients.

The rude observations commenced by the Babylonians were continued with gradually improving instruments, — first by the Greeks and afterward by the Arabs, — but the results failed to afford any insight into the true relation of the earth to the heavens. What was most remarkable in this failure is that, to take a first step forward which would have led on to success, no more was necessary than a course of abstract thinking vastly easier than that required for working out the problems of geometry. That space is infinite is an unexpressed axiom, tacitly assumed by Euclid and his successors. Combining this with the most elementary consideration of the properties of the triangle, it would be seen that a body of any given size could be placed at such a distance in space as to appear to us like a point. Hence a body as large as our earth, which was known to be a globe from the time that the ancient Phœnicians navigated the Mediterranean, if placed in the heavens at a sufficient distance, would look like a star. The obvious conclusion that the stars might be bodies like our globe, shining either by their own light or by that of the sun, would have been a first step to the understanding of the true system of the world.

There is historic evidence that this deduction did not wholly escape the Greek thinkers. It is true that the critical student will assign little weight to the current belief that the vague theory of Pythagoras — that fire was at the centre of all things — implies a conception of the heliocentric theory of the solar system. But the testimony of Archimedes, confused though it is in form, leaves no serious doubt that Aristarchus of Samos not only propounded the view that the earth revolves both on its own axis and around the sun, but that he correctly removed the great stumbling-block in the way of this theory by adding that the distance of the fixed stars was infinitely greater than the dimensions of the earth's orbit. Even the world of philosophy was not yet ready for this conception, and, so far from seeing the reasonableness of the explanation, we find Ptolemy arguing against the rotation of the earth on grounds which careful observations of the phenomena around him would have shown to be ill-founded.

Physical science, if we can apply that term to an uncoördinated body of facts, was successfully cultivated from the earliest times. Something must have been known of the properties of metals, and the art of extracting them from their ores must have been practiced, from the time that coins and medals were first stamped. The properties of the most common compounds were discovered by alchemists in their vain search for the philosopher's stone, but no actual progress worthy of the name rewarded the practitioners of the black art.

Perhaps the first approach to a correct method was that of Archimedes, who by much thinking worked out the law of the lever, reached the conception of the centre of gravity, and demonstrated the first principles of hydrostatics. It is remarkable that he did not extend his researches into the phenomena of motion, whether spontaneous or produced by force. The stationary condition of the human intellect is most strikingly illustrated by the fact that not until the time of Leonardo was any substantial advance made on his discovery. To sum up in one sentence the most characteristic feature of ancient and medieval science, we see a notable contrast between the precision of thought implied in the construction and demonstration of geometrical theorems and the vague indefinite character of the ideas of natural phenomena generally, a contrast which did not disappear until the foundations of modern science began to be laid.

We should miss the most essential point of the difference between medieval and modern learning if we looked upon it as mainly a difference either in the precision or the amount of knowledge. The development of both of these qualities would, under any circumstances, have been slow and gradual, but sure. We can hardly suppose that any one generation, or even any one century, would have seen the complete substitution of exact for inexact ideas. Slowness of growth is as inevitable in the case of knowledge as in that of a growing organism. The most essential point of difference is one of those seemingly slight ones, the importance of which we are too apt to overlook. It was like the drop of blood in the wrong place, which some one has told us makes all the difference between a philosopher and a maniac. It was all the difference between a living tree and a dead one, between an inert mass and a growing organism. The transition of knowledge from the dead to the living form must, in any complete review of the subject, be looked upon as the really great event of modern times.

Before this event the intellect was bound down by a scholasticism which regarded knowledge as a rounded whole, the parts of which were written in books and carried in the minds of learned men. The student was taught from the beginning of his work to look upon authority as the foundation of his beliefs. The older the authority the greater the weight it carried. So effective was this teaching that it seems never to have occurred to individual men that they had all the opportunities ever enjoyed by Aristotle of discovering truth, with the added advantage of all his knowledge to begin with. Advanced as was the development of formal logic, that practical logic was wanting which could see that the last of a series of authorities, every one of which rested on those which preceded it, could never form a surer foundation for any doctrine than that supplied by its original propounder.

The result of this view of knowledge was that, although during the fifteen centuries following the death of the geometer of Syracuse great universities were founded at which generations of professors expounded all the learning of their time, neither professor nor student ever suspected what latent possibilities of good were concealed in the most familiar operations of nature. Every one felt the wind blow, saw water boil, and heard the thunder crash, but never thought of investigating the forces here at play. Up to the middle of the fifteenth century the most acute observer could scarcely have seen the dawn of a new era.

In view of this state of things, it must be regarded as one of the most remarkable facts in evolutionary history that four or five men, whose mental constitution was either typical of the new order of things or who were powerful agents in bringing it about, were all born during the fifteenth century, four of them at least at so nearly the same time as to be contemporaries.

Leonardo da Vinci, whose artistic genius has charmed succeeding generations, was also the first practical engineer of his time, and the first man after Archimedes to make a substantial advance in developing the laws of motion. That the world was not prepared to make use of his scientific discoveries does not detract from the significance which must attach to the period of his birth.

Shortly after him was born the great navigator whose bold spirit was to make known a new world, thus giving to commercial enterprise that impetus which was so powerful an agent in bringing about a revolution in the thoughts of men.

The birth of Columbus was soon followed by that of Copernicus, the first after Aristarchus to demonstrate the true system of the world. In him more than in any of his contemporaries do we see the struggle between the old forms of thought and the new. It seems almost pathetic and is certainly most suggestive of the general view of knowledge taken at that time that, instead of claiming credit for bringing to light great truths before unknown, he made a labored attempt to show that, after all, there was nothing really new in his system, which he claimed to date from Pythagoras and Philolaus. In this connection it is curious that he makes no mention of Aristarchus, who I think will be regarded by conservative historians as his only demonstrated predecessor. To the hold of the older ideas upon his mind we must attribute the fact that in constructing his system he took great pains to make as little change as possible in ancient conceptions.

Luther, the greatest thought-stirrer of them all, practically of the same generation with Copernicus, Leonardo, and Columbus, does not come in as a scientific investigator, but as the great loosener of chains which had so fettered the intellect of men that they dared not think otherwise than as the authorities thought.

Almost coeval with the advent of these intellects was the invention of printing with movable type. Gutenberg was born during the first decade of the century, and his associates and others credited with the invention not many years afterward. If we accept the principle on which I am basing my argument, that we should assign the first place to the birth of those psychic agencies which started men on new lines of thought, then surely was the fifteenth the wonderful century.

Let us not forget that, in assigning the actors then born to their places, we are not narrating history, but studying a special phase of evolution. It matters not for us that no university invited Leonardo to its halls, and that his science was valued by his contemporaries only as an adjunct to the art of engineering. The great fact still is that he was the first of mankind to propound laws of motion. It is not for anything in Luther's doctrines that he finds a place in our scheme. No matter for us whether they were sound or not. What he did toward the evolution of the scientific investigator was to show by his example that a man might question the best-established and most venerable authority and still live — still preserve his intellectual integrity — still command a hearing from nations and their rulers. It matters not for us whether Columbus ever knew that he had discovered a new continent. His work was to teach that neither hydra, chimera, nor abyss — neither divine injunction nor infernal machination — was in the way of men visiting every part of the globe, and that the problem of conquering the world reduced itself to one of sails and rigging, hull and compass. The better part of Copernicus was to direct man to a viewpoint whence he should see that the heavens were of like matter with the earth. All this done, the acorn was planted from which the oak of our civilization should spring. The mad quest for gold which followed the discovery of Columbus, the questionings which absorbed the attention of the learned, the indignation excited by the seeming vagaries of a Paracelsus, the fear and trembling lest the strange doctrine of Copernicus should undermine the faith of centuries, were all helps to the germination of the seed — stimuli to thought which urged it on to explore the new fields opened up to its occupation. This given, all that has since followed came out in regular order of development, and need be here considered only in those phases having a special relation to the purpose of our present meeting.

So slow was the growth at first that the sixteenth century may scarcely have recognized the inauguration of a new era. Torricelli and Benedetti were of the third generation after Leonardo, and Galileo, the first to make a substantial advance upon his theory, was born more than a century after him. Only two or three men appeared in a generation who, working alone, could make real progress in discovery, and even these could do little in leavening the minds of their fellow men with the new ideas.

Up to the middle of the seventeenth century an agent which all experience since that time shows to be necessary to the most productive intellectual activity was wanting. This was the attraction of like minds, making suggestions to each other, criticising, comparing, and reasoning. This element was introduced by the organization of the Royal Society of London and the Academy of Sciences of Paris.

The members of these two bodies seem like ingenious youth suddenly thrown into a new world of interesting objects, the purposes and relations of which they had to discover. The novelty of the situation is strikingly shown in the questions which occupied the minds of the incipient investigators. One natural result of British maritime enterprise was that the aspirations of the Fellows of the Royal Society were not confined to any continent or hemisphere. Inquiries were sent all the way to Batavia to know "whether there be a hill in Sumatra which burneth continually, and a fountain which runneth pure balsam." The astronomical precision with which it seemed possible that physiological operations might go on was evinced by the inquiry whether the Indians can so prepare that stupefying herb Datura that "they make it lie several days, months, years, according as they will, in a man's body without doing him any harm, and at the end kill him without missing an hour's time." Of this continent one of the inquiries was whether there be a tree in Mexico that yields water, wine, vinegar, milk, honey, wax, thread, and needles.

Among the problems before the Paris Academy of Sciences those of physiology and biology took a prominent place. The distillation of compounds had long been practiced, and the fact that the more spirituous elements of certain substances were thus separated naturally led to the question whether the essential essences of life might not be discoverable in the same way. In order that all might participate in the experiments, they were conducted in open session of the Academy, thus guarding against the danger of any one member obtaining for his exclusive personal use a possible elixir of life. A wide range of the animal and vegetable kingdom, including cats, dogs, and birds of various species, were thus analyzed. The practice of dissection was introduced on a large scale. That of the cadaver of an elephant occupied several sessions, and was of such interest that the monarch himself was a spectator.

To the same epoch with the formation and first work of these two bodies belongs the invention of a mathematical method which in its importance to the advance of exact science may be classed with the invention of the alphabet in its relation to the progress of society at large. The use of algebraic symbols to represent quantities had its origin before the commencement of the new era, and gradually grew into a highly developed form during the first two centuries of that era. But this method could represent quantities only as fixed. It is true that the elasticity inherent in the use of such symbols permitted of their being applied to any and every quantity; yet, in any one application, the quantity was considered as fixed and definite. But most of the magnitudes of nature are in a state of continual variation; indeed, since all motion is variation, the latter is a universal characteristic of all phenomena. No serious advance could be made in the application of algebraic language to the expression of physical phenomena until it could be so extended as to express variation in quantities, as well as the quantities themselves. This extension, worked out independently by Newton and Leibnitz, may be classed as the most fruitful of conceptions in exact science. With it the way was opened for the unimpeded and continually accelerated progress of the last two centuries.

The feature of this period which has the closest relation to the purpose of our coming together is the seemingly unending subdivision of knowledge into specialties, many of which are becoming so minute and so isolated that they seem to have no interest for any but their few pursuers. Happily science itself has afforded a corrective for its own tendency in this direction. The careful thinker will see that in these seemingly diverging branches common elements and common principles are coming more and more to light. There is an increasing recognition of methods of research, and of deduction, which are common to large branches, or to the whole of science. We are more and more recognizing the principle that progress in knowledge implies its reduction to more exact forms, and the expression of its ideas in language more or less mathematical. The problem before the organizers of this Congress was, therefore, to bring the sciences together, and seek for the unity which we believe underlies their infinite diversity.

The assembling of such a body as now fills this hall was scarcely possible in any preceding generation, and is made possible now only through the agency of science itself. It differs from all preceding international meetings by the universality of its scope, which aims to include the whole of knowledge. It is also unique in that none but leaders have been sought out as members. It is unique in that so many lands have delegated their choicest intellects to carry on its work. They come from the country to which our republic is indebted for a third of its territory, including the ground on which we stand; from the land which has taught us that the most scholarly devotion to the languages and learning of the cloistered past is compatible with leadership in the practical application of modern science to the arts of life; from the island whose language and literature have found a new field and a vigorous growth in this region; from the last seat of the holy Roman Empire; from the country which, remembering a monarch who made an astronomical observation at the Greenwich Observatory, has enthroned science in one of the highest places in its government; from the peninsula so learned that we have invited one of its scholars to come and tell us of our own language; from the land which gave birth to Leonardo, Galileo, Torricelli, Columbus, Volta — what an array of immortal names! — from the little republic of glorious history which, breeding men rugged as its eternal snow-peaks, has yet been the seat of scientific investigation since the day of the Bernoullis; from the land whose heroic dwellers did not hesitate to use the ocean itself to protect it against invaders, and which now makes us marvel at the amount of erudition compressed within its little area; from the nation across the Pacific, which, by half a century of unequaled progress in the arts of life, has made an important contribution to evolutionary science through demonstrating the falsity of the theory that the most ancient races are doomed to be left in the rear of the advancing age — in a word, from every great centre of intellectual activity on the globe I see before me eminent representatives of that world-advance in knowledge which we have met to celebrate. May we not confidently hope that the discussions of such an assemblage will prove pregnant of a future for science which shall outshine even its brilliant past?

Gentlemen and scholars all! You do not visit our shores to find great collections in which centuries of humanity have given expression on canvas and in marble to their hopes, fears, and aspirations. Nor do you expect institutions and buildings hoary with age. But as you feel the vigor latent in the fresh air of these expansive prairies, which has collected the products of human genius by which we are here surrounded, and, I may add, brought us together; as you study the institutions which we have founded for the benefit, not only of our own people, but of humanity at large; as you meet the men who, in the short space of one century, have transformed this valley from a savage wilderness into what it is to-day — then may you find compensation for the want of a past like yours by seeing with prophetic eye a future world-power of which this region shall be the seat. If such is to be the outcome of the institutions which we are now building up, then may your present visit be a blessing both to your posterity and ours by making that power one for good to all mankind. Your deliberations will help to demonstrate to us and to the world at large that the reign of law must supplant that of brute force in the relations of the nations, just as it has supplanted it in the relations of individuals. You will help to show that the war which science is now waging against the sources of diseases, pain, and misery offers an even nobler field for the exercise of heroic qualities than can that of battle. We hope that when, after your all too fleeting sojourn in our midst, you return to your own shores, you will long feel the influence of the new air you have breathed in an infusion of increased vigor in pursuing your varied labors. And if a new impetus is thus given to the great intellectual movement of the past century, resulting not only in promoting the unification of knowledge, but in widening its field through new combinations of effort on the part of its votaries, the projectors, organizers, and supporters of this Congress of Arts and Science will be justified of their labors.

DIVISION A — NORMATIVE SCIENCE

SPEAKER : PROFESSOR JOSIAH ROYCE, Harvard University

(*Hall* 6, *September* 20, 10 *a. m.*)

THE SCIENCES OF THE IDEAL

BY JOSIAH ROYCE

[**Josiah Royce**, Professor of History of Philosophy, Harvard University, since 1892. **b.** Grass Valley, Nevada County, California, November 20, 1855. A.B. University of California, 1875; Ph.D. Johns Hopkins, 1878; LL.D. University of Aberdeen, Scotland; LL.D. Johns Hopkins. Instructor in English Literature and Logic, University of California, 1878–82. Instructor and Assistant Professor, Harvard University, 1882–92. **Author of** *Religious Aspect of Philosophy; History of California; The Feud of Oakfield Creek; The Spirit of Modern Philosophy; Studies of Good and Evil; The World and the Individual; Gifford Lectures;* and numerous other works and memoirs.]

I SHALL not attempt, in this address, either to justify or to criticise the name, normative science, under which the doctrines which constitute this division are grouped. It is enough for my purpose to recognize at the outset that I am required, by the plans of this Congress, to explain what scientific interests seem to me to be common to the work of the philosophers and of the mathematicians. The task is one which makes severe demands upon the indulgence of the listener, and upon the expository powers of the speaker, but it is a task for which the present age has well prepared the way. The spirit which Descartes and Leibnitz illustrated seems likely soon to become, in a new and higher sense, prominent in science. The mathematicians are becoming more and more philosophical. The philosophers, in the near future, will become, I believe, more and more mathematical. It is my office to indicate, as well as the brief time and my poor powers may permit, why this ought to be so.

To this end I shall first point out what is that most general community of interest which unites all the sciences that belong to our division. Then I shall indicate what type of recent and special scientific work most obviously bears upon the tasks of all of us alike. Thirdly, I shall state some results and problems to which this type of scientific work has given rise, and shall try to show what promise we have of an early increase of insight regarding our common interests.

I

The most general community of interest which unites the various scientific activities that belong to our division is this: We are all concerned with what may be called ideal truth, as distinct from physical truth. Some of us also have a strong interest in physical truth; but none of us lack a notable and scientific concern for the realm of ideas, viewed as ideas.

Let me explain what I mean by these terms. Whoever studies physical truth (taking that term in its most general sense) seeks to observe, to collate, and, in the end, to control, facts which he regards as external to his own thought. But instead of thus looking mainly without, it is possible for a man chiefly to take account, let us say, of the consequences of his own hypothetical assumptions — assumptions which may possess but a very remote relation to the physical world. Or again, it is possible for such a student to be mainly devoted to reflecting upon the formal validity of his own inferences, or upon the meaning of his own presuppositions, or upon the value and the interrelation of human ideals. Any such scientific work, reflective, considerate principally of the thinker's own constructions and purposes, or of the constructions and purposes of humanity in general, is a pursuit of ideal truth. The searcher who is mainly devoted to the inquiry into what he regards as external facts, is indeed active; but his activity is moulded by an order of existence which he conceives as complete apart from his activity. He is thoughtful; but a power not himself assigns to him the problems about which he thinks. He is guided by ideals; but his principal ideal takes the form of an acceptance of the world as it is, independently of his ideals. His dealings are with nature. His aim is the conquest of a foreign realm. But the student of what may be called, in general terms, ideal truth, while he is devoted as his fellow, the observer of outer nature, to the general purpose of being faithful to the verity as he finds it, is still aware that his own way of finding, or his own creative activity as an inventor of hypotheses, or his own powers of inference, or his conscious ideals, constitute in the main the object into which he is inquiring, and so form an essential aspect of the sort of verity which he is endeavoring to discover. The guide, then, of such a student is, in a peculiar sense, his own reason. His goal is the comprehension of his own meaning, the conscious and thoughtful conquest of himself. His great enemy is not the mystery of outer nature, but the imperfection of his reflective powers. He is, indeed, as unwilling as is any scientific worker to trust private caprices. He feels as little as does the observer of outer facts, that he is merely noting down, as they pass, the chance products of his arbitrary fantasy. For him, as for any scientific student, truth is indeed objective; and the standards to which he conforms are eternal. But his method is that of an inner considerateness rather than of a curiosity about external phenomena. His objective world is at the same time an essentially ideal world, and the eternal verity in whose light he seeks to live has, throughout his undertakings, a peculiarly intimate relation to the purposes of his own constructive will.

One may then sum up the difference of attitude which is here in question by saying that, while the student of outer nature is explicitly conforming his plans of action, his ideas, his ideals, to an order of truth which he takes to be foreign to himself — the student of the other sort of truth, here especially in question, is attempting to understand his own plans of action, that is, to develop his ideas, or to define his ideals, or else to do both these things.

Now it is not hard to see that this search for some sort of ideal truth is indeed characteristic of every one of the investigations which have been grouped together in our division of the normative sciences. Pure mathematics shares in common with philosophy this type of scientific interest in ideal, as distinct from physical or phenomenal truth. There is, to be sure, a marked contrast between the ways in which the mathematician and the philosopher approach, select, and elaborate their respective sorts of problems. But there is also a close relation between the two types of investigation in question. Let us next consider both the contrast and the analogy in some of their other most general features.

Pure mathematics is concerned with the investigation of the logical consequences of certain exactly stateable postulates or hypotheses — such, for instance, as the postulates upon which arithmetic and analysis are founded, or such as the postulates that lie at the basis of any type of geometry. For the pure mathematician, the truth of these hypotheses or postulates depends, not upon the fact that physical nature contains phenomena answering to the postulates, but solely upon the fact that the mathematician is able, with rational consistency, to state these assumed first principles, and to develop their consequences. Dedekind, in his famous essay, " Was Sind und Was Sollen die Zahlen," called the whole numbers "freie Schöpfungen des Menschlichen Geistes;" and, in fact, we need not enter into any discussion of the psychology of our number concept in order to be able to assert that, however we men first came by our conception of the whole numbers, for the mathematician the theory of numerical truth must appear simply as the logical development of the consequences of a few fundamental first principles, such as those which Dedekind himself, or Peano, or other recent writers upon this topic, have, in various forms, stated. A similar formal freedom marks the development of any other theory in the realm of pure mathematics. Pure geometry, from the modern point of view, is neither a doctrine forced upon the human mind by the constitution of any primal form of intuition, nor yet a branch of physical science, limited to describing the spatial arrangement of phenomena in the external world. Pure geometry is the theory of the consequences of certain postulates which the geometer is at liberty consistently to make; so that there are as many types of geometry as there are consistent systems of postulates of that generic type of which the geometer takes account. As is also now well known, it has long been impossible to define pure mathematics as the science of quantity, or to limit the range of the exactly stateable hypotheses or postulates with which the mathematician deals to the world of those objects which, ideally speaking, can be viewed as measurable. For the ideally defined measurable objects are by no means the only ones whose properties can be stated in the form of exact postulates or hypotheses; and the possible range of pure mathematics, if taken in the abstract, and viewed apart from any question as to the value of given lines of research, appears to be identical with the whole realm of the consequences of exactly stateable ideal hypotheses of every type.

One limitation must, however, be mentioned, to which the assertion just made is, in practice, obviously subject. And this is, indeed, a momentous limitation. The exactly stated ideal hypotheses whose consequences the mathematician develops must possess, as is sometimes said, sufficient intrinsic importance to be worthy of scientific treatment. They must not be trivial hypotheses. The mathematician is not, like the solver of chess problems, merely displaying his skill in dealing with the arbitrary fictions of an ideal game. His truth is, indeed, ideal; his world is, indeed, treated by his science as if this world were the creation of his postulates a "freie Schöpfung." But he does not thus create for mere sport. On the contrary, he reports a significant order of truth. As a fact, the ideal systems of the pure mathematician are customarily defined with an obvious, even though often highly abstract and remote, relation to the structure of our ordinary empirical world. Thus the various algebras which have been actually developed have, in the main, definite relations to the structure of the space world of our physical experience. The different systems of ideal geometry, even in all their ideality, still cluster, so to speak, about the suggestions which our daily experience of space and of matter give us. Yet I suppose that no mathematician would be disposed, at the present time, to accept any brief definition of the degree of closeness or remoteness of relation to ordinary experience which shall serve to distinguish a trivial from a genuinely significant branch of mathematical theory. In general, a mathematician who is devoted to the theory of functions, or to group theory, appears to spend little time in attempting to show why the development of the consequences of his postulates is a significant enterprise. The concrete mathematical interest of his inquiry sustains him in his labors, and wins for him the sympathy of his fellows. To the questions, "Why consider the ideal structure of just this system of object at all?" "Why study various sorts of numbers, or the properties of functions, or of groups, or the system of points in projective geometry?" — the pure mathematician in general, cares to reply only, that the topic of his special investigation appears to him to possess sufficient mathematical interest. The freedom of his science thus justifies his enterprise. Yet, as I just pointed out, this freedom is never mere caprice. This ideal interest is not without a general relation to the concerns even of common sense. In brief, as it seems at once fair to say, the pure mathematician is working under the influence of more or less clearly conscious philosophical motives. He does not usually attempt to define what distinguishes a significant from a trivial system of postulates, or what constitutes a problem worth attacking from the point of view of pure mathematics. But he practically recognizes such a distinction between the trivial and the significant regions of the world of ideal truth, and since philosophy is concerned with the significance of ideas, this recognition brings the mathematician near in spirit to the philosopher.

Such, then, is the position of the pure mathematician. What, by way of contrast, is that of the philosopher? We may reply that to state the formal consequences of exact assumptions is one thing; to reflect upon the mutual relations, and the whole significance of such assumptions, does indeed involve other interests; and these other interests are the ones which directly carry us over to the realm of philosophy. If the theory of numbers belongs to pure mathematics, the study of the place of the number concept in the system of human ideas belongs to philosophy. Like the mathematician, the philosopher deals directly with a realm of ideal truth. But to unify our knowledge, to comprehend its sources, its meaning, and its relations to the whole of human life, these aims constitute the proper goal of the philosopher. In order, however, to accomplish his aims, the philosopher must, indeed, take account of the results of the special physical science; but he must also turn from the world of outer phenomena to an ideal world. For the unity of things is never, for us mortals, anything that we find given in our experience. You cannot see the unity of knowledge; you cannot describe it as a phenomenon. It is for us now, an ideal. And precisely so, the meaning of things, the relation of knowledge to life, the significance of our ideals, their bearing upon one another — these are never, for us men, phenomenally present data. Hence the philosopher, however much he ought, as indeed he ought, to take account of phenomena, and of the results of the special physical sciences, is quite as deeply interested in his own way, as the mathematician is interested in his way, in the consideration of an ideal realm. Only, unlike the mathematician, the philosopher does not first abstract from the empirical suggestions upon which his exact ideas are actually based, and then content himself merely with developing the logical consequences of these ideas. On the contrary, his main interest is not in any idea or fact in so far as it is viewed by itself, but rather in the interrelations, in the common significance, in the unity, of all fundamental ideas, and in their relations both to the phenomenal facts and to life! On the whole, he, therefore, neither consents, like the student of a special science of experience, to seek his freedom solely through conformity to the phenomena which are to be described; nor is he content, like the pure mathematician, to win his truth solely through the exact definition of the formal consequences of his freely defined hypotheses. He is making an effort to discover the sense and the unity of the business of his own life.

It is no part of my purpose to attempt to show here how this general philosophical interest differentiates into the various interests of metaphysics, of the philosophy of religion, of ethics, of æsthetics, of logic. Enough — I have tried to illustrate how, while both the philosopher and the mathematician have an interest in the meaning of ideas rather than in the description of external facts, still there is a contrast which does, indeed, keep their work in large measure asunder, namely, the contrast due to the fact that the mathematician is directly concerned with developing the consequences of certain freely assumed systems of postulates or hypotheses; while the philosopher is interested in the significance, in the unity, and in the relation to life, of all the fundamental ideals and postulates of the human mind.

Yet not even thus do we sufficiently state how closely related the two tasks are. For this very contrast, as we have also suggested, is, even within its own limits, no final or perfectly sharp contrast. There is a deep analogy between the two tasks. For the mathematician, as we have just seen, is not evenly interested in developing the consequences of any and every system of freely assumed postulates. He is no mere solver of arbitrary ideal puzzles in general. His systems of postulates are so chosen as to be not trivial, but significant. They are, therefore, in fact, but abstractly defined aspects of the very system of eternal truth whose expression is the universe. In this sense the mathematician is as genuinely interested as is the philosopher in the significant use of his scientific freedom. On the other hand, the philosopher, in reflecting upon the significance and the unity of fundamental ideas, can only do so with success in case he makes due inquiry into the logical consequences of given ideas. And this he can accomplish only if, upon occasion, he employs the exact methods of the mathematician, and develops his systems of ideal truth with the precision of which only mathematical research is capable. As a fact, then, the mathematician and the philosopher deal with ideal truth in ways which are not only contrasted, but profoundly interconnected. The mathematician, in so far as he consciously distinguishes significant from trivial problems, and ideal systems, is a philosopher. The philosopher, in so far as he seeks exactness of logical method, in his reflection, must meanwhile aim to be, within his own limits, a mathematician. He, indeed, will not in future, like Spinoza, seek to reduce philosophy to the mere development, in mathematical form, of the consequences of certain arbitrary hypotheses. He will distinguish between a reflection upon the unity of the system of truth and an abstract development of this or that selected aspect of the system. But he will see more and more that, in so far as he undertakes to be exact, he must aim to become, in his own way, and with due regard to his own purposes, mathematical; and thus the union of mathematical and philosophical inquiries, in the future, will tend to become closer and closer.

II

So far, then, I have dwelt upon extremely general considerations relating to the unity and the contrast of mathematical and philosophical inquiries. I can well conceive, however, that the individual worker in any one of the numerous branches of investigation which are represented by the body of students whom I am privileged to address, may at this point mentally interpose the objection that all these considerations are, indeed, far too general to be of practical interest to any of us. Of course, all we who study these so-called normative sciences are, indeed, interested in ideas, for their own sakes — in ideas so distinct from, although of course also somehow related to, phenomena. Of course, some of us are rather devoted to the development of the consequences of exactly stated ideal hypotheses, and others to reflecting as we can upon what certain ideas and ideals are good for, and upon what the unity is of all ideas and ideals. Of course, if we are wise enough to do so, we have much to learn from one another. But, you will say, the assertion of all these things is a commonplace. The expression of the desire for further mutual coöperation is a pious wish. You will insist upon asking further: "Is there just now any concrete instance in a modern type of research which furnishes results such as are of interest to all of us? Are we actually doing any productive work in common? Are the philosophers contributing anything to human knowledge which has a genuine bearing upon the interests of mathematical science? Are the mathematicians contributing anything to philosophy?"

These questions are perfectly fair. Moreover, as it happens, they can be distinctly answered in the affirmative. The present age is one of a rapid advance in the actual unification of the fields of investigation which are included within the scope of this present division. What little time remains to me must be devoted to indicating, as well as I can, in what sense this is true. I shall have still to deal in very broad generalities. I shall try to make these generalities definite enough to be not wholly unfruitful.

We have already emphasized one question which may be said to interest, in a very direct way, both the mathematician and the philosopher. The ideal postulates, whose consequences mathematical science undertakes to develop, must be, we have said, significant postulates, involving ideas whose exact definition and exposition repay the labor of scientific scrutiny. Number, space, continuity, functional correspondence or dependence, group-structure — these are examples of such significant ideas; the postulates or ideal assumptions upon which the theory of such ideas depends are significant postulates, and are not the mere conventions of an arbitrary game. But now what constitutes the significance of an idea, or of an abstract mathematical theory? What gives an idea a worthy place in the whole scheme of human ideas? Is it the possibility of finding a physical application for a mathematical theory which for us decides what is the value of the theory? No, the theory of functions, the theory of numbers, group theory, have a significance which no mathematician would consent to measure in terms of the present applicability or non-applicability of these theories in physical science? In vain, then, does one attempt to use the test of applied mathematics as the main criticism of the value of a theory of pure mathematics. The value of an idea, for the sciences which constitute our division, is dependent upon the place which this idea occupies in the whole organized scheme or system of human ideas. The idea of number, for instance, familiar as its applications are, does not derive its main value from the fact that eggs and dollars and star-clusters can be counted, but rather from the fact that the idea of numbers has those relations to other fundamental ideas which recent logical theory has made prominent — relations, for instance, to the concept of order, to the theory of classes or collections of objects viewed in general, and to the metaphysical concept of the self. Relations of this sort, which the discussions of the number concept by Dedekind, Cantor, Peano, and Russell have recently brought to light — such relations, I say, constitute what truly justified Gauss in calling the theory of numbers a "divine science." As against such deeper relations, the countless applications of the number concept in ordinary life, and in science, are, from the truly philosophical point of view, of comparatively small moment. What we want, in the work of our division of the sciences, is to bring to light the unity of truth, either, as in mathematics, by developing systems of truth which are significant by virtue of their actual relations to this unity, or, as in philosophy, by explicitly seeking the central idea about which all the many ideas cluster.

Now, an ancient and fundamental problem for the philosophers is that which has been called the problem of the categories. This problem of the categories is simply the more formal aspect of the whole philosophical problem just defined. The philosopher aims to comprehend the unity of the system of human ideas and ideals. Well, then, what are the primal ideas? Upon what group of concepts do the other concepts of human science logically depend? About what central interests is the system of human ideals clustered? In ancient thought Aristotle already approached this problem in one way. Kant, in the eighteenth century, dealt with it in another. We students of philosophy are accustomed to regret what we call the excessive formalism of Kant, to lament that Kant was so much the slave of his own relatively superficial and accidental table of categories, and that he made the treatment of every sort of philosophical problem turn upon his own schematism. Yet we cannot doubt that Kant was right in maintaining that philosophy needs, for the successful development of every one of its departments, a well-devised and substantially complete system of categories. Our objection to Kant's over-confidence in the virtues of his own schematism is due to the fact that we do not now accept his table of categories as an adequate view of the fundamental concepts. The efforts of philosophers since Kant have been repeatedly devoted to the task of replacing his scheme of categories by a more adequate one. I am far from regarding these purely philosophical efforts made since Kant as fruitless, but they have remained, so far, very incomplete, and they have been held back from their due fullness of success by the lack of a sufficiently careful survey and analysis of the processes of thought as these have come to be embodied in the living sciences. Such concepts as number, quantity, space, time, cause, continuity, have been dealt with by the pure philosophers far too summarily and superficially. A more thoroughgoing analysis has been needed. But now, in comparatively recent times, there has developed a region of inquiry which one may call by the general name of modern logic. To the constitution of this new region of inquiry men have principally contributed who began as mathematicians, but who, in the course of their work, have been led to become more and more philosophers. Of late, however, various philosophers, who were originally in no sense mathematicians, becoming aware of the importance of the new type of research, are in their turn attempting both to assimilate and to supplement the undertakings which were begun from the mathematical side. As a result, the logical problem of the categories has to-day become almost equally a problem for the logicians of mathematics and for those students of philosophy who take any serious interest in exactness of method in their own branch of work. The result of this actual coöperation of men from both sides is that, as I think, we are to-day, for the first time, in sight of what is still, as I freely admit, a somewhat distant goal, namely, the relatively complete rational analysis and tabulation of the fundamental categories of human thought. That the student of ethics is as much interested in such an investigation as is the metaphysician, that the philosopher of religion needs a well-completed table of categories quite as much as does the pure logician, every competent student of such topics ought to admit. And that the enterprise in question keenly interests the mathematicians is shown by the prominent part which some of them have taken in the researches in question. Here, then, is the type of recent scientific work whose results most obviously bear upon the tasks of all of us alike.

A catalogue of the names of the workers in this wide field of modern logic would be out of place here. Yet one must, indeed, indicate what lines of research are especially in question. From the purely mathematical side, the investigations of the type to which I now refer may be viewed (somewhat arbitrarily) as beginning with that famous examination into one of the postulates of Euclid's geometry which gave rise to the so-called non-Euclidean geometry. The question here originally at issue was one of a comparatively limited scope, namely, the question whether Euclid's parallel-line postulate was a logical consequence of the other geometrical principles. But the investigation rapidly develops into a general study of the foundations of geometry — a study to which contributions are still almost constantly appearing. Somewhat independently of this line of inquiry there grew up, during the latter half of the nineteenth century, that reëxamination of the bases of arithmetic and analysis which is associated with the names of Dedekind, Weierstrass, and George Cantor. At the present time, the labors of a number of other inquirers (amongst whom we may mention the school of Peano and Pieri in Italy, and men such as Poincaré and Couturat in France, Hilbert in Germany, Bertrand Russell and Whitehead in England, and an energetic group of our American mathematicians — men such as Professor Moore, Professor Halsted, Dr. Huntington, Dr. Veblen, and a considerable number of others) have been added to the earlier researches. The result is that we have recently come for the first time to be able to see, with some completeness, what the assumed first principles of pure mathematics actually are. As was to be expected, these principles are capable of more than one formulation, according as they are approached from one side or from another. As was also to be expected, the entire edifice of pure mathematics, so far as it has yet been erected, actually rests upon a very few fundamental concepts and postulates, however you may formulate them. What was not observed, however, by the earlier, and especially by the philosophical, students of the categories, is the form which these postulates tend to assume when they are rigidly analyzed.

This form depends upon the precise definition and classification of certain types of relations. The whole of geometry, for instance, including metrical geometry, can be developed from a set of postulates which demand the existence of points that stand in certain ordinal relationships. The ordinal relationships can be reduced, according as the series of points considered is open or closed, either to the well-known relationship in which three points stand when one is between the other two upon a right line, or else to the ordinal relationship in which four points stand when they are separated by pairs; and these two ordinal relationships, by means of various logical devices, can be regarded as variations of a single fundamental form. Cayley and Klein founded the logical theory of geometry here in question. Russell, and in another way Dr. Veblen, have given it its most recent expressions. In the same way, the theory of whole numbers can be reduced to sets of principles which demand the existence of certain ideal objects in certain simple ordinal relations. Dedekind and Peano have worked out such ordinal theories of the number concept. In another development of the theory of the cardinal whole numbers, which Russell and Whitehead have worked out, ordinal concepts are introduced only secondarily, and the theory depends upon the fundamental relation of the equivalence or non-equivalence of collections of objects. But here also a certain simple type of relation determines the definitions and the development of the whole theory.

Two results follow from such a fashion of logically analyzing the first principles of mathematical science. In the first place, as just pointed out, we learn *how few and simple are the conceptions and postulates* upon which the actual edifice of exact science rests. Pure mathematics, we have said, is free to assume what it chooses. Yet the assumptions whose presence as the foundation principles of the actually existent pure mathematics an exhaustive examination thus reveals, show by their fewness that the ideal freedom of the mathematician to assume and to construct what he pleases, is indeed, in practice, a very decidedly limited freedom. The limitation is, as we have already seen, a limitation which has to do with the essential significance of the fundamental concepts in question. And so the result of this analysis of the bases of the actually developed and significant branches of mathematics, constitutes a sort of empirical revelation of what categories the exact sciences have practically found to be of such significance as to be worthy of exhaustive treatment. Thus the instinctive sense for significant truth, which has all along been guiding the development of mathematics, comes at least to a clear and philosophical consciousness. And meanwhile the essential categories of thought are seen in a new light.

The second result still more directly concerns a philosophical logic. It is this: Since the few types of relations which this sort of analysis reveals as the fundamental ones in exact science are of such importance, the logic of the present day is especially required to face the questions: *What is the nature of our concept of relations?* What are the various possible types of relations? Upon what does the variety of these types depend? What unity lies beneath the variety?

As a fact, logic, in its modern forms, namely, first that symbolic logic which Boole first formulated, which Mr. Charles S. Peirce and his pupils have in this country already so highly developed, and which Schroeder in Germany, Peano's school in Italy, and a number of recent English writers have so effectively furthered — and secondly, the logic of scientific method, which is now so actively pursued, in France, in Germany, and in the English-speaking countries — this whole movement in modern logic, as I hold, is rapidly approaching *new solutions of the problem of the fundamental nature and the logic of relations.* The problem is one in which we are all equally interested. To De Morgan in England, in an earlier generation, and, in our time, to Charles Peirce in this country, very important stages in the growth of these problems are due. Russell, in his work on the *Principles of Mathematics* has very lately undertaken to sum up the results of the logic of relations, as thus far developed, and to add his own interpretations. Yet I think that Russell has failed to get as near to the foundations of the theory of relations as the present state of the discussion permits. For Russell has failed to take account of what I hold to be the most fundamentally important generalization yet reached in the general theory of relations. This is the generalization set forth as early as 1890, by Mr. A. B. Kempe, of London, in a pair of wonderful but too much neglected, papers, entitled, respectively, *The Theory of Mathematical Form*, and *The Analogy between the Logical Theory of Classes and the Geometrical Theory of Points.* A mere hint first as to the more precise formulation of the problem at issue, and then later as to Kempe's special contribution to that problem, may be in order here, despite the impossibility of any adequate statement.

III

The two most obviously and universally important kinds of relations known to the exact sciences, as these sciences at present exist, are: (1) The relations of the type of equality or equivalence; and

(2) the relations of the type of before and after, or greater and less. The first of these two classes of relations, namely, the class represented, although by no means exhausted, by the various relations actually called, in different branches of science by the one name equality, this class I say, might well be named, as I myself have proposed, the leveling relations. A collection of objects between any two of which some one relation of this type holds, may be said to be a collection whose members, in some defined sense or other, are on the same level. The second of these two classes of relations, namely, those of the type of before and after, or greater and less — this class of relations, I say, consists of what are nowadays often called the serial relations. And a collection of objects such that, if any pair of these objects be chosen, a determinate one of this pair stands to the other one of the same pair in some determinate relation of this second type, and in a relation which remains constant for all the pairs that can be thus formed out of the members of this collection — any such collection, I say, constitutes a one-dimensional open series. Thus, in case of a file of men, if you choose any pair of men belonging to the file, a determinate one of them is, in the file, before the other. In the number series, of any two numbers, a determinate one is greater than the other. Wherever such a state of affairs exists, one has a series.

Now these two classes of relations, the leveling relations and the serial relations, agree with one another, and differ from one another in very momentous ways. They *agree* with one another in that both the leveling and the serial relations are what is technically called *transitive;* that is, both classes conform to what Professor James has called the law of "skipped intermediaries." Thus, if A is equal to B, and B is equal to C, it follows that A is equal to C. If A is before B, and B is before C, then A is before C. And this property, which enables you in your reasonings about these relations to skip middle terms, and so to perform some operation of elimination, is the property which is meant when one calls relations of this type transitive. But, on the other hand, these two classes of relations *differ* from each other in that the leveling relations are, while the serial relations are not, *symmetrical* or reciprocal. Thus, if A is equal to B, B is equal to A. But if X is greater than Y, then Y is not greater than X, but less than X. So the leveling relations are symmetrical transitive relations. But the serial relations are transitive relations which are not symmetrical.

All this is now well known. It is notable, however, that nearly all the processes of our exact sciences, as at present developed, can be said to be essentially such as lead either to the placing of sets or classes of objects on the same level, by means of the use of symmetrical transitive relations, or else to the arranging of objects in orderly rows or series, by means of the use of transitive relations which are not symmetrical. This holds also of all the applications of the exact sciences. Whatever else you do in science (or, for that matter, in art), you always lead, in the end, either to the arranging of objects, or of ideas, or of acts, or of movements, in rows or series, or else to the placing of objects or ideas of some sort on the same level, by virtue of some equivalence, or of some invariant character. Thus numbers, functions, lines in geometry, give you examples of serial relations. Equations in mathematics are classic instances of leveling relations. So, of course, are invariants. Thus, again, the whole modern theory of energy consists of two parts, one of which has to do with levels of energy, in so far as the quantity of energy of a closed system remains invariant through all the transformations of the system, while the other part has to do with the irreversible serial order of the transformations of energy themselves, which follow a set of unsymmetrical relations, in so far as energy tends to fall from higher to lower levels of intensity within the same system.

The entire conceivable universe then, and all of our present exact science, can be viewed, if you choose, as a collection of objects or of ideas that, whatever other types of relations may exist, are at least largely characterized either by the leveling relations, or by the serial relations, or by complexes of both sorts of relations. Here, then, we are plainly dealing with very fundamental categories. The "between" relations of geometry can of course be defined, if you choose, in terms of transitive relations that are not symmetrical. There are, to be sure, some other relations present in exact science, but the two types, the serial and leveling relations, are especially notable.

So far the modern logicians have for some time been in substantial agreement. Russell's brilliant book is a development of the logic of mathematics very largely in terms of the two types of relations which, in my own way, I have just characterized; although Russell gives due regard, of course, to certain other types of relations.

But hereupon the question arises, "Are these two types of relations what Russell holds them to be, namely, ultimate and irreducible logical facts, unanalyzable categories — mere data for the thinker? Or can we reduce them still further, and thus simplify yet again our view of the categories?

Here is where Kempe's generalization begins to come into sight. These two categories, in at least one very fundamental realm of exact thought, can be reduced to one. There is, namely, a world of ideal objects which especially interest the logician. It is the world of a *totality of possible logical classes*, or again, it is the ideal world, equivalent in formal structure to the foregoing, but composed of a *totality of possible statements*, or thirdly, it is the world, equivalent once more, in formal structure, to the foregoing, but consisting of a *totality of possible acts of will*, of possible decisions. When we proceed to consider the relational structure of such a world, taken merely in the abstract as such a structure, a relation comes into sight which at once appears to be peculiarly general in its nature. It is the so-called illative relation, the relation which obtains between two classes when one is subsumed under the other, or between two statements, or two decisions, when one implies or entails the other. This relation is transitive, but may be either symmetrical or not symmetrical; so that, according as it is symmetrical or not, it may be used either to establish levels or to generate series. In the order system of the logician's world, the relational structure is thus, in any case, a highly general and fundamental one.

But this is not all. In this the logician's world of classes, or of statements, or of decisions, there is also another relation observable. This is the relation of exclusion or mutual opposition. This is a purely symmetrical or reciprocal relation. It has two forms — obverse or contradictory opposition, that is, negation proper, and contrary opposition. But both these forms are purely symmetrical. And by proper devices each of them can be stated in terms of the other, or reduced to the other. And further, as Kempe incidentally shows, and as Mrs. Ladd Franklin has also substantially shown in her important theory of the syllogism, *it is possible to state every proposition, or complex of propositions involving the illative relation, in terms of this purely symmetrical relation of opposition.* Hence, so far as mere relational form is concerned, the illative relation itself may be wholly reduced to the symmetrical relation of opposition. This is our first result as to the relational structure of the realm of pure logic, that is, the realm of classes, of statements, or of decisions.

It follows that, in describing the logician's world of possible classes or of possible decisions, *all unsymmetrical, and so all serial, relations can be stated solely in terms of symmetrical relations, and can be entirely reduced to such relations.* Moreover, as Kempe has also very prettily shown, the relation of opposition, in its two forms, just mentioned, need not be interpreted as obtaining merely between pairs of objects. It may and does obtain between triads, tetrads, *n*-ads of logical entities; and so all that is true of the relations of logical classes may consequently be stated merely by ascribing certain perfectly symmetrical and homogeneous predicates to pairs, triads, tetrads, *n*-ads of logical objects. The essential contrast between symmetrical and unsymmetrical relations thus, in this ideal realm of the logician, simply vanishes. The categories of the logician's world of classes, of statements, or of decisions, are marvelously simple. All the relations present may be viewed as variations of the mere conception of opposition as distinct from non-opposition.

All this holds, of course, so far, merely for the logician's world of classes or of decisions. There, at least, all serial order can actually be derived from wholly symmetrical relations. But Kempe now very beautifully shows (and here lies his great and original contribution to our topic) — he shows, I say, that the ordinal relations of geometry, as well as of the number-system, can all be regarded as indistinguishable from *mere variations of those relations which, in pure logic, one finds to be the symmetrical relations obtaining within pairs or triads of classes or of statements.* The formal identity of the geometrical relation called "between" with a purely logical relation which one can define as existing or as not existing amongst the members of a given triad of logical classes, or of logical statements, is shown by Kempe in a fashion that I cannot here attempt to expound. But Kempe's result thus enables one, as I believe, to simplify the theory of relations far beyond the point which Russell in his brilliant book has reached. For Kempe's triadic relation in question can be stated, in what he calls its obverse form, in perfectly symmetrical terms. And he proves very exactly that the resulting logical relation is precisely identical, in all its properties, with the fundamental ordinal relation of geometry.

Thus the order-systems of geometry and analysis appear simply as special cases of the more general order-system of pure logic. The whole, both of analysis and of geometry, can be regarded as a description of certain selected groups of entities, which are chosen, according to special rules, from a single ideal world. This general and inclusive ideal world consists simply of *all the objects which can stand to one another in those symmetrical relations wherein the pure logician finds various statements, or various decisions inevitably standing.* "Let me," says in substance Kempe, "choose from the logician's ideal world of classes or decisions, what entities I will; and I will show you a collection of objects that are in their relational structure, precisely identical with the points of a geometer's space of n dimensions." In other words, all of the geometer's figures and relations can be precisely pictured by the relational structure of a selected system of classes or of statements, whose relations are wholly and explicitly logical relations, such as opposition, and whose relations may all be regarded, accordingly, as reducible to a single type of purely symmetrical relation.

Thus, for *all* exact science, and not merely for the logician's special realm, the contrast between symmetrical and unsymmetrical relations proves to be, after all, superficial and derived. The purely logical categories, such as opposition, and such as hold within the calculus of statements, are, apparently, the basal categories of all the exact science that has yet been developed. Series and levels are relational structures that, sharply as they are contrasted, can be derived from a single root.

I have restated Kempe's generalization in my own way. I think it the most promising step towards new light as to the categories that we have made for some generations.

In the field of modern logic, I say, then, work is doing which is rapidly tending towards the unification of the tasks of our entire division. For this problem of the categories, in all its abstractness, is still a common problem for all of us. Do you ask, however, what such researches can do to furnish more special aid to the workers in metaphysics, in the philosophy of religion, in ethics, or in æsthetics, beyond merely helping towards the formulation of a table of categories — then I reply that we are already not without evidence that such general researches, abstract though they may seem, are bearing fruits which have much more than a merely special interest. Apart from its most general problems, that analysis of mathematical concepts to which I have referred has in any case revealed numerous unexpected connections between departments of thought which had seemed to be very widely sundered. One instance of such a connection I myself have elsewhere discussed at length, in its general metaphysical bearings. I refer to the logical identity which Dedekind first pointed out between the mathematical concept of the ordinal number of series and the philosophical concept of the formal structure of an ideally completed self. I have maintained that this formal identity throws light upon problems which have as genuine an interest for the student of the philosophy of religion as for the logician of arithmetic. In the same connection it may be remarked that, as Couturat and Russell, amongst other writers, have very clearly and beautifully shown, the argument of the Kantian mathematical antinomies needs to be explicitly and totally revised in the light of Cantor's modern theory of infinite collections. To pass at once to another, and a very different instance: The modern mathematical conceptions of what is called group theory have already received very wide and significant applications, and promise to bring into unity regions of research which, until recently, appeared to have little or nothing to do with one another. Quite lately, however, there are signs that group theory will soon prove to be of importance for the definition of some of the fundamental concepts of that most refractory branch of philosophical inquiry, æsthetics. Dr. Emch, in an important paper in the *Monist*, called attention, some time since, to the symmetry groups to which certain æsthetically pleasing forms belong, and endeavored to point out the empirical relations between these groups and the æsthetic effects in question.

The grounds for such a connection between the groups in question and the observed æsthetic effects, seemed, in the paper of Dr. Emch to be left largely in the dark. But certain papers recently published in the country by Miss Ethel Puffer, bearing upon the psychology of the beautiful (although the author has approached the subject without being in the least consciously influenced, as I understand, by the conceptions of the mathematical group theory), still actually lead, if I correctly grasp the writer's meaning, to the doctrine that the æsthetic object, viewed as a psychological whole, must possess a structure closely, if not precisely, equivalent to the ideal structure of what the mathematician calls a group. I myself have no authority regarding æsthetic concepts, and speak subject to correction. But the unexpected, and in case of Miss Puffer's research, quite unintended, appearance of group theory in recent æsthetic analysis is to me an impressive instance of the use of relatively new mathematical conceptions in philosophical regions which *seem*, at first sight, very remote from mathematics.

That both the group concept and the concept of the self just suggested are sure to have also a wide application in the ethics of the future, I am myself well convinced. In fact, no branch of philosophy is without close relations to all such studies of fundamental categories.

These are but hints and examples. They suffice, I hope, to show that the workers in this division have deep common interests, and will do well, in future, to study the arts of coöperation, and to regard one another's progress with a watchful and cordial sympathy. In a word: Our common problem is the theory of the categories. That problem can be solved only by the coöperation of the mathematicians and of the philosophers.

DEPARTMENT I — PHILOSOPHY

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(*Hall 6, September 20, 11.15 a. m.*)

CHAIRMAN: PROFESSOR BORDEN P. BOWNE, Boston University.
SPEAKERS: PROFESSOR GEORGE H. HOWISON, University of California.
PROFESSOR GEORGE T. LADD, Yale University.

IN opening the Department of Philosophy, the Chairman, Professor Borden P. Bowne, LL.D., of Boston University, made an interesting address on the Philosophical Outlook. Professor Bowne said in part: —

I congratulate the members of the Philosophical Section on the improved outlook in philosophy. In the generation just passed, philosophy was somewhat at a discount. The great and rapid development of physical science and invention, together with the profound changes in biological thought, produced for a time a kind of chaos. New facts were showered upon us in great abundance, and we had no adequate philosophical preparation for dealing with them. Such a condition is always disturbing. The old mental equilibrium is overthrown and readjustment is a slow process. Besides, the shallow sense philosophy of that time readily lent itself to mechanical and materialistic interpretations, and for a while it seemed as if all the higher faiths of humanity were permanently discredited. All this has passed away. Philosophical criticism began its work and the naïve dogmatism of materialistic naturalism was soon disposed of. It quickly appeared that our trouble was not due to the new facts, but to the superficial philosophy by which they had been interpreted. Now that we have a better philosophy, we have come to live in perfect peace with the facts once thought disturbing, and even to welcome them as valuable additions to knowledge. . . .

The brief naturalistic episode was not without instruction for us. It showed conclusively the great practical importance of philosophy. Had we had thirty years ago the current philosophical insight, the great development of the physical and biological sciences would have made no disturbance whatever. But being interpreted by a crude scheme of thought, it produced somewhat of a storm. Philosophy may not contribute much of positive value, but it certainly has an important negative function in the way of suppressing pretentious dogmatism and fictitious knowledge, which often lead men astray. It is these things which produce conflicts of science and religion or which find in evolution the solvent of all mysteries and the source of all knowledge.

Concerning the partition of territory between science and philosophy, there are two distinct questions respecting the facts of experience. First, we need to know the facts in their temporal and spatial order, and the way they hang together in a system of law. To get this knowledge is the function of science, and in this work science has inalienable rights and a most important practical function. This work cannot be done by speculation nor interfered with by authority of any kind. It is not surprising, then, that scientists in their sense of contact with reality should be indignant with, or feel contempt for, any who seek to limit or proscribe their research. But supposing this work all done, there remains another question respecting the causality and interpretation of the facts. This question belongs to philosophy. Science describes and registers the facts with their temporal and spatial laws; philosophy studies their causality and significance. And while the scientist justly ignores the philosopher who interferes with his inquiries, so the philosopher may justly reproach the scientist who fails to see that the scientific question does not touch the philosophic one. . . .

In the field of metaphysics proper I note a strong tendency toward personal idealism, or as it might be called, Personalism; that is, the doctrine that substantial reality can be conceived only under the personal form and that all else is phenomenal. This is quite distinct from the traditional idealisms of mere conceptionism. It holds the essential fact to be a community of persons with a Supreme Person at their head while the phenomenal world is only expression and means of communication. And to this view we are led by the failure of philosophizing on the impersonal plane, which is sure to lose itself in contradiction and impossibility. Under the form of mechanical naturalism, with its tendencies to materialism and atheism, impersonalism has once more been judged and found wanting. We are not likely to have a recurrence of this view unless there be a return to philosophical barbarism. But impersonalism at the opposite pole in the form of abstract categories of being, causality, unity, identity, continuity, sufficient reason, etc., is equally untenable. Criticism shows that these categories when abstractly and impersonally taken cancel themselves. On the impersonal plane we can never reach unity from plurality, or plurality from unity; and we can never find change in identity, or identity in change. Continuity in time becomes mere succession without the notion of potentiality, and this in turn is empty. Existence itself is dispersed into nothingness through the infinite divisibility of space and time, while the law of the sufficient reason loses itself in barren tautology and the infinite regress. The necessary logical equivalence of cause and effect in any impersonal scheme makes all real explanation and progress impossible, and shuts us up to an unintelligible oscillation between potentiality and actuality, to which there is no corresponding thought. . . .

Philosophy is still militant and has much work before it, but the omens are auspicious, the problems are better understood, and we are coming to a synthesis of the results of past generations of thinking which will be a very distinct progress. Philosophy has already done good service, and never better than in recent times, by destroying pretended knowledge and making room for the higher faiths of humanity. It has also done good service in helping these faiths to better rational form, and thus securing them against the defilements of superstition and the cavilings of hostile critics. With all its aberrations and shortcomings, philosophy deserves well of humanity.

PHILOSOPHY: ITS FUNDAMENTAL CONCEPTIONS AND ITS METHODS

BY GEORGE HOLMES HOWISON

[**George Holmes Howison,** Mills Professor of Intellectual and Moral Philosophy and Civil Polity, University of California. **b.** Montgomery County, Maryland, 1834. A.B. Marietta College, 1852; M.A. 1855; LL.D. *ibid.* 1883. Post-graduate, Lane Theological Seminary, University of Berlin, and Oxford. Headmaster High School, Salem, Mass., 1862–64; Assistant Professor of Mathematics, Washington University, St. Louis, 1864–66; Tileston Professor of Political Economy, *ibid.* 1866–69; Professor of Logic and the Philosophy of Science, Massachusetts Institute of Technology, 1871–79; Lecturer on Ethics, Harvard University, 1879–80; Lecturer on Logic and Speculative Philosophy, University of Michigan, 1883–84. Member and vice-president St. Louis Philosophical Society; member California Historical Society; American Historical Association; American Association for the Advancement of Science; National Geographic Society, etc. **Author of** *Treatise on Analytic Geometry*, 1869; *The Limits of Evolution*, 1901, 2d edition, 1904; joint author and editor of *The Conception of God*, 1897, etc. **Editor** Philosophical Publications of University of California; American Editorial Representative *Hibbert Journal*, London.]

The duty has been assigned me, honored colleagues, of addressing you on the Fundamental Conceptions and the Methods of our common pursuit — philosophy. In endeavoring to deal with the subject in a way not unworthy of its depth and its extent, I have found it impossible to bring the essential material within less compass than would occupy, in reading, at least four times the period granted by our programme. I have therefore complied with the rule of the Congress which directs that, if a more extended writing be left with the authorities for publication, the reading must be restricted to such a portion of it as will not exceed the allotted time. I will accordingly read to you, first, a brief summary of my entire discussion, by way of introduction, and then an excerpt from the larger document, which may serve for a *specimen,* as our scholastic predecessors used to say, of the whole inquiry I have carried out. The impression will, of course, be fragmentary, and I must ask beforehand for your most benevolent allowances, to prevent a judgment too unfavorable.

The discussion naturally falls into two main parts: the first dealing with the Fundamental Conceptions; and the second, with the Methods.

In the former, after presenting the conception of philosophy itself, as *the consideration of things in the light of the whole,* I take up the involved Fundamental Concepts in the following order: —

I. Whole and Part;
II. Subject and Object (Knowing and Being, Mind and Matter; Dualism, Materialism, Idealism);
III. Reality and Appearance (Noumenon and Phenomenon);

IV. Cause and Effect (Ground and Consequence; Causal System);
V. One and Many (Number System; Monism and Pluralism);
VI. Time and Space (their relation to Number; their Origin and Real Meaning);
VII. Unconditioned and Conditioned (Soul, World, God; their Reinterpretation in terms of Pluralism);
VIII. The True, the Beautiful, the Good (their relation to the question between Monism and Pluralism).

These are successively dealt with as they rise one out of the other in the process of interpreting them and applying them in the actual creation of philosophy, as this goes on in the historic schools. The theoretic progress of philosophy is in this way explained by them, in its movement from natural dualism, or realism, through the successive forms of monism, materialistic, agnostic, and idealistic, until it reaches the issue, now coming so strongly forward within the school of idealism, between the adherents of monism and those of pluralism.

The importance of the Fundamental Concepts is shown to increase as we pass along the list, till on reaching Cause and Effect, and entering upon its full interpretation into the complete System of Causes, we arrive at the very significant conception of the RECIPROCITY OF FIRST CAUSES, and through it come to the PRIMACY OF FINAL CAUSE, and the derivative position of the other forms of cause, Material, Formal, Efficient. The philosophic strength of idealism, but especially of idealistic pluralism, comes into clear light as the result of this stage of the inquiry. But it appears yet more decidedly when One and Many, Time and Space, and their interrelations, are subjected to analysis. So the discussion next passes to the higher conceptions, Soul, World, God, by the pathway of the correlation Unconditioned and Conditioned, and its kindred contrasts Absolute and Relative, Necessary and Contingent, Infinite and Finite, corroborating and reinforcing the import of idealism, and, still more decidedly, that of its plural form. Finally, the strong and favorable bearing of this last on the dissolution of agnosticism and the habilitation of the ideals, the True, the Beautiful, and the Good, in a heightened meaning, is brought out.

This carries the inquiry to the second part of it, that of the Philosophical Methods. Here I recount these in a series of six: the Dogmatic, the Skeptical, the Critical, the Pragmatic, the Genetic, the Dialectic. These, I show, in spite of the tendency of the earlier members in the series to over-emphasis, all have their place and function in the development of a complete philosophy, and in fact form an ascending series in methodic effectiveness, all that precede the last being taken up into the comprehensive Critical Rationalism of the last. Methodology thus passes upward, over the ascending and widening roadways of (1) Intuition and Deduction; (2) Experience and Induction; (3) Intuition and Experience adjusted by Critical Limits; (4) Skepticism reinforced and made *quasi*-affirmative by Desire and Will; (5) Empiricism enlarged by substitution of cosmic and psychic history for subjective consciousness; (6) Enlightened return to a Rationalism critically established by the inclusion of the preceding elements, and by the sifting and the grading of the Fundamental Concepts through their behavior when tested by the effort to make them universal. In this way, the methods fall into a System, the organic principle of which is this principle of Dialectic, which proves itself alone able to establish *necessary* truths; that is, *truths indeed*, — judgments that are seen to exclude their opposites, because, in the attempt to substitute the opposite, the place of it is still filled by the judgment which it aims to dislodge.

And now, with your favoring leave, I will read the excerpt from my larger text.

The task to which, in an especial sense, the cultivators of philosophy are summoned by the plans of the present Congress of Arts and Science, is certainly such as to stir an ambition to achieve it. At the same time, it tempers eagerness by its vast difficulty, and the apprehension lest this may prove insuperable. The task, the officers of the Congress tell us, is no less than to promote the unification of all human knowledge. It requires, then, the reduction of the enormous detail in our present miscellany of sciences and arts, which to a general glance, or even to a more intimate view, presents a confusion of differences that seems overwhelming, to a system nevertheless clearly harmonious, — founded, that is to say, upon universal principles which control all differences by explaining them, and which therefore, in the last resort, themselves flow lucidly from a single supreme principle. Simply to state this meaning of the task set us, is enough to awaken the doubt of its practicability.

This doubt, we are bound to confess, has more and more impressed itself upon the general mind, the farther this has advanced in the experience of scientific discovery. The very increase in the multiplicity and complexity of facts and their causal groupings increases the feeling that at the root of things there is "a final inexplicability" — total reality seems, more and more, too vast, too profound, for us to grasp or to fathom. And yet, strangely enough, this increasing sense of mysterious vastness has not in the least prevented the modern mind from more and more asserting, with a steadily increasing insistence, the essential and unchangeable unity of that whole of things which to our ordinary experience, and even to all our sciences, appears such an endless and impenetrable complex of differences, — yes, of contradictions. In fact, this assertion of the unity of all things, under the favorite name of the Unity of Nature, is the pet dogma of modern science; or, rather, to speak with right accuracy, it is the stock-in-trade of a *philosophy* of science, current among many of the leaders of modern science; for every such assertion, covering, as it tacitly and unavoidably does, a view about the absolute whole, is an assertion belonging to the province of philosophy, before whose tribunal it must come for the assessment of its value. The presuppositions of all the special sciences, and, above all, this presupposition of the Unity and Uniformity of Nature, common to all of them, must thus come back for justification and requisite definition to philosophy — that uppermost and all-inclusive form of cognition which addresses itself to the whole as whole. In their common assertion of the Unity of Nature, the exponents of modern science come unawares out of their own province into quite another and a higher; and in doing so they show how unawares they come, by presenting in most instances the curious spectacle of proclaiming at once their increasing belief in the unity of things, and their increasing disbelief in its penetrability by our intelligence: —

In's Innere der Natur,
Dringt kein erschaffner Geist,

is their chosen poet's expression of their philosophic mood. Curious we have the right to call this state of the scientific mind, because it is to critical reflection so certainly self-contradictory. How can there be a real unity belonging to what is inscrutable? — what evidence of unity can there be, except in intelligible and explanatory continuity?

But, at all events, this very mood of agnostic self-contradiction, into which the development of the sciences casts such a multitude of minds, brings them, — brings all of us, — as already indicated, into that court of philosophy where alone such issues lawfully belong, and where alone they can be adjudicated. If the unification of the sciences can be made out to be real by making out its sole sufficient condition, namely, that there is a genuine, and not a merely nominal, unity in the whole of reality itself, — a unity that explains because it is itself, not simply intelligible, but the only completely intelligible of things, — this desirable result must be the work of philosophy. However difficult the task may be, it is rightly put upon us who belong to the Department listed first among the twenty-four in the programme of this representative Congress.

I cannot but express my own satisfaction, as a member of this Department, nor fail to extend my congratulations to you who are my colleagues in it, that the Congress, in its programme, takes openly the affirmative on this question of the possible unification of knowledge. The Congress has thus declared beforehand for the practicability of the task it sets. It has even declared for its not distant accomplishment; indeed, not impossibly, its accomplishment through the transactions of the Congress itself; and it indicates, by no uncertain signs, the leading, the determining part that philosophy must have in the achievement. In fact, the authorities of the Congress themselves suggest a solution of their own for their problem. In their programme we see a renewed Hierarchy of the Sciences, and at the summit of this appears now again, after so long a period of humiliating obscuration, the figure of Philosophy, raised anew to that supremacy, as Queen of the Sciences, which had been hers from the days of Plato to those of Copernicus, but which she began to lose when modern physical and historical research entered upon its course of sudden development, and which, until recently, she has continued more and more to lose as the sciences have advanced in their career of discoveries, — ever more unexpected, more astonishing, yet more convincing and more helpful to the welfare of mankind. May this sign of her recovered empire not fail! If we rejoice at the token, the Congress has made it our part to see that the title is vindicated. It is ours to show this normative function of philosophy, this power to reign as the unifying discipline in the entire realm of our possible knowledge; to show it by showing that the very nature of philosophy — its elemental concepts and its directing ideals, its methods taken in their systematic succession — is such as must result in a view of universal reality that will supply the principle at once giving rise to all the sciences and connecting them all into one harmonious whole.

Such, and so grave, my honored colleagues, is the duty assigned to this hour. Sincerely can I say, Would it had fallen to stronger hands than mine! But since to mine it has been committed, I will undertake it in no disheartened spirit; rather, in that temper of animated hope in which the whole Congress has been conceived and planned. And I draw encouragement from the place, and its associations, where we are assembled — from its historic connections not only with the external expansion of our country, but with its growth in culture, and especially with its growth in the cultivation of philosophy. For your speaker, at least, can never forget that here in St. Louis, the metropolis of the region by which our national domain was in the Louisiana Purchase so enlarged, — here was the centre of a movement in philosophic study that has proved to be of national import. It is fitting that we all, here to-day, near to the scene itself, commemorate the public service done by our present National Commissioner of Education and his group of enthusiastic associates, in beginning here, in the middle years of the preceding century, those studies of Kant and his great idealistic successors that unexpectedly became the nucleus of a wider and more penetrating study of philosophy in all parts of our country. It is with quickened memories belonging to the spot where, more than five-and-thirty years ago, it was my happy fortune to take some part with Dr. Harris and his companions, that I begin the task assigned me. The undertaking seems less hopeless when I can here recall the names and the congenial labors of Harris, of Davidson, of Brockmeyer, of Snider, of Watters, of Jones, — half of them now gone from life. They "builded better than they knew;" and, humbly as they may themselves have estimated their ingenuous efforts to gain acquaintance with the greatest thoughts, history will not fail to take note of what they did, as marking one of the turning-points in the culture of our nation. The publication of the *Journal of Speculative Philosophy*, granting all the subtractions claimed by its critics on the score of defects (of which its conductors were perhaps only too sensible), was an influence that told in all our circles of philosophical study, and thence in the whole of our social as well as our academic life.

[Here I enter upon the discussion of the subject proper, beginning, as above indicated, with the Fundamental Conceptions. Having followed these through the contrasts Whole and Part, Subject and Object, Reality and Appearance (or Noumenon and Phenomenon), and developed the bearing of these on the procedure of thought from the dualism of natural realism to materialism and thence to idealism, with the issue now coming on, in this last, between monism and pluralism, I strike into the contrast Cause and Effect, and, noting its unfolding into the more comprehensive form of Ground and Consequence, go on thence as follows:]

It is plain that the contrast Ground and Consequence will enable us to state the new issue with closer precision and pertinence than Reality and Appearance, Noumenon and Phenomenon, can supply; while, at the same time, Ground and Consequence exhibits Cause and Effect as presenting a contrast that only fulfills what Noumenon and Phenomenon foretold and strove towards; in fact, what was more remotely, but not less surely, also indicated by Whole and Part, Knowing and Being, Subject and Object. For in penetrating to the coherent meaning of these conceptions, the philosophic movement, as we saw, advanced steadily to the fuller and fuller translating of each of them into the reality that unifies *by explanation*, instead of pretending to explain by merely unifying; and this, of course, will now be put forward explicitly, in the clarified category of Cause and Effect, transfigured from a physical into a purely logical relation. What idealism now says, in terms of this, is that the Cause (or, as we now read it, the Ground) of all that exists is the Subject; is Mind, the intelligently Self-conscious; and that all things else, the *mere* objects, material things, are its Consequence, its Outcome, —

in that sense its Effect. And what the new pluralistic idealism says, is that the *assemblage of individual minds* — intelligence being essentially personal and individual, and never merely universal and collective — is the true total Cause of all, and that every mind thus belongs to the order of First Causes; nevertheless, that part, and the most significant part, of the nature of every mind, essential to its personality and its reason, is *its recognition of other minds in the very act of its own self-definition.* That is to say, a mind by its spontaneous nature as intelligence, by its intrinsic rational or logical genius, puts itself as member of a *system* of minds; all minds are put by each other as Ends — completely standard and sacred Objects, as much parts of the system of true Causes as each is, in its capacity of Subject; and we have a noumenal Reality that is properly to be described as the eternal Federal Republic of Spirits.

Consequently, the relation of Cause and Effect now expands and heightens into a system of the RECIPROCITY OF FIRST CAUSES; causes, that is, which, while all coefficients in the existence and explanation of that natural world of experience which forms their passive effect, their objects of mere perception, are themselves related only in the higher way of Final Causes — that is, Defining-Bases and Ends — of each other, making them the logical Complements, and the Objects of conduct, all for each, and each for all. Hence, the system of causation undergoes a signal transformation, and proves to be organized by Final Cause as its basis and root, instead of by Efficient Cause, or Originating Ground, as the earlier stages of thinking had always assumed.

The causal relation between the absolute or primary realities being purely Final, or Defining and Purposive; that is to say, the uncoercive influence of recognition and ideality; all the other forms of cause, as grouped by Aristotle, — Material, Formal, and Efficient, — are seen to be the derivatives of Final Cause, as being supplied by the action of the minds that, as absolute or underived realities, exist only in the relation of mutual Complements and Ends. Accordingly, Efficient Cause operates only from minds, as noumena, to matter, as their phenomenon, their presented contents of experience; or, in a secondary and derivative sense, from one phenomenon to another, or from one group of phenomena to another group, these playing the part of transmitters, or (as some logicians would say) Instrumental Causes, or Means. Cause, as Material, is hence defined as the elementary phenomenon, and the combinations of this; and therefore, strictly taken, is merely Effect (or Outcome) of the self-active consciousness, whose spontaneous forms of conception and perception become the Formal Cause that organizes the sum of phenomena into cosmic harmony or unity.

Here, accordingly, comes into view the further and in some respects deeper conceptual pair, Many and One. The history of philosophic thought proves that this antithesis is darkly obscure and deeply ambiguous; for about it have centred a large part of the conflicts of doctrine. This pair has already been used, implicitly, in exhibiting the development of the preceding group, Cause and Effect; and in so using it we have supplied ourselves with a partial clarification of it, and with one possible solution of its ambiguity. We have seen, namely, how our strong natural persuasion that philosophy guided by the fundamental concept Cause must become the search for the One amid the wilderness of the Many, and that this search cannot be satisfied and ended except in an all-inclusive Unit, in which the Many is embraced as the integral and originated parts, completely determined, subjected, and controlled, may give way to another and less oppressive conception of unity; a conception of it as the harmony among many free and independent primary realities, a harmony founded on their intelligent and reasonable mutual recognition. This conception casts at least *some* clearing light upon the long and dreary disputes over the Many and the One; for it exposes, plainly, the main source of them. They have arisen out of two chief ambiguities, — the ambiguity of the concept One, and the ambiguity of the concept Cause in its supreme meaning. The normal contrast between the One and the Many is a clear and simple contrast: the One is the single unit, and the Many is the repetition of the unit, or is the collection of the several units. But if we go on to suppose that there is a collection or sum of all possible units, and call this the Whole, then, since there can be no second such, we call it also "one" (or the One, by way of preëminence), overlooking the fact that it differs from the simple one, or unit, *in genere;* that it is in fact not a unit at all, not an elementary member of a series, but the annulment of all series; that our name "one" has profoundly changed its meaning, and now stands for the Sole, the Only. Thus, by our forgetfulness of differences, we fall into deep water, and, with the confused illusions of the drowning, dream of the One and All as the single *punctum originationis* of all things, the Source and Begetter of the very units of which it is in reality only the resultant and the derivative. Or, from another point of view, and in another mood, we rightly enough take the One to mean the coherent, the intelligible, the consistent, the harmonious; and putting the Many, on the misleading hint of its contrast to the unit, in antithesis to this One of harmony, we fall into the belief that the Many cannot be harmonious, is intrinsically a cluster of repulsions or of collisions, incapable of giving rise to accord; indeed, essentially hostile to it. So, as accord is the aim and the essence of our reason, we are caught in the snare of monism, pluralism having apparently become the equivalent of chaos, and thus the *bête noir* of rational metaphysics. Nay, in the opposed camp itself, some of the most ardent adherents of pluralism, the liveliest of wit, the most exuberant in literary resources, are the abjectest believers in the hopeless disjunction and capriciousness of the plural, and hold there is a rift in the texture of reality that no intelligence, "even though you dub it 'the Absolute,'" can mend or reach across. Yet surely there is nothing in the Many, as a sum of units, the least at war with the One as a system of harmony. On the contrary, even in the pure form of the Number Series, the Many is impossible except on the principle of harmony, — the units can be collected and summed (that is, constitute the Many), only if they cohere in a community of intrinsic kindred. Consequently the whole question of the chaotic or the harmonic nature of a plural world turns on the nature of the genus which we find characteristic of the absolutely (*i.e.*, the unreservedly) real, and which is to be taken as the common denomination enabling us to count them and to sum them. When minds are seen to be necessarily the primary realities, but *also necessarily federal* as well as individual, the illusion about the essential disjunction and non-coherence of the plurally real dissolves away, and a primordial world of manifold persons is seen to involve no fundamental or hopeless anarchy of individualism, irreducible in caprice, but an indwelling principle of harmony, rather, that from the springs of individual being intends the control and composure of all the disorders that mark the world of experiential appearance, and so must tend perpetually to effect this.

The other main source of our confusions over the Many and the One is the variety of meaning hidden in the concept Cause, and our propensity to take its most obvious but least significant sense for its supreme intent. Closest at hand, in experience, is our productive causation of changes in our sense-world, and hence most obvious is that reading of Cause which takes it as the producer of changes and, with a deeper comprehension of it, of the inalterable linkage between changes, whereby one follows regularly and surely upon another. Thus what we have in philosophy agreed to call Efficient Cause comes to be mistaken for the profoundest and the supreme form of cause, and all the other modes of cause, the Material (or Stuff), the Form (or Conception), and the End (or Purpose), its consequent and derivative auxiliaries. Under the influence of this strong impression, we either assume total reality to be One Whole, all-embracing and all-producing of its manifold modes, or else view it as a duality, consisting of One Creator and his manifold creatures. So it has come about that metaphysics has hitherto been chiefly a contention between pantheism and monotheism, or, as the latter should for greater accuracy be called, monarchotheism; and, it must be acknowledged, this struggle has been attended by a continued (though not continual) decline of this later dualistic theory before the steadfast front and unyielding advance of the older monism. Thus persistent has been the assumption that harmony can only be assured by the unity given in some single productive causation: the only serious uncertainty has been about the most rational way of conceiving the operation of this Sole Cause; and this doubt has thus far, on the whole, declined in favor of the Elder Oriental or monistic conception, as against the Hebraic conception of extraneous creation by fiat. The frankly confessed mystery of the latter, its open appeal to miracle, places it at a fatal disadvantage with the Elder Orientalism, when the appeal is to reason and intelligibility. It is therefore no occasion for wonder that, especially since the rise of the scientific doctrine of Evolution, with its postulate of a universal unity, self-varying yet self-fulfilling, even the leaders of theology are more and more falling into the monistic line and swelling the ever-growing ranks of pantheism. If it be asked here, *And why not? — where is the harm of it? — is not the whole question simply of what is true?* the answer is, *The mortal harm of the destruction of personality, which lives or dies with the preservation or destruction of individual responsibility; while the completer truth is, that there are other and profounder (or, if you please, higher) truths than this of explanation by Efficient Cause.* In fact, there is a higher conception of Cause itself than this of production, or efficiency; for, of course, as we well might say, that alone can be the supreme conception of Cause which can subsist between absolute or unreserved realities, and such must exclude their production or their necessitating control by others. So that we ought long since to have realized that Final Cause, the recognized presence to each other as unconditioned realities, or Defining Auxiliaries and Ends, is the sole causal relation that can hold among primary realities; though among such it *can* hold, and in fact must.

For the absolute reality of personal intelligences, at once individual and universally recognizant of others, is called for by other conceptions fundamental to philosophy. These other fundamental concepts can no more be counted out or ignored than those we have hitherto considered; and when we take them up, we shall see how vastly more significant they are. They alone will prove supreme, truly organizing, normative; they alone can introduce gradation in truths, for they alone introduce the judgment of worth, of valuation; they alone can give us counsels of perfection, for they alone rise from those elements in our being which deal with ideals and with veritable Ideas. So let us proceed to them.

Our path into their presence, however, is through another pair, not so plainly antithetic as those we have thus far considered. This pair that I now mean is Time and Space, which, though not obviously antinomic, yet owes its existence, as can now be shown, to that profoundest of concept-contrasts which we earlier considered under the head of Subject and Object, when the Object takes on its only adequate form of Other Subject. But in passing from the contrast One and Many towards its rational transformation into the moral society of Mind and Companion Minds, we break into this pair of Time and Space, and must make our way through it by taking in its full meaning.

Time and Space play an enormous part in all our empirical thinking, our actual use of thought in our sense-perceptive life. And no wonder; for, in coöperation, they form the postulate and condition of all our possible sensuous consciousness. Only on them as backgrounds can thought take on the peculiar clearness of an image or a picture; only on the screens which they supply can we literally *depict* an object. And this clarity of outline and boundary is so dear to our ordinary consciousness, that we are prone to say there is no sufficient, no real clearness, unless we can clarify by the bounds either of place or of date, or of both. In this mood, we are led to deny the reality and validity of thought altogether, when it cannot be defined in the metes and bounds afforded by Time or by Space: that which has no date nor place, we say, — no extent and no duration, — cannot be real; it is but a pseudo-thought, a pretense and a delusion. Here is the extremely plausible foundation of the philosophy known as sensationism, the refined or second-thought form of materialism, in which it begins its euthanasia into idealism.

Without delaying here to criticise this, let us notice the part that Time and Space play in reference to the conceptual pair we last considered, the One and the Many; for not otherwise shall we find our way beyond them to the still more fundamental conceptions which we are now aiming to reach. Indeed, it is through our surface-apprehension of the pair One and Many, as this illumines experience, that we most naturally come at the pair Time and Space; so that these are at first taken for mere generalizations and abstractions, the purely nominal representatives of the actual distinctions between the members of the Many by our sense-perception of this from that, of here from there, of now from then. It is not till our reflective attention is fixed on the fact that *there* and *here*, *now* and *then*, are *peculiar* distinctions, wholly different from other contrasts of this with that, — which may be made in all sorts of ways, by difference of quality, or of quantity, or of relations quite other than place and date, — it is not till we realize this *peculiar* character of the Time-contrast and the Space-contrast, that we see these singular differential *qualia* cannot be derived from others, not even from the contrast One and Many, but are independent, are themselves underived and spontaneous utterances of our intelligent, our percipient nature. But when Kant first helped mankind to the realization of this spontaneous (or *a priori*) character of this pair of perceptive conditions, or Sense-Forms, he fell into the persuasion, and led the philosophic world into it, that though Time and Space are not derivatives of the One and the Many read as the numerical aspect of our perceptive experiences, yet there *is* between the two pairs a connection of dependence as intimate as that first supposed, but in exactly the opposite sense; namely, that the One and the Many are conditioned by Time and Space, or, when it comes to the last resort, are at any rate completely dependent upon Time. By a series of units, this view means, we really understand a set of items discriminated and related either as points or as instants: in the last analysis, as instants: that is, it is impossible to apprehend a unit, or to count and sum units, unless the unit is taken as an instant, and the units as so many instants. Numbers, Kant holds, are no doubt pure (or quite unsensuous) percepts, — discerned particulars, — therefore spontaneous products of the mind *a priori*, but made possible only by the primary pure percept Time, or, again, through the mediation of this, by the conjoined pure percept Space; so that the numbers, in their own pure character, are simply the instants in their series. As the instants, and therefore the numbers, are pure percepts, — particulars discerned without the help of sense, — so pure percepts, in a primal and comprehensive sense, argues Kant, must their conditioning postulates Time and Space be, to supply the "element," or "medium," that will render such pure percepts possible.

This doctrine of Kant's is certainly plausible; indeed, it is impressively so; and it has taken a vast hold in the world of science, and has reinforced the popular belief in the unreality of thought apart from Time and Space; an unreality which it is an essential part of Kant's system to establish critically. But as a graver result, it has certainly tended to discredit the belief in personal identity as an abiding and immutable reality, enthroned over the mutations of things in Time and Space; since all that is in these is numbered and is mutable, and is rather many than one, yet nothing is believed real except as it falls under them, at any rate under Time. And with this decline of the belief in a changeless self, has declined, almost as rapidly and extensively, the belief in immortality. Or, rather, the permanence and the identity of the person has faded into a question regarded as unanswerable; though none the less does this agnostic state of belief tend to take personality, in any responsible sense of the word, out of the region of practical concern. With what is unknowable, even if existing, we can have no active traffic; 't is for our conduct as if it were not.

So it behooves us to search if this prevalent view about the relation of One and Many to Time and Space is trustworthy and exact. What place and function in philosophy must Space and Time be given? — for they certainly have a place and function; they certainly are among the inexpugnable conceptions with which thought has to concern itself when it undertakes to gain a view of the whole. But it may be easy to give them a larger place and function than belong to them by right. Is it true, then, that the One and the Many — that the system of Numbers, in short — are unthinkable except as in Space and Time, or, at any rate, in Time? Or, to put the question more exactly, as well as more gravely and more pertinently, Are Space and Time the true *principia individui*, and is Time preëminently the ultimate *principium individuationis ?* Is there accordingly no individuality, and no society, no associative assemblage, except in the fleeting world of phenomena, dated and placed? Simply to ask the question, and thus bring out the full drift of this Kantian doctrine, is almost to expose the absurdity of it. Such a doctrine, though it may be wisely refusing to confound personality, true individuality, with the mere logical singular; nay, worse, with a limited and special illustration of the singular, the one *here* or the one *there*, the one *now* or the one *then ;* nevertheless, by confining numerability to things material and sensible, makes personal identity something unmeaning or impossible, and destroys part of the foundation for the relations of moral responsibility. Though the vital trait of the person, his genuine individuality, doubtless lies, not in his being exactly numerable, but in his being aboriginal and originative; in a word, in his self-activity, in his being a centre of autonomous social recognition; yet exactly numerable he indeed is, and must be, not confusable with any other, else his professed autonomy, his claim of rights and his sense of duty, can have no significance, must vanish in the universal confusion belonging to the indefinite. Nor, on the other hand, is it at all true that a number has to be a point or an instant, nor that things when numbered and counted are implicitly pinned upon points or, at all events, upon instants. It may well enough be the fact that in our empirical use of number we have to employ Time, or even Space, but it is a gaping *non sequitur* to conclude that we therefore can count nothing but the placed and the dated. Certainly we count whenever we *distinguish*, — by whatever means, on whatever ground. To think is, in general, at least to "distinguish the things that differ;" but this will not avail except we keep account of the differences; hence the One and the Many lie in the very bosom of intelligence, and this fundamental and spontaneous contrast can not only rive Time and Space into expressions of it, in instants and in points, but travels with thought from its start to its goal, and as organic factor in mathematical science does indeed, as Plato in the *Republic* said, deal with absolute being, if yet dreamwise; so that One and Many, and Many as the sum of the ones, makes part of the measure of that primally real world which the world of minds alone can be. If the contrast One and Many can pass the bounds of the merely phenomenal, by passing the temporal and the spatial; if it applies to universal being, to the noumenal as well as to the phenomenal; then the absolutely real world, so far as concerns this essential condition, can be a world of genuine individuals, identifiable, free, abiding, responsible, and there can be a real moral order; if not, then there can be no such moral world, and the deeper thought-conceptions to which we now approach must be regarded, at the best, as fair illusions, bare ideals, which the serious devotee of truth must shun, except in such moments of vacancy and leisure as he may venture to surrender, at intervals, to purely hedonic uses. But if the One and the Many are not dependent on Time and Space, their universal validity is possible; and it has already been shown that they are not so dependent, are not thus restricted.

And now it remains to show their actual universality, by exhibiting their place in the structure of the absolutely real; since nobody calls in question their pertinence to the world of phenomena. But their noumenal applicability follows from their essential implication with all and every difference: no difference, no distinction, that does not carry counting; and this is quite as true as that there can be no counting without difference. The One and the Many thus root in Identity and Difference, pass up into fuller expression in Universal and Particular, hold forward into Cause and Effect, attain their commanding presentation in the Reciprocity of First Causes, and so keep record of the contrast between Necessity and Contingency. In short, they are founded in, and in their turn help (indispensably) to express, *all* the categories, — Quality, Quantity, Relation, Modality. Nor do they suffer arrest there; they hold in the ideals, the True, the Beautiful, the Good, and in the primary Ideas, the Self, the World, and God. For all of these differ, however close their logical linkage may be; and in so far as they differ, each of them is a counted unit, and so they are many. And, most profoundly of all, One and Many take footing in absolute reality so soon as we realize that nothing short of intelligent being can be primordially real, underived, and truly causal, and that intelligence is, by its idea, at once an *I*-thinking and a universal recognizant outlook upon others that think *I*.

Hence Number, so far from being the derivative of Time and Space, founds, at the bottom, in the self-definition and social recognition of intelligent beings, and so finds *a priori* a valid expression in Time and in Space, as well as in every other primitive and spontaneous form in which intelligence utters itself. The Pythagorean doctrine of the rank of Number in the scale of realities is only one remove from the truth: though the numbers are indeed not the Prime Beings, they do enter into the essential nature of the Prime Beings; are, so to speak, the organ of their definite reality and identity, and for that reason go forward into the entire defining procedure by which these intelligences organize their world of experiences. And the popular impression that Time and Space are derivatives from Number, is in one aspect the truth, rather than the doctrine of Kant is; for though they are not mere generalizations and abstractions from numbered dates and durations, places and extents, they do exist as relating-principles which minds simply *put*, as the conditions *of perceptive experiences ;* which by the nature of intelligence they must number in order to have and to master; while Number itself, the contrast of One and Many, enters into the very being of minds, and therefore still holds in Time and in Space, which are the organs, or *media*, not of the whole being of the mind, but only of that region of it constituted by sensation, — the material, the disjunct, the empirical. Besides, the logical priority of Number is implied in the fact that minds in putting Time and Space *a priori* must count them as two, since they discriminate them with complete clearness, so that it is impossible to work up Space out of Time (as Berkeley and Stuart Mill so adroitly, but so vainly, attempted to do), or Time out of Space (as Hegel, with so little adroitness and such patent failure, attempted to do). No; there Time and Space stand, fixed and inconfusable, incapable of mutual transmutation, and thus the ground of an abiding difference between the inner or psychic sense-world and the outer or physical, between the subjective and the (sensibly) objective. By means of them, the world of minds discerns and bounds securely between the privacy of each and the publicity, the life "out of doors," which is common to all; between the cohering isolation of the individual and the communicating action of the society. Indeed, as from this attained point of view we can now clearly see, the real ground of the difference between Time and Space, and hence between subjective perception and the objective existence of physical things, is in the fact that a mind, in *being* such, — in its very act of self-definition, — correlates itself with a *society* of minds, and so, to fulfill its nature, in so far as this includes a world of experiences, must form its experience socially as well as privately, and hence will put forth a condition of sensuous communication, as well as a condition of inner sensation. Thus the dualization of the sense-world into inner and outer, psychic and physical, subjective and objective, rests at last on the intrinsically social nature of conscious being; rests on the twofold structure, logically dichotomous, of the self-defining act; and we get the explanation, from the nature of intelligence as such, why the Sense-Forms are necessarily two, and only two. It is no accident that we experience all things sensible in Time or in Space, or in both together; it is the natural expression of our primally intelligent being, concerned as that is, directly and only, with our self and its logically necessary complement, the other selves; and so the natural order, in its two discriminated but complemental portions, the inner and the outer, is founded in that moral order which is given in the fundamental act of our intelligence. It is this resting of Space upon our veritable Objects, the Other Subjects, that imparts to it its externalizing quality, so that things in it are referred to the testing of all minds, not to ours only, and are reckoned external because measured by that which is alone indeed other than we.

In this way we may burst the restricting limit which so much of philosophy, and so much more of ordinary opinion, has drawn about our mental powers in view of this contrast Time and Space, especially with reference to the One and the Many, and to the persuasion that plural distinctions, at any rate, cannot belong in the region of absolute reality. Ordinary opinion either inclines to support a philosophy that is skeptical of either Unity or Plurality being pertinent beyond Time and Space, and thus to hold by agnosticism, or, if it affects affirmative metaphysics, tends to prefer monism to pluralism, when the number-category is carried up into immutable regions: to represent the absolutely real as One, somehow seems less contradictory of the "fitness of things" than to represent it as Many; moreover, carrying the Many into that supreme region, by implying the belonging there of mortals such as we, seems shocking to customary piety, and full of extravagant presumption. Still, nothing short of this can really satisfy our deep demand for a moral order, a personal responsibility, nay, an adequate logical fulfillment of our conception of a self as an *intelligence;* while the clarification which a rational pluralism supplies for such ingrained puzzles in the theory of knowledge as that of the source and finality of the contrast Time and Space, to mention no others, should afford a strong corroborative evidence in its behalf. And, as already said, this view enables us to pass the limit which Time and Space are so often supposed to put, hopelessly, upon our concepts of the ideal grade, the springs of all our aspiration. To these, then, we may now pass.

We reach them through the doorways of the Necessary *vs.* the Contingent, the Unconditioned *vs.* the Conditioned, the Infinite *vs.* the Finite, the Absolute *vs.* the Relative; and we recognize them as our profoundest foundation-concepts, alone deserving, as Kant so pertinently said, the name of IDEAS, — the Soul, the World, and God. Associated with them are what we may call our three Forms of the Ideal, — the True, the Beautiful, the Good. These Ideas and their affiliated ideals have the highest directive and settling function in the organization of philosophy; they determine its schools and its history, by forming the centre of all its controlling problems; they prescribe its great subdivisions, breaking it up into Metaphysics, Æsthetics, and Ethics, and Metaphysics, again, into Psychology Cosmology, and Ontology, — or Theology in the classic sense, which, in the modern sense, becomes the Philosophy of Religion; they call into existence, as essential preparatory and auxiliary disciplines, Logic and the Theory of Knowledge, or Epistemology. They thus provide the true distinctions between philosophy and the sciences of experience, and present these sciences as the carrying out, upon experiential details, of the methodological principles which philosophy alone can supply; hence they lead us to view all the sciences as in fact the applied branches, the completing organs of philosophy, instead of its hostile competitors.

As for the controlling questions which they start, these are such as follow: Are the ideals but bare ideals, serving only to cast "a light that never was, on land or sea?" — are the Ideas only bare ideas, without any objective being of their own, without any footing in the real, serving only to enhance the dull facts of experience with auroral illusions? The philosophic thinker answers affirmatively, or with complete skeptical dubiety, or with a convinced and uplifting negative, according to his less or greater penetration into the real meaning of these deepest concepts, and depending on his view into the nature and thought-effect of the Necessary and the Contingent, the Unconditioned and the Conditioned, the Infinite and the Finite, the Absolute and the Relative.

And what, now, are the accurate, the adequate meanings of the three Ideas? — what *does* our profoundest thought intend by the Soul, by the World, by God? We know how Kant construed them, in consequence of the course by which he came critically (as he supposed) upon them, — as respectively the paramount Subject of experiences; the paramount Object of experiences, or the Causal Unity of the possible series of sensible objects; and the complete Totality of Conditions for experience and its objects, itself therefore the Unconditioned. It is worth our notice, that especially by his construing the idea of God in this way, thus rehabilitating the classical and scholastic conception of God as the Sum of all Realities, he laid the foundation for that very transfiguration of mysticism, that idealistic monism, which he himself repudiated, but which his three noted successors in their several ways so ardently accepted, and which has since so pervaded the philosophic world. But suppose Kant's alleged critical analysis of the three Ideas and their logical basis is in fact far from critical, far from "exactly discriminative," — and I believe there is the clearest warrant for declaring that it is, — then the assumed "undeniable critical basis" for idealistic monism will be dislodged, and it will be open to us to interpret the Ideas with accuracy and consistency — an interpretation which may prove to establish, not at all any monism, but a rational pluralism. And this will also reveal to us, I think, that our prevalent construing of the Unconditioned and the Conditioned, the Necessary and the Contingent, the Infinite and the Finite, the Absolute and the Relative, suffers from an equal inaccuracy of analysis, and precisely for this reason gives a plausible but in fact untrustworthy support to the monistic interpretation of God, and Soul, and World; or, as Hegel and his chief adherents prefer to name them, God, Mind, and Nature. If the Kantian analysis stands, then it seems to follow, clearly enough, that God is the Inclusive Unit which at once embraces Mind and Nature, Soul and World, expresses itself in them, and imparts to them their meaning; and the plain dictate then is, that Kant's personal prejudice, and the personal prejudices of others like him, in favor of a transcendent God, must give way to that conception of the Divine, as immanent and inclusive, which is alone consistent with its being indeed the Totality of Conditions, — the Necessary Postulate, and the Sufficient Reason, for both Subject and Object.

But will Kant's analysis stand? Have we not here another of his few but fatal slips, — like his doctrine of the dependence of Number upon Time and Space, and its consequent subjection to them? It surely seems so. If the veritable postulate of categorical syllogizing be, as Kant thinks it is, merely the Subject, the self as experiencer of presented phenomena, in contrast to the Object, the causally united sum of possible phenomena; and if the true postulate of conditional syllogizing is this cosmic Object, as contrasted with the correlate Subject, then it would seem we cannot avoid certain pertinent questions. Is such a postulate Subject any fit and adequate account of the whole Self, of the Soul? — is there not a vital difference between this subject-self and the Self as Person? — does not Kant himself imply so, in his doctrine of the primacy of the Practical Reason? Again: Is not the World, as explained in Kant's analysis, and as afterwards made by him the solution of the Cosmological Antinomies, simply the supplemental factor necessarily correlate to the subjective aspect of the conscious life, and reduced from its uncritical rôle of thing-in-itself to the intelligible subordination required by Kant's theory of Transcendental Idealism? — and can this be any adequate account of the Idea that is to stand in sufficing contrast to the whole Self, the Person? — what less than the Society of Persons can meet the World-Idea for that? Further: If with Kant we take the World to mean no more than this object-factor in self-consciousness, must not the Soul, the total Self, from which, according to Kant's Transcendental Idealism, both Space and Time issue, supplying the basis for the immutable contrast between the experiencing subject and the really experienced objects, — must not this *whole* Self be the real meaning of the "Totality of Conditions, itself unconditioned," which comes into view as simply the postulate of disjunctive syllogizing? How in the world can disjunctive syllogizing, the confessed act of the *I*-thinking intelligence, really postulate anything as Totality of Conditions, in any other sense than the total of conditions for such syllogizing? — namely, the conditioning *I* that organizes and does the reasoning? There is surely no warrant for calling this total, which simply transcends and conditions the subject and the object of sensible experiences, by any loftier name than that which Kant had already given it in the Deduction of the Categories, when he designated it the "originally synthetic unity of apperception (self-consciousness)," or "the *I*-thinking (*das ich-denke*) that must accompany all my mental presentations," — that is to say, the whole Self, or thinking Person, idealistically interpreted. The use of the name God in this connection, where Kant is in fact only seeking the roots of the three orders of the syllogism *when reasoning has by supposition been restricted to the subject-matter of experience*, is assuredly without warrant; yes, without excuse. In fact, it is because Kant sees that the third Idea, as reached through his analysis, is intrinsically immanent, — resident in the self that syllogizes disjunctively, and, because so resident, incapable of passing the bounds of possible experience, — while he also sees that the idea of God should mean a Being transcendent of every other thinker, himself a distinct individual consciousness, though not an empirically limited one, — it is, I say, precisely because he sees all this, that he pronounces the Idea, though named with the name of God, utterly without pertinence to indicate God's existence, and so enters upon that part of his Transcendental Dialectic which is, in chief, directed to exposing the transcendental illusion involved in the celebrated Ontological Proof. Consistently, Kant in this famous analytic of the syllogism should be talking, not of the Soul, the World, and God, but of the Subject (as uniting-principle of its sense-*perceptions*), the Object (as uniting-principle of all possible sense-*percepts*), and the Self (the whole *I* presiding over experience in both its aspects, as these are discriminated in Time and Space). By what rational title — even granting for the sake of argument that they are the genuine postulates of categorical and of conditional syllogizing — can this Subject and this Object, these correlate factors in the Self, rank as Ideas with the Idea of their conditioning Whole — the Self, that in its still unaltered identity fulfills, in Practical Reason, the high rôle of Person? If *this* no more than meets the standard of Idea, how can *they* meet it? How can two somethings, neither of which is the Totality of Conditions, and both of which are therefore in fact conditioned, deserve the same title with that which is intrinsically the Totality of Conditions, and, as such, unconditioned? To call the conditioned and the unconditioned alike Ideas is a confounding of dignities that Pure Reason should not tolerate, whether the procedure be read as a leveling down or a leveling up. Distributing the titles conferred by Pure Reason in this democratic fashion reminds us too much, unhappily for Kant, of the Cartesian performances with Substance; whereby God, mind, and matter became alike "substances," though only God could in truth be said to "require nothing for his existence save himself," while mind and matter, though absolutely dependent on God, and derivative from him, were still to be called substances in the "modified" and Pickwickian sense of being underived from each other.

But if Kant's naming his third syllogistic postulate the Idea of God is inconsequent upon his analysis; or if, when the analysis is made consequent by taking the third Idea to mean the whole Self, the first and second postulates sink in conceptual rank, so that they cannot with any pertinence be called Ideas, unless we are willing to keep the same name when its meaning must be changed *in genere*, — a procedure that can only encumber philosophy instead of clearing its way, — these difficulties do not close the account; we shall find other curious things in this noted passage, upon which part of the characteristic outcome of Kant's philosophizing so much depends. Besides the misnaming of the third Idea, we have already had to question, in view of the path by which he reaches it, the fitness of his calling the first by the title of the Soul; and likewise, though for other and higher reasons, of his calling the second by the name of the World. In fact, it comes home to us that all of the Ideas are, in one way or another, misnomers; Kant's whole procedure with them, in fine, has already appeared inexact, inconsistent, and therefore uncritical. But now we shall become aware of certain other inconsistencies. In coming to the Subject, as the postulate of categorical syllogizing, Kant, you remember, does so by the path of the relation Subject and Predicate, arguing that the chain of categorical prosyllogisms has for its limiting concept and logical motor the notion of an absolute subject that cannot be a predicate; and as no subject of a judgment can of itself give assurance of fulfilling this condition, he concludes this motor-limit of judgment-subjects to be identical with the Subject as thinker, upon whom, at the last, all judgments depend, and who, therefore, and who alone, can never be a predicate merely. In similar fashion, he finds as the motor-limit of the series of conditional prosyllogisms, which is governed by the relation Cause and Effect, the notion of an absolute cause — a cause, that is, incapable of being an effect; and this, as undiscoverable in the chain of phenomenal causes, which are all in turn effects, he concludes is a pure Idea, the reason's native conception of a necessary linkage among all changes in Space, or of a Cosmic Unity among physical phenomena. In both conceptions, then, whether of the unity of the Subject or of the World, we seem to have a case of the unconditioned, as each, surely, is a totality of conditions: the one, for all possible syllogisms by Subject and Predicate; the other, for all possible syllogisms from Cause and Effect. Until it can be shown that the syllogisms of the first sort and the syllogisms of the second are both conditioned by the system of disjunctive syllogisms, so that the Idea alleged to be the totality of conditions for this system becomes the conditioning principle for both the others, there appears to be no ground for contrasting the totality of conditions presented in it with those presented in the others, as if it were the absolute Totality of all Conditions, while the two others are only "relative totalities," — which would be as much as to say they were only pseudo-totalities, both being conditioned instead of being unconditioned. But there seems to be no evidence, not even an indication, that disjunctive reasoning conditions categorical or conditional — that it constitutes the whole kingdom, in which the other two orders of reasoning form dependent provinces, or that for final validation these must appeal to the disjunctive series and the Idea that controls it. On the contrary, any such relation seems disproved by the fact that the three types of syllogism apply alike in all subject-matter, psychic or physical, subjective or objective, concerning the Self or concerning the World, — yes, concerning other Selves or even concerning God; whereas, if the relation were a fact, it would require that only disjunctive reasoning can deal with the Unconditioned, and that conditional must confine itself to cosmic material, while categorical pertains only to the things of inner sense.

Such considerations cannot but shake our confidence in the inquisition to which Kant has submitted the Ideas of Reason, both as regards what they really mean and how they are to be correlated. At all events, the analysis of logical procedure and connection on which his account of them is based is full of the confusions and oversights that have now been pointed out, and justifies us in saying that his case is not established. Hence we are not bound to follow when his three successors, or their later adherents, proceed in acceptance of his results, and advance into various forms of idealism, all of the monistic type, as if the general relation between the three Ideas had been demonstrably settled by Kant in the monist sense, despite his not knowing this, and that all we have to do is to disregard his recorded protests, and render his results consistent, and our idealism "absolute," by casting out from his doctrine the distinction between the Theoretical and the Practical Reason, with the "primacy" of the latter, through making an end of his assumed world of *Dinge an sich*, or "things in themselves." This movement, I repeat, we are not bound to follow: a rectification of view as to the meaning of the three Ideas becomes possible as soon as we are freed from Kant's entangled method of discovering and defining them; and when this rectification is effected, we shall find that the question between monism and rational or harmonic pluralism is at least open, to say no more. Nay, we are not to forget that by the results of our analysis of the concepts One and Many, Time and Space, and the real relation between them, plural metaphysics has already won a precedence in this contest.

THE DEVELOPMENT OF PHILOSOPHY IN THE NINETEENTH CENTURY

BY GEORGE TRUMBULL LADD

[**George Trumbull Ladd,** Professor of Philosophy, Yale University. **b.** January 19, 1842, Painesville, Ohio. B.A. Western Reserve College, 1864; B.D. Andover Theological Seminary, 1869; D.D. Western Reserve, 1879; M.A. Yale, 1881; LL.D. Western Reserve, 1895; LL.D. Princeton, 1896. Decorated with the 3d Degree of the Order of the Rising Sun of Japan, 1899; Pastor, Edinburg, Ohio, 1869–71; *ibid.*, Milwaukee, Wis., 1871–79; Professor of Philosophy, Bowdoin College, 1879–81; *ibid.*, Yale University, 1881——; Lecturer, Harvard, Tokio, Bombay, etc., 1885——. Member American Psychological Association, American Society of Naturalists, American Philosophical Association, American Oriental Society, Imperial Educational Society of Japan, Connecticut Academy. **Author of** *Elements of Physiological Psychology*; *Philosophy of Knowledge*; *Philosophy of Mind*; *A Theory of Reality*; and many other noted scientific works and papers.]

THE history of man's critical and reflective thought upon the more ultimate problems of nature and of his own life has, indeed, its period of quickened progress, relative stagnation, and apparent decline. Great thinkers are born and die, "schools of philosophy," so-called, arise, flourish, and become discredited; and tendencies of various characteristics mark the national or more general Zeitgeist of the particular centuries. And always, a certain deep undercurrent, or powerful stream of the rational evolution of humanity, flows silently onward. But these periods of philosophical development do not correspond to those which have been marked off for man by the rhythmic motion of the heavenly bodies, or by himself for purposes of greater convenience in practical affairs. The proposal, therefore, to treat any century of philosophical development as though it could be taken out of, and considered apart from, this constant unfolding of man's rational life is, of necessity, doomed to failure. And, indeed, the nineteenth century is no exception to the general truth.

There is, however, one important and historical fact which makes more definite, and more feasible, the attempt to present in outline the history of the philosophical development of the nineteenth century. This fact is the death of Immanuel Kant, February 12, 1804. In a very unusual way this event marks the close of the development of philosophy in the eighteenth century. In a yet more unusual way the same event defines the beginning of the philosophical development of the nineteenth century. The proposal is, therefore, not artificial, but in accordance with the truth of history, if we consider the problems, movements, results, and present condition of this development, so far as the fulfillment of our general purpose is concerned, in the light of the critical philosophy of Kant. This purpose may then be further defined in the following way: to trace the history of the evolution of critical and reflective thought over the more ultimate problems of Nature and of human life, in the Western World during the last hundred years, and from the standpoint of the conclusions, both negative and positive, which are best embodied in the works of the philosopher of Königsberg. This purpose we shall try to fulfill in these four divisions of our theme: (1) A statement of the problems of philosophy as they were given over to the nineteenth century by the Kantian Critique; (2) a brief description of the lines of movement along which the attempts at the improved solution of these problems have proceeded, and of the principal influences contributing to these attempts; (3) a summary of the principal results of these movements — the items, so to say, of progress in philosophy which may be credited to the last century; and finally, (4) a survey of the present state of these problems as they are now to be handed down by the nineteenth to the twentieth century. Truly an immensely difficult, if not an impossible task, is involved in this purpose!

I. The problems which the critical philosophy undertook definitively to solve may be divided into three classes. The first is the epistemological problem, or the problem offered by human knowledge — its essential nature, its fixed limitations, if such there be, and its ontological validity. It was this problem which Kant brought to the front in such a manner that certain subsequent writers on philosophy have claimed it to be, not only the primary and most important branch of philosophical discipline, but to comprise the sum-total of what human reflection and critical thought can successfully compass. "We call philosophy self-knowledge," says one of these writers. "The theory of knowledge is the true *prima philosophia*," says another. Kant himself regarded it as the most imperative demand of reason to establish a science that shall "determine *a priori* the possibility, the principles, and the extent of all cognitions." The burden of the epistemological problem has pressed heavily upon the thought of the nineteenth century; the different attitudes toward this problem, and its different alleged solutions, have been most influential factors in determining the philosophical discussions, divisions, schools, and permanent or transitory achievements of the century.

In the epistemological problem as offered by the Kantian philosophy of cognition there is involved the subordinate but highly important question as to the proper method of philosophy. Is the method of criticism, as that method was employed in the three Critiques of Kant, the exclusive, the sole appropriate and productive way of advancing human philosophical thought? I do not think that the experience of the nineteenth century warrants an affirmative answer to this question of method. This experience has certainly, however, resulted in demonstrating the need of a more thorough, consistent, and fundamental use of the critical method than that in which it was employed by Kant. And this improved use of the critical method has induced a more profound study of the psychology of cognition, and of the historical development of philosophy in the branch of epistemology. More especially, however, it has led to the reinstatement of the value-judgments, as means of cognition, in their right relations of harmony with the judgments of fact and of law.

The second of the greater problems which the critical philosophy of the eighteenth handed on to the nineteenth century is the ontological problem. This problem, even far more than the epistemological, has excited the intensest interest, and called for the profoundest thought, of reflective minds during the last hundred years. This problem engages in the inquiry as to what Reality is; for to define philosophy from the ontological point of view renders it "the rational science of reality;" or, at least, "the science of the supreme and most important realities." In spite of the fact that the period immediately following the conclusion of the Kantian criticism was the age when the people were singing

> "*Da die Metaphysik vor Kurzem unbeerbt abging,*
> *Werden die Dinge an sich jetzo sub hasta verkauft,*"

the cultivation of the ontological problem, and the growth of systematic metaphysics in the nineteenth century, had never previously been surpassed. In spite of, or rather because of, the fact that Kant left the ancient body of metaphysics so dismembered and discredited, and his own ontological structure in such hopeless confusion, all the several buildings both of Idealism and of Realism either rose quickly or were erected upon the foundations made bare by the critical philosophy.

But especially unsatisfactory to the thought of the first quarter of the nineteenth century was the Kantian position with reference to the problem in which, after all, both the few who cultivate philosophy and the multitude who share in its fruits are always most truly interested; and this is the ethico-religious problem. In the judgment of the generation which followed him, Kant had achieved for those who accepted his points of view, his method of philosophizing, and his results, much greater success in "removing knowledge" than in "finding room for faith." For he seemed to have left the positive truths of Ethics so involved in the negative positions of his critique of knowledge as greatly to endanger them; and to have entangled the conceptions of religion with those of morality in a manner to throw doubt upon them both.

The breach between the human cognitive faculties and the ontological doctrines and conceptions on which morality and religion had been supposed to rest firmly, the elaborately argued distrust and skepticism which had been aimed against the ability of human reason to reach reality, and the consequent danger which threatened the most precious judgments of worth and the ontological value of ethical and æsthetical sentiments, could not remain unnoticed, or fail to promote ceaseless and earnest efforts to heal it. The hitherto accepted solutions of the problems of cognition, of being, and of man's ethico-religious experience, could not survive the critical philosophy. But the solutions which the critical philosophy itself offered could not fail to excite opposition and to stimulate further criticism. Moreover, certain factors in human nature, certain interests in human social life, and certain needs of humanity, not fully recognized and indeed scarcely noticed by criticism, could not fail to revive and to enforce their ancient, perennial, and valid claims.

In a word, Kant left the main problems of philosophy involved in numerous contradictions. The result of his penetrating but excessive analysis was unwarrantably to contrast sense with understanding; to divide reason as constitutive from reason as regulative; to divorce the moral law from our concrete experience of the results of good and bad conduct, true morality from many of the noblest desires and sentiments, and to set in opposition phenomena and noumena, order and freedom, knowledge and faith, science and religion. Now the highest aim of philosophy is reconciliation. What wonder, then, that the beginning of the last century felt the stimulus of the unreconciled condition of the problems of philosophy at the end of the preceding century! The greatest, most stimulating inheritance of the philosophy of the nineteenth century from the philosophy of the eighteenth century was the "post-Kantian problems."

II. The lines of the movement of philosophical thought and the principal contributory influences which belong to the nineteenth century may be roughly divided into two classes; namely, (1) those which tended in the direction of carrying to the utmost extreme the negative and destructive criticism of Kant, and (2) those which, either mainly favoring or mainly antagonizing the conclusions of the Kantian criticism, endeavored to place the positive answer to all three of these great problems of philosophy upon more comprehensive, scientifically defensible, and permanently sure foundations. The one class so far completed the attempt to remove the knowledge at which philosophy aims as, by the end of the first half of the century, to have left no rational ground for any kind of faith. The other class had not, even by the end of the second half of the century, as yet agreed upon any one scheme for harmonizing the various theories of knowledge, of reality, and of the ground of morality and religion. There appeared, however, — especially during the last two decades of the century, — certain signs of convergence upon positions, to occupy which is favorable for agreement upon such a scheme, and which now promise a new constructive era for philosophy. The terminus of the destructive movement has been reached in our present-day positivism and philosophical skepticism. For this movement there would appear to be no more beyond in the same direction. The terminus of the other movement can only be somewhat dimly descried. It may perhaps be predicted with a reasonable degree of confidence as some form of ontological Idealism (if we may use such a phrase) that shall be at once more thoroughly grounded in man's total experience, as interpreted by modern science, and also more satisfactory to human ethical, æsthetical, and religious ideals, than any form of systematic philosophy has hitherto been. But to say even this much is perhaps unduly to anticipate.

If we attempt to fathom and estimate the force of the various streams of influence which have shaped the history of the philosophical development of the nineteenth century, I think there can be no doubt that the profoundest and the most powerful is the one influence which must be recognized and reckoned with in all the centuries. This influence is humanity's undying interest in its moral, civil, and religious ideals, and in the civil and religious institutions which give a faithful but temporary expression to these ideals. In the long run, every fragmentary or systematic attempt at the solution of the problem of philosophy must sustain the test of an ability to contribute something of value to the realization of these ideals. The test which the past century has proposed for its own thinkers, and for its various schools of philosophy, is by far the severest which has ever been proposed. For the most part unostentatiously and in large measure silently, the thoughtful few and the comparatively thoughtless multitude have been contributing, either destructively or constructively, to the effort at satisfaction for the rising spiritual life of man. And if in some vague but impressive manner we speak of this thirst for spiritual satisfaction as characteristic of any period of human history, we may say, I believe, that it has been peculiarly characteristic and especially powerful as an influence during the last hundred years. The opinions, sentiments, and ideals which shape the development of the institutions of the church and state, and the freer activities of the same opinions, sentiments, and ideals, have been in this century, as they have been in every century, the principal factors in determining the character of its philosophical development.

But a more definite and visible kind of influence has constantly proceeded from the centres of the higher education. The universities — especially of Germany, next, perhaps of Scotland, but also of England and the United States, and even in less degree of France and Italy — have both fostered and shaped the evolution of critical and reflective thought, and of its product as philosophy. In Germany during the eighteenth century the greater universities had been emancipating themselves from the stricter forms of political and court favoritism and of ecclesiastical protection and control. This emancipation had already operated at the beginning of the nineteenth century, and it continued more and more to operate throughout this century, for participation in that free thought whose spirit is absolutely essential to the flourishing of true philosophy. All the other colleges and universities can scarcely repay the debt which modern philosophy owes to the universities of Germany. The institutions of the higher education which are moulded after this spirit, and which have a generous share of this spirit, have everywhere been *schools of thought* as well as schools of learning and research. Without the increasing numbers and growing encouragement of such centres for the cultivation of the discipline of critical and reflective thinking, it is difficult to conjecture how much the philosophical development of the nineteenth century would have lost. *Libertas docendi* and *Academische Freiheit* — without these philosophy has one of its wings fatally wounded or severely clipped.

Not all the philosophy of the last century, however, was born and developed in academical centres and under academical influences. In Germany, Great Britain, and France, the various so-called "Academies" or other unacademical associations of men of scientific interests and attainments — notably, the Berlin Academy, which has been called "the seat of an anti-scholastic popular philosophy" — were during the first half of the nineteenth century contributing by their conspicuous failures as well as by their less conspicuous successes, important factors to the constructive new thought of the latter half of the nineteenth century. In general, although these men decried system and were themselves inadequately prepared to treat the problems of philosophy, whether from the historical or the speculative and critical point of view, they cannot be wholly neglected in estimating its development. Clever reasoning, and witty and epigrammatic writing on scientific or other allied subjects, cannot indeed be called *philosophy* in the stricter meaning of the word. But this so-called "popular philosophy" has greatly helped in a way to free thought from its too close bondage to scholastic tradition. And even the despite of philosophy, and sneering references to its "barrenness," which formerly characterized the meetings and the writings of this class of its critics, but which now are happily much less frequent, have been on the whole both a valuable check and a stimulus to her devotees. He would be too narrow and sour a disciple of scholastic metaphysics and systematic philosophy, who, because of the levity or scorning of "outsiders," should refuse them all credit. Indeed, the lesson of the close of the nineteenth century may well enough be the motto for the beginning of the twentieth century: *In philosophy — since to philosophize is natural and inevitable for all rational beings — there really are no outsiders.*

In this connection it is most interesting to notice how men of the type just referred to, were at the end of the eighteenth century found grouped around such thinkers as Mendelssohn, Lessing, F. Nicolai, — representing a somewhat decided reaction from the French realism to the German idealism. The work of the Academicians in the criticism of Kant was carried forward by Jacobi, who, at the time of his death, was the pensioned president of the Academy at Munich. Some of these same critics of the Kantian philosophy showed a rather decided preference for the "common-sense" philosophy of the Scottish School.

But both inside and outside of the Universities and Academies the scientific spirit and acquisitions of the nineteenth century have most profoundly, and on the whole favorably, affected the development of its philosophy. In the wider meaning of the word, "science," — the meaning, namely, in which science=*Wissenschaft*, — philosophy aims to be scientific; and science can never be indifferent to philosophy. In their common aim at a rational and unitary system of principles, which shall explain and give its due significance to the totality of human experience, science and philosophy can never remain long in antagonism; they ought never even temporarily to be divided in interests, or in the spirit which leads each generously to recognize the importance of the other. The early part of the last century was, indeed, too much under the influence of that almost exclusively speculative *Natur-philosophie*, of which Schelling and Hegel were the most prominent exponents. On the other hand, the conception of nature as a vast interconnected and unitary system of a rational order, unfolding itself in accordance with teleological principles, — however manifold and obscure, — is a noble conception and not destined to pass away.

On the continent — at least in France, where it had attained its highest development — the scientific spirit was, at the close of the eighteenth century, on the whole opposed to systematization. The impulse to both science and philosophy during both the eighteenth and the nineteenth centuries, over the entire continent of Europe, was chiefly due to the epoch-making work of that greatest of all titles in the modern scientific development of the Western World, the *Principia* of Newton. In mathematics and the physical sciences, during the early third or half of the last century, Great Britain also has a roll of distinguished names which compares most favorably with that of either France or Germany. But in England, France, and the United States, during the whole century, science has lacked the breadth and philosophic spirit which it had in Germany during the first three quarters of this period. During all that time the German man of science was, as a rule, a scholar, an investigator, a teacher, *and a philosopher*. Science and philosophy thrived better, however, in Scotland than elsewhere outside of Germany, so far as their relations in interdependence were concerned. Into the Scottish universities Playfair introduced some of the continental suggestions toward the end of the eighteenth century, so that there was less of exclusiveness and unfriendly rivalry between science and philosophy; and both profited thereby. In the United States, during the first half or more of the century, so dominant were the theological and practical interests and influences that there was little free development of either science or philosophy, — if we interpret the one as the equivalent of *Wissenschaft* and understand the other in the stricter meaning of the word.

The history of the development of the scientific spirit and of the achievements of the particular sciences is not the theme of this paper. To trace in detail, or even in its large outlines, the reciprocal influence of science and philosophy during the past hundred years, would itself require far more than the space allotted to me. It must suffice to say that the various advances in the efforts of the particular sciences to enlarge and to define the conceptions and principles employed to portray the Being of the World in its totality, have somewhat steadily grown more and more completely metaphysical, and more and more of positive importance for the reconstruction of systematic philosophy. The latter has not simply been disciplined by science, compelled to improve its method, and to examine all its previous claims. But philosophy has also been greatly enriched by science with respect to its material awaiting synthesis, and it has been not a little profited by the unsuccessful attempts of the current scientific theories to give themselves a truly satisfactory account of that Ultimate Reality which, to understand the better, is no unworthy aim of their combined efforts.

During the nineteenth century science has seen many important additions to that Ideal of Nature and her processes, to form which in a unitary and harmonizing but comprehensive way is the philosophical goal of science. The gross mechanical conception of nature which prevailed in the earlier part of the eighteenth century has long since been abandoned, as quite inadequate to our experience with her facts, forces, and laws. The kinetic view, which began with Huygens, Euler, and Ampère, and which was so amplified by Lord Kelvin and Clerk-Maxwell in England, and by Helmholtz and others in Germany, on account of its success in explaining the phenomena of light, of gases, etc., very naturally led to the attempt to develop a kinetic theory, a doctrine of energetics, which should explain all phenomena. But the conception of "that which moves," the experience of important and persistent qualitative *differentiae*, and the need of assuming ends and purposes served by the movement, are troublesome obstacles in the way of giving such a completeness to this theory of the Being of the World. Yet again the amazing success which the theory of evolution has shown in explaining the phenomena with which the various biological sciences concern themselves, has lent favor during the latter half of the century to the vitalistic and genetic view of nature. For all our most elaborate and advanced kinetic theories seem utterly to fail us as explanatory when we, through the higher powers of the microscope, stand wondering and face to face with the evolution of a single living cell. But from such a view of the essential Being of the World as evolution suggests to the psycho-physical theory of nature is not an impassable gulf. And thus, under its growing wealth of knowledge, science may be leading up to an Ideal of the Ultimate Reality, in which philosophy will gratefully and gladly coincide. At any rate, the modern conception of nature and the modern conception of God are not so far apart from each other, as either of these conceptions is now removed from the conceptions covered by the same terms, some centuries gone by.

There is one of the positive sciences, however, with which the development of philosophy during the last century has been particularly allied. This science is psychology. To speak of its history is not the theme of this paper. But it should be noted in passing how the development of psychology has brought into connection with the physical and biological sciences the development of philosophy. This union, whether it be for better or for worse, — and, on the whole, I believe it to be for better rather than for worse, — has been in a very special way the result of the last century. In tracing its details we should have to speak of the dependence of certain branches of psychology on physiology, and upon Sir Charles Bell's discovery of the difference between the sensory and the motor nerves. This discovery was the contribution of the beginning of the century to an entire line of discoveries, which have ended at the close of the century with putting the localization of cerebral function upon a firm experimental basis. Of scarcely less importance has been the cellular theory as applied (1838) by Matthias Schleiden, a pupil of Fries in philosophy, to plants, and by Theodor Schwann about the same time to animal organisms. To these must be added the researches of Johannes Müller (1801–1858), the great biologist, a listener to Hegel's lectures, whose law of *specific energies* brings him into connection with psychology and, through psychology, to philosophy. Even more true is this of Helmholtz, whose *Lehre von den Tonempfindungen* (1862) and *Physiologische Optik* (1867) placed him in even closer, though still mediate, relations to philosophy. But perhaps especially Gustav Theodor Fechner (1801–1887), whose researches in psycho-physics laid the foundations of whatever, either as psychology or as philosophy, goes under this name; and whether the doctrine have reference to the relation of man's mind and body, or to the wider relations of spirit and matter.

In my judgment it cannot be affirmed that the attempts of the latter half of the nineteenth century to develop an experimental science of psychology in independence of philosophical criticism and metaphysical assumption, or the claims of this science to have thrown any wholly new light upon the statement, or upon the solution of philosophical problems, have been largely successful. But certain more definitely psychological questions have been to a commendable degree better analyzed and elucidated; the new experimental methods, where confined within their legitimate sphere, have been amply justified; and certain *quasi*-metaphysical views respecting the nature of the human mind, and even, if you will, the nature of the Spirit in general—have been placed in a more favorable and scientifically engaging attitude toward speculative philosophy. This seems to me to be especially true with respect to two problems in which both empirical psychology and philosophy have a common and profound interest. These are (1) the complex synthesis of mental functions involved in every act of true cognition, together with the bearing which the psychology of cognition has upon epistemological problems; and (2) the yet more complex and profound analysis, from the psychological point of view, of what it is to be a self-conscious and self-determining Will, a true Self, together with the bearing which the psychology of selfhood has upon all the problems of ethics, æsthetics, and religion.

The more obvious and easily traceable influences which have operated to incite and direct the philosophical development of the nineteenth century are, of course, dependent upon the teachings and writings of philosophers, and the schools of philosophy which they have founded. To speak of these influences even in outline would be to write a manual of the history of philosophy during that hundred of years, which has been of all others by far the most fruitful in material results, whatever estimate may be put upon the separate or combined values of the individual thinkers and their so-called schools. No fewer than seven or eight relatively independent or partially antagonistic movements, which may be traced back either directly or more indirectly to the critical philosophy, and to the form in which the problems of philosophy were left by Kant, sprung up during the century. In Germany chiefly, there arose the Faith-philosophy, the Romantic School, and Rational Idealism; in France, Eclecticism and Positivism (if, indeed, the latter can be called *a* philosophy); in Scotland, a naïve and crude form of Realism, which served well for the time as an antagonist of a skeptical idealism, but which itself contributed to an improved form of Idealism; and in the United States, or rather in New England, a peculiar kind of Transcendentalism of the sentimental type. But all these movements of thought, and others lying somewhere midway between, in a pair composed of any two, together with a steadfast remainder of almost every sort of Dogmatism, and all degrees and kinds of Skepticism, have been intermixed and contending with one another, in all these countries. Such has been the varied, undefinable, and yet intensely stimulating and interesting character of the development of systematic and scholastic philosophy, during the nineteenth century.

The early opposition to Kant in Germany was, in the main, twofold: — both to his peculiar extreme analysis with its philosophical conclusions, and also to all systematic as distinguished from a more popular and literary form of philosophizing. Toward the close of the eighteenth century a group of men had been writing upon philosophical questions in a spirit and method quite foreign to that held in respect by the critical philosophy. It is not wholly without significance that Lessing, whose aim had been to use common sense and literary skill in clearing up obscure ideas and improving and illumining the life of man, died in the very year of the appearance of Kant's *Kritik der reinen Vernunft.* Of this class of men an historian dealing with this period has said, "There is hardly one who does not quote somewhere or other Pope's saying, 'The proper study of mankind is man.'" To this class belong Hamann (1730–1788), the inspirer of Herder and Jacobi. The former, who was essentially a poet and a friend of Goethe, controverted Kant with regard to his doctrine of reason, his antithesis between the individual and the race, and his schism between things as empirically known and the known unity in the Ground of their being and becoming. Herder's path to truth was highly colored with flowers of rhetoric; but the promise was that he would lead men back to the heavenly city. Jacobi, too, with due allowance made for the injury wrought by his divorce of the two philosophies, — that of faith and that of science, — and his excessive estimate of the value-judgments which repose in the mist of a feeling-faith, added something of worth by way of exposing the barrenness of the Kantian doctrine of an unknowable "Thing-in-itself."

From men like Fr. Schlegel (1772–1829), whose valid protest against the sharp separation of speculative philosophy from the æsthetical, social, and ethical life, assumed the "standpoint of irony," little real result in the discovery of truth could be expected. But Schleiermacher (1768–1834), in spite of that mixture of unfused elements which has made his philosophy "a rendezvous for the most diverse systems," contributed valuable factors to the century's philosophical development, both of a negative and of a positive character. This thinker was peculiarly fortunate in the enrichment of the conception of experience as warranting a justifiable confidence in the ontological value of ethical, æsthetical, and religious sentiment and ideas; but he was most unfortunate in reviving and perpetuating the unjustifiable Kantian distinction between cognition and faith in the field of experience. On the whole, therefore, the Faith-philosophy and the Romantic School can easily be said to have contributed more than a negative and modifying influence to the development of the philosophy of the nineteenth century. Its more modern revival toward the close of the same century, and its continued hold upon certain minds of the present day, are evidences of the positive but partial truth which its tenets, however vaguely and unsystematically, continue to maintain in an æsthetically and practically attractive way.

The admirers of Kant strove earnestly and with varied success to remedy the defects of his system. Among the earlier, less celebrated and yet important members of this group, were K. G. Reinhold (1758–1823), and Maimon (died, 1800). The former, like Descartes, in that he was educated by the Jesuits, began the attempt, after rejecting some of the arbitrary distinctions of Kant and his barren and self-contradictory "Thing-in-itself," to unify the critical philosophy by reducing it to some one principle. The latter really transcended Kant in his philosophical skepticism, and anticipated the Hamiltonian form of the so-called principle of relativity. Fries (1773–1843), and Hermes (1775–1831) — the latter of whom saw in empirical psychology the only true propædeutic to philosophy — should be mentioned in this connection. In the same group was another, both mathematician and philosopher, who strove more successfully than others of this group to accept the critical standpoint of Kant and yet to transcend his negative conclusions with regard to a theory of knowledge. I refer to Bolzano (Prague, 1781–1848), who stands in the same line of succession with Fries and Hermes, and whose works on the *Science of Religion* (4 vols. 1834) and his *Science of Know-*

ledge (4 vols. 1837) are noteworthy contributions to epistemological doctrine. In the latter we have developed at great length the important thought that the illative character of propositional judgments implies an objective relation; and that in all truths the subject-idea must be objective. In the work on religion there is found as thoroughly dispassionate and rational a defense of Catholic doctrine as exists anywhere in philosophical literature. The limited influence of these works, due in part to their bulk and their technical character, is on the whole, I think, sincerely to be regretted.

It was, however, chiefly that remarkable series of philosophers which may be grouped under the rubric of a "rational Idealism," who filled so full and made so rich the philosophical life of Germany during the first half of the last century; whose philosophical thoughts and systems have spread over the entire Western World, and who are most potent influences in shaping the development of philosophy down to the present hour. Of these we need do little more than that we can do — mention their names. At their head, in time, stands Fichte, who — although Kant is reported to have complained of this disciple because he lied about him so much — really divined a truth which seems to be hovering in the clouds above the master's head, but which, if the critical philosophy truly meant to teach it, needed helpful deliverance in order to appear in perfectly clear light. Fichte, although he divined this truth, did not, however, free it from internal confusion and self-contradiction. It *is* his truth, nevertheless, that in the Self, as a self-positing and self-determining activity, must somehow be found the Ground of all experience and of all Reality.

The important note which Schelling sounded was the demand that philosophy should recognize "Nature" as belonging to the sphere of Reality, and as requiring a measure of reflective thought which should in some sort put it on equal terms with the Ego, for the construction of our conception of the Being of the World. To Schelling it seemed impossible to deduce, as Fichte had done, all the rich concrete development of the world of things from the subjective needs and constitutional forms of functioning which belong to the finite Self. And, indeed, the doctrine which limits the origin, existence, and value of all that is known about this sphere of experience to these needs, and which finds the sufficient account of all experience with nature in these forms of functioning, must always seem inadequate and even grotesque in the sight of the natural sciences. Both Nature and Spirit, thought Schelling, must be allowed to claim actual existence and equally real value; while at the same time philosophy must reconcile the seeming opposition of their claims and unite them in an harmonious and self-explanatory way. In some common substratum, in which, to adopt Hegel's sarcastic criticism, as in the darkness of the night "all cows are black," — that is in the Absolute, as an

Identical Basis of Differences, — the reconciliation was to be accomplished.

But the constructive idealistic movement, in which Fichte and Schelling bore so important a part, could not be satisfied with the positions reached by either of these two philosophers. Neither the physical and psychological sciences, nor the speculative interests of religion, ethics, art, and social life, permitted this movement to stop at this point. In all the subsequent developments of philosophy during the first half or three quarters of the nineteenth century, undoubtedly the influence of Hegel was greatest of all individual thinkers. His *motif* and plan are revealed in his letter of November 2, 1800, to Schelling, namely, to transform what had hitherto been an ideal into a thoroughly elaborate system. And in spite of his obvious obscurities of thought and style, there is real ground for his claim to be the champion of the common consciousness. It is undoubtedly in Hegel's *Phänomenologie des Geistes* (1807), that the distinctive features of the philosophy of the first half of the last century most clearly define themselves. The forces of reflection now abandon the abstract analytic method and positions of the Kantian Critique, and concentrate themselves upon the study of man's spiritual life as an historical evolution, in a more concrete, face-to-face manner. Two important and, in the main, valid assumptions underlie and guide this reflective study: (1) The Ultimate Reality, or principle of all realities, is Mind or Spirit, which is to be recognized and known in its essence, not by analysis into its formal elements (the categories), but as a living development; (2) those formal elements, or categories to which Kant gave validity merely as constitutional forms of the functioning of the human understanding, represent, the rather, the essential structure of Reality.

In spite of these true thoughts, fault was justly found by the particular sciences with both the speculative method of Hegel, which consists in the smooth, harmonious, and systematic arrangement of conceptions in logical or ideal relations to one another; and also with the result, which reduces the Being of the World to terms of thought and dialectical processes merely, and neglects or overlooks the other aspects of racial experience. Therefore, the idealistic movement could not remain satisfied with the Hegelian dialectic. Especially did both the religious and the philosophical party revolt against the important thought underlying Hegel's philosophy of religion; namely, that "the more philosophy approximates to a complete development, the more it exhibits the same need, the same interest, and the same content, as religion itself." This, as they interpreted it, meant the absorption of religion in philosophy.

Next after Hegel, among the great names of this period, stand the names of Herbart and Schopenhauer. The former contributes in an important way to the proper conception of the task and the method of philosophy, and influences greatly the development of psychology, both as a science that is pedagogic to philosophy, and as laying the basis for pedagogical principles and practice. But Herbart commits again the ancient fallacy, under the spell of which so much of the Kantian criticism was bound; and which identifies contradictions that belong to the imperfect or illusory conceptions of individual thinkers with insoluble antinomies inherent in reason itself. In spite of the little worth and misleading character of his view of perception, and the quite complete inadequacy of the method by which, at a single leap, he reaches the one all-explanatory principle of his philosophy, Schopenhauer made a most important contribution to the reflective thought of the century. It is true, as Kuno Fischer has said, that it seems to have occurred to Schopenhauer only twenty-five years after he had propounded his theory, that will, as it appears in consciousness, is as truly phenomenal as is intellect. It is also true that his theory of knowledge and his conception of Reality, as measured by their power to satisfy and explain our total experience, are inflicted with irreconcilable contradictions. Neither can we accord firm confidence or high praise to the "Way of Salvation" which somehow Will can attain to follow by æsthetic contemplation and ascetic self-denial. Yet the philosophy of Schopenhauer rightly insists upon our Idealistic construction of Reality having regard to aspects of experience which his predecessors had quite too much neglected; and even its spiteful and exaggerated reminders of the facts which contradict the tendency of all Idealism to construct a smooth, regular, and altogether pleasing conception of the Being of the World, have been of great benefit to the development of the latter half of the nineteenth century.

In estimating the thoughts and the products of modern Idealism we ought not to forget the larger multitude of thoughtful men, both in Germany and elsewhere, who have contributed toward shaping the course of reflection in the attempt to answer the problems which the critical philosophy left to the nineteenth century. It is a singular comment upon the caprices of fame that, in philosophy as in science, politics, and art, some of those who have really reasoned most soundly and acutely, if not also effectively upon these problems, are little known even by name in the history of the philosophical development of the century. Among the earlier members of this group, did space permit, we should wish to mention Berger, Solger, Steffens, and others, who strove to reconcile the positions of a subjective idealism with a realistic but pantheistic conception of the Being of the World. There are others, who like Weisse, I. H. Fichte, C. P. Fischer, and Braniss, more or less bitterly or moderately and reasonably, opposed the method and the conclusions of the Hegelian dialectic. Still another group earned for themselves the supposedly opprobrious but decidedly vague title of "Dualists," by rejecting what they conceived to be the pantheism of Hegel. Still others, like Fries and Beneke and their successors, strove to parallel philosophy with the particular sciences by grounding it in an empirical but scientific psychology; and thus they instituted a line of closely connected development, to which reference has already been made.

Hegel himself believed that he had permanently effected that reconciliation of the orthodox creed with the cognition of Ultimate Reality at which his dialectic aimed. In all such attempts at reconciliation three great questions are chiefly concerned: (1) the Being of God; (2) the nature of man; (3) the actual and the ideally satisfactory relations between the two. But, as might have been expected, a period of wild, irregular, and confused contention met the attempt to establish this claim. In this conflict of more or less noisy and popular as well as of thoughtful and scholastic philosophy, Hegelians of various degrees of fidelity, anti-Hegelians of various degrees of hostility, and ultra-Hegelians of various degrees of eccentricity, all took a valiant and conspicuous part. We cannot follow its history; but we can learn its lesson. Polemical philosophy, as distinguished from quiet, reflective, and critical but constructive philosophy involves a most uneconomical use of mental force. Yet out of this period of conflict, and in a measure as its result, there came a period of improved relations between science and philosophy and between philosophy and theology, which was the dawn, toward the close of the nineteenth century, of that better illumined day into the middle of which we hope that we are proceeding.

Before leaving this idealistic movement in Germany, and elsewhere as influenced largely by German philosophy, one other name deserves mention. This name is that of Lotze, who combined elements from many previous thinkers with those derived from his own studies and thoughts, — the conceptions of mechanism as applied to physical existences and to psychical life, with the search for some monistic Principle that shall satisfy the æsthetical and ethical, as well as the scientific demands of the human mind. This variety of interests and of culture led to the result of his making important contributions to psychology, logic, metaphysics, and æsthetics. If we find his system of thinking — as I think we must — lacking in certain important elements of consistency and obscured in places by doubts as to his real meaning, this does not prevent us from assigning to Lotze a position which, for versatility of interests, genial quality of reflection and criticism, suggestiveness of thought and charm of style, is second to no other in the history of nineteenth century philosophical development.

In France and in England the first quarter of the last century was far from being productive of great thinkers or great thoughts in the sphere of philosophy. De Biran (1766–1824), in several important respects the forerunner of modern psychology, after revolting from his earlier complacent acceptance of the vagaries of Condillac and Cabanis, made the discovery that the "immediate consciousness of self-activity is the primitive and fundamental principle of human cognition." Meantime it was only a little group of Academicians who were being introduced, in a somewhat superficial way, to the thoughts of the Scottish and the German idealistic Schools by Royer-Collard, Jouffroy, Cousin, and others. A more independent and characteristic movement was that inaugurated by Auguste Comte (1798–1857), who, having felt the marked influence of Saint-Simon when he was only a boy of twenty, in a letter to his friend Valat, in the year 1824, declares: "I shall devote my whole life and all my powers to the founding of positive philosophy." In spite of the impossibility of harmonizing with this point of view the vague and mystical elements which characterize the later thought of Comte, or with its carrying into effect the not altogether intelligent recognition of the synthetic activity of the mind (*tout se réduit toujours à lier*) and certain hints as to "first principles;" and in spite of the small positive contribution to philosophy which Comtism could claim to have made; it has in a way represented the value of two ideas. These are (1) the necessity for philosophy of studying the actual historical forces which have been at work and which are displayed in the facts of history; and (2) the determination not to go by mere unsupported speculation beyond experience in order to discover knowable Reality. There is, however, a kind of subtle irony in the fact that the word "Positivism" should have come to stand so largely for *negative* conclusions, in the very spheres of philosophy, morals, and religion where *affirmative* conclusions are so much desired and sought.

That philosophy in Great Britain was in a nearly complete condition of decadence during the first half or three quarters of the nineteenth century was the combined testimony of writers from such different points of view as Carlyle, Sir William Hamilton, and John Stuart Mill. And yet these very names are also witnesses to the fact that this decadence was not quite complete. In the first quarter of the century Coleridge, although he had failed, on account of weakness both of mind and of character, in his attempt to reconcile religion to the thought of his own age, on the basis of the Kantian distinction between reason and the intellect, had sowed certain seed-thoughts which became fertile in the soil of minds more vigorous, logical, and practical than his own. This was, perhaps, especially true in America, where inquirers after truth were seeking for something more satisfactory than the French skepticism of the revolutionary and following period. Carlyle's mocking sarcasm was also not without wholesome effect.

But it was Sir William Hamilton and John Stuart Mill whose thoughts exercised a more powerful formative influence over the minds of the younger men. The one was the flower of the Scottish Realism, the other of the movement started by Bentham and the elder Mill.

That the Scottish Realism should end by such a combination with the skepticism of the critical philosophy as is implied in Hamilton's law of the relativity of all knowledge, is one of the most curious and interesting turns in the history of modern philosophy. And when this law was so interpreted by Dean Mansel in its application to the fundamental cognitions of religion as to lay the foundations upon which the most imposing structure of agnosticism was built by Herbert Spencer, surely the entire swing around the circle, from Kant to Kant again, has been made complete. The attempt of Hamilton failed, as every similar attempt must always fail. Neither speculative philosophy nor religious faith is satisfied with an abstract conception, about the correlate of which in Reality nothing is known or ever can be known. But every important attempt of this sort serves the double purpose of stimulating other efforts to reconstruct the answer to the problem of philosophy, on a basis of positive experience of an enlarged type; and also of acting as a real, if only temporary practical support to certain value-judgments which the faiths of morality, art, and religion both implicate and, in a measure, validate.

The influence of John Stuart Mill, as it was exerted not only in his conduct of life while a servant of the East India Company, but also in his writings on Logic, Politics, and Philosophy, was, on the whole, a valuable contribution to his generation. In the additions which he made to the Utilitarianism of Bentham we have done, I believe, all that ever can be done in defense of this principle of ethics. And his posthumous confessions of faith in the ontological value of certain great conceptions of religion are the more valuable because of the nature of the man, and of the experience which is their source. Perhaps the most permanent contribution which Mill made to the development of philosophy proper, outside of the sphere of logic, ethics, and politics, was his vigorous polemical criticism of Hamilton's claim for the necessity of faith in an "Unconditioned" whose conception is "only a fasciculus of negations of the Conditioned in its opposite extremes, and bound together merely by the aid of language and their common character of incomprehensibility."

The history of the development of philosophy in America during the nineteenth century, as during the preceding century, has been characterized in the main by three principal tendencies. These may be called the theological, the social, and the eclectic. From the beginning down to the present time the religious influence and the interest in political and social problems have been dominant. And yet withal, the student of these problems in the atmosphere of this country likes, in a way, to do his own thinking and to make his own choices of the thoughts that seem to him true and best fitted for the best form of life. In spite of the fact that the different streams of European thought have flowed in upon us somewhat freely, there has been comparatively little either of the adherence to schools of European philosophy or of the attempt to develop a national school. Doubtless the influence of English and Scottish thinking upon the academical circles of America was greatest for more than one hundred and fifty years after the gift in 1714 by Governor Yale of a copy of Locke's Essay to the college which bore his name, — and especially upon the reflections and published works of Jonathan Edwards touching the fundamental problems of epistemology, ethics, and religion. During the early part of this century these views awakened antagonism from such writers as Dana, Whedon, Hazard, Nathaniel Taylor, Jeremiah Day, Henry P. Tappan, and other opponents of the Edwardean theology, and also from such advocates of so-called "free-thinking," as had derived their *motifs* and their views from English deistical writers like Shaftesbury, or from the skepticism of Hume.

A more definite philosophical movement, however, which had established itself somewhat firmly in scholastic centres by the year 1825, and which maintained itself for more than half a century, went back to the arrival in this country of John Witherspoon, in 1768, to be the president of Princeton, bringing with him a library of three hundred books. It was the appeal of the Scottish School to the "plain man's consciousness" and to so-called "common sense," which was relied upon to controvert all forms of philosophy which seemed to threaten the foundations of religion and of the ethics of politics and sociology. But even during this period, which was characterized by relatively little independent thinking in scholastic circles, a more pronounced productivity was shown by such writers as Francis Wayland, and others; but, perhaps, especially by Laurens P. Hickok, whose works on psychology and cosmology deserve especial recognition: while in psychology, as related to philosophical problems, the principal names of this period are undoubtedly the presidents of Yale and Princeton, — Noah Porter and James McCosh, — both of whom (but especially the former) had their views modified by the more scientific psychology of Europe and the profounder thinking of Germany.

It was Germany's influence, however, both directly and indirectly through Coleridge and a few other English writers, that caused a ferment of impressions and ideas which, in their effort to work themselves clear, resulted in what is known as New England "Transcendentalism." In America this movement can scarcely be called definitely philosophical; much less can it be said to have resulted in a system, or even in a school, of philosophy. It must also be said to have been "inspired but not borrowed" from abroad. Its principal, if not sole, literary survival is to be found in the works of Emerson. As expounded by him, it is not precisely Pantheism — certainly not a consistent and critical development of the pantheistic theory of the Being of the World; it is, rather, a vague, poetical, and pantheistical Idealism of a decidedly mystical type.

The introduction of German philosophy proper, in its nature form, and essential being, to the few interested seriously in critical and reflective thinking upon the ultimate problems of nature and of human life, began with the founding of the *Journal of Speculative Philosophy*, in 1867, under the direction of William T. Harris, then Superintendent of Schools in this city.

With the work of Darwin, and his predecessors and successors, there began a mighty movement of thought which, although it is primarily scientific and more definitely available in biological science, has already exercised, and is doubtless destined to exercise in the future, an enormous influence upon philosophy. Indeed, we are already in the midst of the preliminary confusions and contentions, but most fruitful considerations and discoveries belonging to a so-called philosophy of evolution.

This development has, in the sphere of systematic philosophy, reached its highest expression in the voluminous works produced through the latter half of the nineteenth century by Mr. Herbert Spencer, whose recent death seems to mark the close of the period we have under consideration. The metaphysical assumptions and ontological value of the system of Spencer, as he wished it to be understood and interpreted, have perhaps, though not unnaturally, been quite too much submerged in the more obvious expressions of its agnostic positivism. In its psychology, however, the assumption of "some underlying substance in contrast to all changing forms," distinguishes it from a pure positivism in a very radical way. But more especially in philosophy, the metaphysical postulate of a mysterious Unity of Force that somehow manages to reveal itself, and the law of its operations, to the developed cognition of the nineteenth century philosopher, however much it seems to involve the system in internal contradictions, certainly forbids that we should identify it with the positivism of Auguste Comte. In our judgment, however, it is in his ethical good sense and integrity of judgment, — a good sense and integrity which commits to ethics rather than to sociology the task of determining the highest type of human life, — and in basing the conditions for the prevalence and the development of the highest type of life upon ethical principles and upon the adherence to ethical ideas, that Herbert Spencer will be found most clearly entitled to a lasting honor.

III. The third number of our difficult tasks is to summarize the principal results, to inventory the net profits, as it were, of the development of philosophy during the nineteenth century. This task is made the more difficult by the heterogeneous nature and as yet unclassified condition of the development. With the quickening and diversifying of all kinds and means of intercourse, there has come the breaking-down of national schools and idiosyncrasies of method and of thought. In philosophy, Germany, France, Great Britain, and indeed, Italy, have come to intermingle their streams of influence; and from all these countries these streams have been flowing in upon America. In psychology, especially, as well as in all the other sciences, but also to some degree in philosophy, returning streams of influence from America have, during the last decade or two, been felt in Europe itself.

It must also be admitted that the attempts at a reconstruction of systematic philosophy which have followed the rapid disintegration of the Hegelian system, and the enormous accumulations of new material due to the extension of historical studies and of the particular sciences, — including especially the so-called "new psychology," — have not as yet been fruitful of large results. In philosophy, as in art, politics, and even scientific theory, the spirit and the opportunity of the time are more favorable to the gathering of material and to the projecting of a bewildering variety of new opinions, or old opinions put forth under new names, than to that candid, patient, and prolonged reflection and balancing of judgment which a worthy system-building inexorably requires. The age of breaking up the old, without assimilating the new, has not yet passed away. And whatever is new, startling, large, even monstrous, has in many quarters the seeming preference, in philosophy's building as in other architecture. To the confusion which reigns even in scholastic circles, contributions have been arriving from the outside, from philosophers like Nietzsche, and from men great in literature like Tolstoi. Nor has the matter been helped by the more recent extreme developments of positivism and skepticism, which often enough, without any consciousness of their origin and without the respect for morality and religion which Kant always evinced, really go back to the critical philosophy.

In spite of all this, however, the last two decades or more have shown certain hopeful tendencies and notable achievements, looking toward the reconstruction of systematic philosophy. In this attempt to bring order out of confusion, to enable calm, prolonged, and reflective thinking to build into its structure the riches of the new material which the evolution of the race has secured, a place of honor ought to be given to France, where so much has been done of late to blend with clearness of style and independence of thought that calm reflective and critical judgment which looks all sides of human experience sympathetically but bravely in the face. In psychology Ribot, and in philosophy, Fouillée, Renouvier, Secrétan, and others, deserve grateful recognition. No friend of philosophy can, I think, fail to recognize the probable benefits to be derived from that movement with which such names as Mach and Ostwald in Germany are connected, and which is sounding the call to the men of science to clear up the really distressing obscurity and confusion which has so long clung to their fundamental conceptions; and to examine anew the significance of their assumptions, with a view to the construction of a new and improved doctrine of the Being of the World. And if to these names we add those of the numerous distinguished investigators of psychology as pedagogic to philosophy, and, in philosophy, of Deussen, Eucken, von Hartmann, Riehl, Wundt, and others, we may well affirm that new light will continue to break forth from that country which so powerfully aroused the whole Western World at the end of the eighteenth and beginning of the nineteenth centuries. In Great Britain the name and works of Thomas Hill Green have influenced the attempts at a reconstruction of systematic philosophy in a manner to satisfy at one and the same time both the facts and laws of science and the æsthetical, ethical, and religious ideals of the age, in a very considerable degree. And in this attempt, both as it expresses itself in theoretical psychology and in the various branches of philosophical discipline, writers like Bradley, Fraser, Flint, Hodgson, Seth, Stout, Ward, and others, have taken a conspicuous part. Nor are there wanting in Holland, Italy, and even in Sweden and Russia, thinkers equally worthy of recognition, and recognized, in however limited and unworthy fashion, in their own land. The names of those in America who have labored most faithfully, and succeeded best, in this enormous task of reconstructing philosophy in a systematic way, and upon a basis of history and of modern science, I do not need to mention; they are known, or they surely ought to be known, to us all.

In attempting to summarize the gains of philosophy during the last hundred years, we should remind ourselves that progress in philosophy does not consist in the final settlement, and so in the "solving" of any of its great problems. Indeed, the relations of philosophy to its grounds in experience, and the nature of its method and of its ideal, are such that its progress can never be expected to put an end to itself. But the content of the total experience of humanity has been greatly enriched during the last century; and the critical and reflective thought of trained minds has been led toward a more profound and comprehensive theory of Reality, and toward a doctrine of values that shall be more available for the improvement of man's political, social, and religious life.

In view of this truth respecting the limitations of systematic philosophy, I think we may hold that certain negative results, which are customarily adduced as unfavorable to the claims of philosophical progress, are really signs of improvement during the latter half of the nineteenth century. One is an increased spirit of reserve and caution, and an increased modesty of claims. This result is perhaps significant of riper wisdom and more trustworthy maturity. Kant believed himself to have established for philosophy a system of apodeictic conclusions, which were as completely forever to have displaced the old dogmatism as Copernicus had displaced the Ptolemaic astronomy. But the steady pressure of historical and scientific studies has made it increasingly difficult for any sane thinker to claim for any system of thinking such demonstrable validity. May we not hope that the students of the particular sciences, to whom philosophy owes so much of its enforced sanity and sane modesty, will themselves soon share freely of the philosophic spirit with regard to their own metaphysics and ethical and religious standpoints, touching the Ultimate Reality? Even when the recoil from the overweening self-satisfaction and crass complacency of the earlier part of the last century takes the form of melancholy, or of acute sadness, or even of a mild despair of philosophy, I am not sure that the last state of that man is not better than the first.

In connection with this improvement in spirit, we may also note an improvement in the method of philosophy. The purely speculative method, with its intensely interesting but indefensible disregard of concrete facts, and of the conclusions of the particular sciences, is no longer in favor even among the most ardent devotees and advocates of the superiority of philosophy to those sciences. At the same time, philosophy may quite properly continue to maintain its position of independent critic, as well as of docile pupil, toward the particular sciences.

In the same connection must be mentioned the hopeful fact that the last two or three decades have shown a decided improvement in the relations of philosophy toward the positive sciences. There are plain signs of late that the attitude of antagonism, or of neglect, which prevailed so largely during the second and third quarters of the nineteenth century, is to be replaced by one of friendship and mutual helpfulness. And, indeed, science and philosophy cannot long or greatly flourish without reciprocal aid, if by science we mean a true *Wissenschaft* and if we also mean to base philosophy upon our total experience. For science and philosophy are really engaged upon the same task, — *to understand and to appreciate the totality of man's* *experience.* They, therefore, have essential and permanent relations of dependence for material, for inspiration and correction, and for other forms of helpfulness. While, then, their respective spheres have been more clearly delimited during the last century, their interdependence has been more forcefully exhibited. Both of them have been developing a systematic exposition of the universe. Both of them desire to enlarge and deepen the conception of the Being of the World, as made known to the totality of human experience, in its Unity of nature and significance. We cannot believe that the end of the nineteenth century would sustain the charge which Fontenelle made in the closing years of the seventeenth century: "*L'Académie des Sciences ne prend la nature que par petites parcelles.*" Science itself now bids us regard the Universe as a dynamical Unity, teleologically conceived, because in a process of evolution under the control of immanent ideas. Philosophy assumes the same point of view, rather at the beginning than at the end of defining its purpose; and so feels a certain glad leap at its heart-strings, and an impulse to hold out the hand to science, when it hears such an utterance as that of Poincaré: *Ce n'est pas le méchanisme le vrai, le seul but ; c'est l'unité.*

Shall we not say, then, that this double-faced but wholly true lesson has been learned: namely, that the so-called philosophy of nature has no sound foundation and no safeguard against vagaries of every sort, unless it follows the lead of the positive sciences of nature; but that the sciences themselves can never afford a full satisfaction to the legitimate aspirations of human reason unless they, too, contribute to the philosophy of nature — writ large and conceived of as a real-ideal Unity.

That nature, as known and knowable by man, is a great artist, and that man's æsthetical consciousness may be trusted as having a certain ontological value, is the postulate properly derived from the considerations advanced in the latest, and in some respects the most satisfactory, of the three Critiques of Kant. The ideal way of looking at natural phenomena which so delighted the mind of Goethe has now been placed on broad and sound foundations by the fruitful industries of many workmen, — such as Karl Ernst von Baer and Charles Darwin, — whose morphological and evolutionary conceptions of the universe have transformed the current conceptions of cosmic processes. But the world of physical and natural phenomena has thereby been rendered not less, but more, of a Cosmos, an orderly totality.

In addition to these more general but somewhat vague evaluations of the progress of philosophy during the nineteenth century, we are certainly called upon to face the question whether, after all, any advance has been made toward the more satisfactory solution of the definite problems which the Kantian criticism left unsolved. To this question I believe an affirmative answer may be given in accordance with the facts of history. It will be remembered that the first of these problems was the epistemological. Certainly no little improvement has been made in the psychology of cognition. We can no longer repeat the mistakes of Kant, either with respect to the uncritical assumptions he makes regarding the origin of knowledge in the so-called "faculties" of the human mind or regarding the analysis of those faculties and their interdependent relations. It is not the Scottish philosophy alone which has led to the conclusion that, in the word of the late Professor Adamson, "What are called acts or states of consciousness are *not* rightly conceived of as having for their objects their own modes of existence as ways in which a subject is modified." And in the larger manner both science and philosophy, in their negations and their affirmations, and even in their points of view, have better grounds for the faith of human reason in its power progressively to master the knowledge of Reality than was the case a hundred years ago. Nor has the skepticism of the same era, whether by shallow scoffing at repeated failures, or by pious sighs over the limitations of human reason, or by critical analysis of the cognitive faculties "according to well-established principles," succeeded in limiting our speculative pretensions to the sphere of possible experience, — in the Kantian meaning both of "principles" and of "experience." But what both science and philosophy are compelled to agree upon as a common underlying principle is this: The proof of the most fundamental presuppositions, as well as of the latest more scientifically established conclusions, of both science and philosophy, is the assistance they afford in the satisfactory explanation of the totality of racial experience.

In the evolution of the ontological problem, as compared with the form in which it was left by the critical philosophy, the past century has also made some notable advances. To deny this would be to discredit the development of human knowledge so far as to say that we know no more about what nature is, and man is, than was known a hundred years ago. To say this, however, would not be to speak truth of fact. And here we may not unnaturally grow somewhat impatient with that metaphysical fallacy which places an impassable gulf between Reality and Experience. No reality is, of course, cognizable or believable by man which does not somehow show its presence in his total experience. But no growth of experience is possible without involving increase of knowledge representing Reality. For Reality is no absent and dead, or statical, Ding-an-Sich. Cognition itself is a commerce of realities. And are there not plain signs that the more thoughtful men of science are becoming less averse to the recognition of the truth of ontological philosophy; namely, that the deeper meaning of their own studies is grasped only when they recognize that they are ever face to face with what they call Energy and we call Will, and with what they call laws and we call Mind as significant of the progressive realization of immanent ideas. This Ultimate Reality is so profound that neither science nor philosophy will ever sound all its depths, and so comprehensive as more than to justify all the categories of both.

Probably, on the whole, there has been less progress made toward a satisfactory solution of the problems offered by the value-judgments of ethics and religion, in the form in which these problems were left by the critical philosophy. The century has illustrated the truth of Falckenberg's statement: "In periods which have given birth to a skeptical philosophy, one never looks in vain for the complementary phenomenon of mysticism." Twice during the century the so-called "faith-philosophy," or philosophy of feeling, has been borne to the front, to raise a bulwark against the advancing hosts of agnostics — occasioned in the first period by the negations of the Kantian criticism, and in the second by the positive conclusions of the physical and biological sciences. This form of protesting against the neglect or disparagement of important factors which belong to man's æsthetical, ethical, and religious experience, is reasonable and must be heard. But the extravagances with which these neglected factors have been posited and appraised, to the neglect of the more definitively scientific and strictly logical, is to be deplored. The great work before the philosophy of the present age is the reconciliation of the historical and scientific conceptions of the Universe with the legitimate sentiments and ideals of art, morality, and religion. But surely neither rationalism nor "faith-philosophy" is justified in pouring out the living child with the muddy water of the bath.

IV. The attempt to survey the present situation of philosophy, and to predict its immediate future, is embarrassed by the fact that we are all immersed in it, are a part of its spirit and present form. But if nearness has its embarrassments, it has also its benefits. Those who are amidst the tides of life may know better, in a way, how these tides are tending and what is their present strength, than do those who survey them from distant, cool, and exalted heights. "*Für jeden einzelnen bildet der Vater und der Sohn eine greifbare Kette von Lebensereignungen und Erfahrungen.*" The very intensely vital and formative but unformed condition of systematic philosophy — its protoplasmic character — contains promises of a new life. If we may believe the view of Hegel that the systematizing of the thought of any age marks the time when the peculiar living thought of that age is passing into a period of decay, we may certainly claim for our present age the prospect of a prolonged vitality.

The nineteenth century has left us with a vast widening of the horizon, — outward into space, backward in time, inward toward the secrets of life, and downward into the depths of Reality. With this there has been an increase in the profundity of the conviction of the spiritual unity of the race. In the consideration of all of its problems in the immediate future and in the coming century — so far as we can see forward into this century — philosophy will have to reckon with certain marked characteristics of the human spirit which form at the same time inspiring stimuli and limiting conditions of its endeavors and achievements. Chief among these are the greater and more firmly established principles of the positive sciences, and the prevalence of the historical spirit and method in the investigation of all manner of problems. These influences have given shape to the conception which, although it is as yet by no means in its final or even in thoroughly self-consistent form, is destined powerfully to affect our philosophical as well as our scientific theories. This conception is that of Development. But philosophy, considered as the product of critical and reflective thinking over the more ultimate problems of nature and of human life, is itself a development. And it is now, more than ever before, a development interdependently connected with all the other great developments.

Philosophy, in order to adapt itself to the spirit of the age, must welcome and cultivate the freest critical inquiry into its own methods and results, and must cheerfully submit itself to the demand for evidences which has its roots in the common and essential experience of the race. Moreover, the growth of the spirit of democracy, which, on the one hand, is distinctly unfavorable to any system of philosophy whose tenets and formulas seem to have only an academic validity or a merely esoteric value, and which, on the other hand, requires for its satisfaction a more tenable, helpful, and universally applicable theory of life and reality, cannot fail, in my judgment, to influence favorably the development of philosophy. In the union of the speculative and the practical; in the harmonizing of the interests of the positive sciences, with their judgments of fact and law, and the interests of art, morality, and religion, with their value-judgments and ideals; in the synthesis of the truths of Realism and Idealism, as they have existed hitherto and now exist in separateness or antagonism; in a union that is not accomplished by a shallow eclecticism, but by a sincere attempt to base philosophy upon the totality of human experience; — in such a union as this must we look for the real progress of philosophy in the coming century.

Just now there seem to be two somewhat heterogeneous and not altogether well-defined tendencies toward the reconstruction of systematic philosophy, both of which are powerful and represent real truths conquered by ages of intellectual industry and conflict. These two, however, need to be internally harmonized, in order to obtain a satisfactory statement of the development of the last century. They may be called the evolutionary and the idealistic. The one tendency lays emphasis on mechanism, the other on spirit. Yet it is most interesting to notice how many of the early workmen in the investigation of the principle of the conservation and correlation of energy took their point of departure from distinctly teleological and spiritual conceptions. "I was led," said Colding, — to take an extreme case, — at the Natural Science Congress at Innsbruck, 1869, "to the idea of the constancy of national forces by the religious conception of life." And even Moleschott, in his *Autobiography*, posthumously published, declares: "I myself was well aware that the whole conception might be converted; for since all matter is a bearer of force, endowed with force or penetrated with spirit, it would be just as correct to call it a spiritualistic conception." On the other hand, the modern, better instructed Idealism is much inclined, both from the psychological and from the more purely philosophical points of view, to regard with duly profound respect all the facts and laws of that mechanism of Reality, which certainly is not merely the dependent construction of the human mind functioning according to a constitution that excludes it from Reality, but is rather the ever increasingly more trustworthy revealer of Reality. This tendency to a union of the claims of both Realism and Idealism is profoundly influencing the solution of each one of these problems which the Kantian criticism left to the philosophy of the nineteenth century. In respect of the epistemological problem, philosophy — as I have already said — is not likely again to repeat the mistakes either of Kant or of the dogmatism which his criticism so effectually overthrew. It was a wise remark of the physician Johann Benjamin Erhard, in a letter dated May 19, 1794, à *propos* of Fichte: "The philosophy which *proceeds* from a *single* fundamental principle, and pretends to deduce everything from it, is and always will remain a piece of artificial sophistry: only that philosophy which *ascends* to the highest principle and exhibits everything else in perfect harmony with it, is the true one." This at least ought — one would say — to have been made clear by the century of discussion over the epistemological problem, since Kant. You cannot *deduce* the Idea from the Reality, or the Reality from the Idea. The problem of knowledge is not, as Fichte held in the form of a fundamental assumption, an alternative of this sort. The Idea *and* Reality are, the rather already there, and to be recognized as in a living unity, in every cognitive experience. Psychology is constantly adding something toward the problem of cognition as a problem in synthesis; and is then in a way contributing to the better scientific understanding of the philosophical postulate which is the confidence of human reason in its ability, by the harmonious use of all its powers, progressively to reach a better and fuller knowledge of Reality.

The ontological problem will necessarily always remain the unsolved, in the sense of the very incompletely solved problem of philosophy. But as long as human experience develops, and as long as philosophy bestows upon experience the earnest and candid efforts of reflecting minds, the solution of the ontological problem will be approached, but never fully reached. That Being of the World which Kant, in the negative and critical part of his work, left as an *X*, unknown and unknowable, the last century has filled with a new and far richer content than it ever had before. Especially has this century changed the conception of the Unity of the Universe in such manner that it can never return again to its ancient form. On the one hand, this Unity cannot be made comprehensible in terms of any one scientific or philosophical principle or law. Science and philosophy are both moving farther and farther away from the hope of comprehending the variety and infinite manifoldness of the Absolute in terms of any one side or aspect of man's complex experience. But, on the other hand, the confidence in this essential Unity is not diminished, but is the rather confirmed. As humanity itself develops, as the Selfhood of man grows in the experience of the world which is its own environment, and of the world within which it is its own true Self, humanity may reasonably hope to win an increased, and increasingly valid, cognition of the Being of the World as the Absolute Self.

Closely connected, and in a way essentially identical with the ontological problem, is that of the origin, validity, and rational value of the ideas of humanity. May it not be said that the nineteenth century transfers to the twentieth an increased interest in and a heightened appreciation of the so-called practical problems of philosophy. Science and philosophy certainly ought to combine — and are they not ready to combine? — in the effort to secure a more nearly satisfactory understanding and solution of the problems afforded by the æsthetical, ethical, and religious sentiments and ideals of the race. To philosophy this combination means that it shall be more fruitful than ever before in promoting the uplift and betterment of mankind. The fulfillment of the practical mission of philosophy involves the application of its conceptions and principles to education, politics, morals, as a matter of law and of custom, and to religion as matter both of rational faith and of the conduct of life.

How, then, can this brief and imperfect sketch of the outline of the development of philosophy in the nineteenth century better come to a close than by words of encouragement and of exhortation as well. There are, in my judgment, the plainest signs that the somewhat too destructive and even nihilistic tendencies of the second and third quarters of the nineteenth century have reached their limit; that the strife of science and philosophy, and of both with religion, is lessening, and is being rapidly displaced by the spirit of mutual fairness and reciprocal helpfulness; and that reasonable hopes of a new and a splendid era of reconstruction in philosophy may be entertained. For I cannot agree with the *dictum* of a recent writer on the subject, that "the sciences are coming less and less to admit of a synthesis, and not at all of a synthetic philosopher."

On the contrary, I hold that, with an increased confidence in the capacity of human reason to discover and validate the most secret and profound, as well as the most comprehensive, of truths, philosophy may well put aside some of its shyness and hesitancy, and may resume more of that audacity of imagination, sustained by ontological convictions, which characterized its work during the first half of the nineteenth century. And if the latter half of the twentieth century does for the constructions of the first half of the same century, what the latter half of the nineteenth century did for the first half of that century, this new criticism will only be to illustrate the way in which the human spirit makes every form of its progress.

Therefore, a summons of all helpers, in critical but fraternal spirit, to this work of reconstruction, for which two generations of enormous advance in the positive sciences has gathered new material, and for the better accomplishment of which both the successes and the failures of the philosophy of the nineteenth century have prepared the men of the twentieth century, is the winsome and imperative voice of the hour.

SECTION A — METAPHYSICS

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(*Hall 6, September 21, 10 a. m.*)

CHAIRMAN: PROFESSOR A. C. ARMSTRONG, Wesleyan University.
SPEAKERS: PROFESSOR A. E. TAYLOR, McGill University, Montreal.
PROFESSOR ALEXANDER T. ORMOND, Princeton University.
SECRETARY: PROFESSOR A. O. LOVEJOY, Washington University.

The Chairman of the Section, Professor A. C. Armstrong, of Wesleyan University, in opening the meeting referred to the continued vitality of metaphysics as shown by its repeated revivals after the many destructive attacks upon it in the later modern times: he congratulated the Section on the fact that the principal speakers were scholars who had made notable contributions to metaphysical theory.

THE RELATIONS BETWEEN METAPHYSICS AND THE OTHER SCIENCES

BY PROFESSOR ALFRED EDWARD TAYLOR

[**Alfred Edward Taylor**, Frothingham Professor of Philosophy, McGill University, Montreal, Canada. **b.** Oundle, England, December 22, 1869. M.A. Oxford. Fellow, Merton College, Oxford, 1891–98, 1902–; Lecturer in Greek and Philosophy, Owens College, Manchester, 1896–1903; Assistant Examiner to University of Wales, 1899–1903; Green Moral Philosophy Prizeman, Oxford, 1899; Frothingham Professor of Philosophy, McGill University, 1903–; Member Philosophical Society, Owens College, American Philosophical Association. **Author of** *The Problem of Conduct; Elements of Metaphysics*.]

WHEN we seek to determine the place of metaphysics in the general scheme of human knowledge, we are at once confronted by an initial difficulty of some magnitude. There seems, in fact, to be no one universally accepted definition of our study, and even no very general consensus among its votaries as to the problems with which the metaphysician ought to concern himself. This difficulty, serious as it is, does not, however, justify the suspicion that our science is, like alchemy or astrology, an illusion, and its high-sounding title a mere "idol of the market-place," one of those *nomina rerum quae non sunt* against which the Chancellor Bacon has so eloquently warned mankind. If it is hard to determine precisely the scope of metaphysics, it is no less difficult to do the same thing for the undoubtedly legitimate sciences of logic and mathematics. And in all three cases the absence of definition merely shows that we are dealing with branches of knowledge which are, so to say, still in the making. It is not until the first principles of science are already firmly laid beyond the possibility of cavil that we must look for general agreement as to its boundary lines, though excellent work may be done, long before this point has been reached, in the establishment of individual principles and deduction of consequences from them. To revert to the parallel cases I have just cited, many mathematical principles of the highest importance are formulated in the *Elements* of Euclid, and many logical principles in the *Organon* of Aristotle; yet it is only in our own time that it has become possible to offer a general definition either of logic or of mathematics, and even now it would probably be true to say that the majority of logicians and mathematicians trouble themselves very little about the precise definition of their respective studies.

The state of our science then compels me to begin this address with a more or less arbitrary, because provisional, definition of the term metaphysics, for which I claim no more than that it may serve to indicate with approximate accuracy the class of problems which I shall have in view in my subsequent use of the word. By metaphysics, then, I propose to understand the inquiry which used formerly to be known as ontology, that is, the investigation into the general character which belongs to real Being as such, the science, in Aristotelian phraseology, of ὄντα ᾗ ὄντα. Or, if the term "real" be objected against as ambiguous, I would suggest as an alternative account the statement that metaphysics is the inquiry into the general character by which the content of *true* assertions is distinguished from that of *false* assertions. The two definitions here offered will, I think, be found equivalent when it is borne in mind that what the second of them speaks of is exclusively the *content* which is asserted as true in a true proposition, not the process of true assertion, which, like all other processes in the highest cerebral centres, falls under the consideration of the vastly different sciences of psychology and cerebral physiology. Of the two equivalent forms of statement, the former has perhaps the advantage of making it most clear that it is ultimately upon the objective distinction between the reality and the unreality of that which is asserted for truth, and not upon any psychological peculiarity in the process of assertion itself that the distinction between true and untrue rests, while the second may be useful in guarding against misconceptions that might be suggested by too narrow an interpretation of the term "reality," such as, *e. g.*, the identification of the "real" with what is revealed by sensuous perception.

From the acceptance of such a definition two important consequences would follow. (1) The first is that metaphysics is at once sharply discriminated from any study of the psychical *process* of knowledge, if indeed, there can be any such study distinct from the psychology of conception and belief, which is clearly not itself the science we have in view. For the psychological laws of the formation of concepts and beliefs are exemplified equally in the discovery and propagation of truth and of error. And thus it is in vain to look to them for any explanation of the difference between the two. Nor does the otherwise promising extension of Darwinian conceptions of the "struggle for existence" and the "survival of the fittest" to the field of opinions and convictions appear to affect this conclusion. Such considerations may indeed assist us to understand how true convictions in virtue of their "usefulness" gradually come to be established and extended, but they require to presume the truth of these convictions as an antecedent condition of their "usefulness" and consequent establishment. I should infer, then, that it is a mistake in principle to seek to replace ontology by a "theory of knowledge," and should even be inclined to question the very possibility of such a theory as distinct from metaphysics on the one hand and empirical psychology on the other. (2) The second consequence is of even greater importance. The inquiry into the general character by which the contents of true assertions are discriminated from the contents of false assertions must be carefully distinguished from any investigation into the truth or falsehood of special assertions. To ask how in the end truth differs from falsehood is to raise an entirely different problem from that created by asking whether a given statement is to be regarded as true or false. The distinction becomes particularly important when we have to deal with what Locke would call assertions of "real existence," *i. e.*, assertions as to the occurrence of particular events in the temporal order. All such assertions depend, in part at least, upon the admission of what we may style "empirical" evidence, the immediate unanalyzed witness of simple apprehension to the occurrence of an alleged matter of fact. Thus it would follow from our proposed conception of metaphysics that metaphysics is in principle incapable either of establishing or refuting any assertion as to the details of our immediate experience of empirical fact, though it may have important bearings upon any theory of the general nature of true Being which we may seek to found upon our alleged experiences. In a word, if our conception be the correct one, the functions of a science of metaphysics in respect of our knowledge of the temporal sequence of events psychical and physical must be purely critical, never constructive, — a point to which I shall presently have to recur.

One more general reflection, and we may pass to the consideration of the relation of metaphysics to the various already organzied branches of human knowledge more in detail. The admission that there is, or may be, such a study as we have described, seems of itself to involve the recognition that definite knowledge about the character of what really "is," is attainable, and thus to commit us to a position of sharp opposition both to consistent and thorough-going agnosticism and also to the latent agnosticism of Kantian and neo-Kantian "critical philosophy." In recognizing ontology as a legitimate investigation, we revert in principle to the "dogmatist" position common, *e. g.*, to Plato, to Spinoza and to Leibniz, that there is genuine truth which can be known, and that this genuine truth is not confined to statements about the process of knowing itself. In fact, the "critical" view that the only certain truth is truth about the process of knowing seems to be inherently self-contradictory. For the knowledge that such a proposition as, *e. g.*, "I know only the laws of my own apprehending activity," is true, would itself be knowledge not about the process of knowing but about the content known. Thus metaphysics, conceived as the science of the general character which distinguishes truth from falsehood, presupposes throughout all knowledge the presence of what we may call a "transcendent object," that is, a content which is never identical with the process by which it is apprehended, though it may no doubt be maintained that the two, the process and its content, if distinct, are yet not ultimately separable. That they are in point of fact not ultimately separable would seem to be the doctrine which, under various forms of statement, is common to and characteristic of all the "idealistic" systems of metaphysics. So much then in defense of a metaphysical point of view which seems to be closely akin to that of Mr. Bradley and of Professor Royce, to mention only two names of contemporary philosophers, and which might, I think, for the purpose of putting it in sharp opposition to the "neo-Kantian" view, not unfairly be called, if it is held to need a name, "neo-Leibnizian."

In passing on to discuss in brief the nature of the boundary lines which divide metaphysics from other branches of study, it seems necessary to start with a clear distinction between the "pure" or "formal" and the "applied" or "empirical" sciences, the more so as in the loose current employment of language the name "science" is frequently given exclusively to the latter. In every-day life, when we are told that a certain person is a "man of science," or as the detestable jargon of our time likes to say, a "scientist," we expect to find that he is, *e. g.*, a geologist, a chemist, a biologist, or an electrician. We should be a little surprised to find on inquiry that our "man of science" was a pure mathematician, and probably more than a little to learn that he was a formal logician. The distinction between the pure and the empirical sciences may be roughly indicated by saying that the latter class comprises all those sciences which yield information about the particular details of the temporal order of events physical and psychical, whereas the pure sciences deal solely with the general characteristics either of all truths, or of all truths of some well-defined class. More exactly we may say that the marks by which an empirical is distinguished from a pure science are two. (1) The empirical sciences one and all imply the presence among their premises of empirical propositions, that is, propositions which assert the actual occurrence of some temporal fact, and depend upon the witness of immediate apprehension, either in the form of sense-perception or in that of what is commonly called self-consciousness. In the vague language made current by Kant, they involve an appeal to some form of unanalyzed "intuition." The pure sciences, on the other hand, contain no empirical propositions either among their premises or their conclusions. The principles which form their premises are self-evidently true propositions, containing no reference to the actual occurrence of any event in the temporal order, and thus involving no appeal to any form of "intuition." And the conclusions established in a pure science are all rigidly logical deductions from such self-evident premises. That the universality of this distinction is still often overlooked even by professed writers on scientific method seems explicable by two simple considerations. On the one hand, it is easy to overlook the important distinction between a principle which is self-evident, that is, which cannot be denied without explicit falsehood, and a proposition affirmed on the warrant of the senses, because, though its denial cannot be seen to be obviously false, the senses appear on each fresh appeal to substantiate the assertion. Thus the Euclidean postulate about parallels was long falsely supposed to possess exactly the same kind of self-evidence as the *dictum de omni* and the principle of identity which are part of the foundations of all logic. And further Kant, writing under the influence of this very confusion, has given wide popularity to the view that the best known of the pure sciences, that of mathematics, depends upon the admission of empirical premises in the form of an appeal to intuition of the kind just described. Fortunately the recent developments of arithmetic at the hands of such men as Weierstrass, Cantor, and Dedekind seem to have definitely refuted the Kantian view as far as general arithmetic, the pure science of number, is concerned, by proving that one and all of its propositions are *analytic* in the strict sense of the word, that is, that they are capable of rigid deduction from self-evident premises, so that, in what regards arithmetic, we may say with Schröder that the famous Kantian question "how are synthetic judgments *a priori* possible?" is now known to be meaningless. As regards geometry, the case appears to a non-mathematician like myself more doubtful. Those who hold with Schröder that geometry essentially involves, as Kant thought it did, an appeal to principles not self-evident and dependent upon an appeal to sensuous "intuition," are logically bound to conclude with him that geometry is an "empirical," or as W. K. Clifford called it, a "physical" science, different in no way from mechanics except in the relative paucity of the empirical premises presupposed, and to class it with the applied sciences. On the other hand, if Mr. Bertrand Russell should be successful in his promised demonstration that all the principles of geometry are deducible from a few premises which include nothing of the nature of an appeal to sensuous diagrams, geometry too would take its place among the pure sciences, but only on condition of our recognizing that its truths, like those of arithmetic, are one and all, as Leibniz held, strictly analytical. Thus we obtain as a first distinction between the pure and the empirical sciences the principle that the propositions of the former class are all analytical, those of the latter all synthetic. It is not the least of the services which France is now rendering to the study of philosophy that we are at last being placed by the labors of M. Couturat in a position to appreciate at their full worth the views of the first and greatest of German philosophers on this distinction, and to understand how marvelously they have been confirmed by the subsequent history of mathematics and of logic.

(2) A consequence of this distinction is that only the pure or formal sciences can be matter of rigid logical demonstration. Since the empirical or applied sciences one and all contain empirical premises, *i. e.*, premises which we admit as true only because they have always appeared to be confirmed by the appeal to "intuition," and not because the denial of them can be shown to lead to falsehood, the conclusions to which they conduct us must one and all depend, in part at least, upon induction from actual observation of particular temporal sequences. This is as much as to say that all propositions in the applied sciences involve somewhere in the course of the reasoning by which they are established the appeal to the calculus of Probabilities, which is our one method of eliciting general results from the statistics supplied by observation or experiment. That this is the case with the more concrete among such applied sciences has long been universally acknowledged. That it is no less true of sciences of such wide range as mechanics may be said, I think, to have been definitely established in our own day by the work of such eminent physicists as Kirchhoff and Mach. In fact, the recent developments of the science of pure number, to which reference has been made in a preceding paragraph, combined with the creation of the "descriptive" theory of mechanics, may fairly be said to have finally vindicated the distinction drawn by Leibniz

long ago between the truths of reason and the truths of empirical fact, a distinction which the Kantian trend of philosophical speculation tended during the greater part of the nineteenth century to obscure, while it was absolutely ignored by the empiricist opponents of metaphysics both in England and in Germany. The philosophical consequences of a revival of the distinction are, I conceive, of far-reaching importance. On the one side, recognition of the empirical and contingent character of all general propositions established by induction appears absolutely fatal to the current mechanistic conception of the universe as a realm of purposeless sequences unequivocally determined by unalterable "laws of nature," a result which has in recent years been admirably illustrated for the English-speaking world by Professor Ward's well-known Gifford lectures on "Naturalism and Agnosticism." Laws of physical nature, on the empiristic view of applied science, can mean no more than observed regularities, obtained by the application of the doctrine of chances, — regularities which we are indeed justified in accepting with confidence as the basis for calculation of the future course of temporal sequence, but which we have no logical warrant for treating as ultimate truths about the final constitution of things. Thus, for example, take the common assumption that our physical environment is composed of a multitude of particles each in every respect the exact counterpart of every other. Reflection upon the nature of the evidence by which this conclusion, if supported at all, has to be supported, should convince us that at most all that the statement ought to mean is that individual differences between the elementary constituents of the physical world need not be allowed for in devising practical formulae for the intelligent anticipation of events. When the proposition is put forward as an absolute truth and treated as a reason for denying the ultimate spirituality of the world, we are well within our rights in declining the consequence on the logical ground that conclusions from an empirical premise must in their own nature be themselves empirical and contingent.

On the other hand, the extreme empiricism which treats all knowledge whatsoever as merely relative to the total psychical state of the knower, and therefore in the end problematic, must, I apprehend, go down before any serious investigation into the nature of the analytic truths of arithmetic, a consequence which seems to be of some relevance in connection with the philosophic view popularly known as Pragmatism. Thus I should look to the coming regeneration of metaphysics, of which there are so many signs at the moment, on the one hand, for emphatic insistence on the right, *e. g.*, of physics and biology and psychology to be treated as purely empirical sciences, and as such freed from the last vestiges of any domination by metaphysical presuppositions and foregone conclusions, and on the other, for an equally salutary purgation of formal studies like logic and arithmetic from the taint of corruption by the irrelevant intrusion of considerations of empirical psychology.

We cannot too persistently bear in mind that there is, corresponding to the logical distinction between the analytic and the synthetic proposition, a deep and broad general difference between the wants of our nature ministered to by the formal and the applied sciences respectively. The formal sciences, incapable of adding anything to our detailed knowledge of the course of events, as we have seen, enlighten us solely as to the general laws of interconnection by which all conceivable systems of true assertions are permeated and bound together. In a different connection it would be interesting to develop further the reflection that the necessity of appealing to such formal principles in all reasoning about empirical matters of fact contains the explanation of the famous Platonic assertion that the "Idea of Good" or supreme principle of organization and order in the universe, is itself not an existent, but something ἔτι ἐπέκεινα τῆς οὐσίας, "transcending even existence," and the very similar declaration of Hegel that the question whether "God" — in the sense of such a supreme principle — exists is frivolous, inasmuch as existence (*Dasein*) is a category entirely inadequate to express the Divine nature. For my present purpose it is enough to remark that the need to which the formal sciences minister is the demand for that purely speculative satisfaction which arises from insight into the order of interconnection between the various truths which compose the totality of true knowledge. Hence it seems a mistake to say, as some theorists have done, that were we born with a complete knowledge of the course of temporal sequences throughout the universe, and a faultless memory, we should have no need of logic or metaphysics, or in fact of inference. For even a mind already in possession of all true propositions concerning the course of events, would still lack one of the requisites for complete intellectual satisfaction unless it were also aware, not only of the individual truths, but of the order of their interdependence. What Aristotle said long ago with reference to a particular instance may be equally said universally of all our empirical knowledge; "even if we stood on the moon and saw the earth intercepting the light of the sun, we should still have to ask for the reason *why*." The purposes ministered to by the empirical sciences, on the other hand, always include some reference to the actual manipulation in advance by human agency of the stream of events. We study mechanics, for instance, not merely that we may perceive the interdependence of truths, but that we may learn how to maintain a system of bodies in equilibrium, or how to move masses in a given direction with a given momentum. Hence it is true of applied science, though untrue of science as a whole, that it would become useless if the whole past and future course of events were from the first familiar to us. And, incidentally it may be observed, it is for the same reason untrue of inference, though true of inductive inference, that it is essentially a passage from the known to the unknown.

In dealing with the relation of metaphysics to the formal sciences generally, the great difficulty which confronts us is that of determining exactly the boundaries which separate one from another. Among such pure sciences we have by universal admission to include at least two, pure formal logic and pure mathematics, as distinguished from the special applications of logic and mathematics to an empirical material. Whether we ought also to recognize ethics and æsthetics, in the sense of the general determination of the nature of the good and the beautiful, as non-empirical sciences, seems to be a more difficult question. It seems clear, for instance, that ethical discussions, such as bulk so largely in our contemporary literature, as to what is the right course of conduct under various conditions, are concerned throughout with an empirical material, namely, the existing peculiarities of human nature as we find it, and must therefore be regarded as capable only of an empirical and therefore problematic solution. Accordingly I was at one time myself tempted to regard ethics as a purely empirical science, and even published a lengthy treatise in defense of that point of view and in opposition to the whole Kantian conception of the possibility of a constructive *Metaphysik der Sitten*. It seems, however, possible to hold that in the question "What do we mean by good?" as distinguished from the question "What in particular is it right to do?" there is no more of a reference to the empirical facts of human psychology than in the question "What do we mean by truth?" and that there must therefore be a non-empirical answer to the problem. The same would of course hold equally true of the question "What *is* beauty?" If there are, however, such a pure science of ethics and again of æsthetics, it must at least be allowed that for the most part these sciences are still undiscovered, and that the ethical and æsthetical results hitherto established are in the main of an empirical nature, and this must be my excuse for confining the remarks of the next two paragraphs to the two great pure sciences of which the general principles may be taken to be now in large measure known.

That metaphysics and logic should sometimes have been absolutely identified, as for instance by Hegel, will not surprise us when we consider how hard it becomes on the view here defended to draw any hard and fast boundary line between them. For metaphysics, according to this conception of its scope, deals with the formulation of the self-evident principles implied, in there being such a thing as truth and the deductions which these principles warrant us in drawing.

Thus it might be fairly said to be the supreme science of *order*, and it would not be hard to show that all the special questions commonly included in its range, as to the nature of space, time, causation, continuity, and so forth, are all branches of the general question, how many types of order among concepts are there, and what is their nature. A completed metaphysics would thus appear as the realization of Plato's splendid conception of dialectic as the ultimate reduction of the contents of knowledge to order by their continuous deduction from a supreme principle (or, we may add, principles). Now such a view seems to make it almost impossible to draw any ultimate distinction between logic and metaphysics. For logic is strictly the science of the mutual implication of propositions, as we see as soon as we carefully exclude from it all psychological accretions. In the question what are the conditions under which one proposition or group of propositions imply another, we exhaust the whole scope of logic pure and proper, as distinguished from its various empirical applications. This is the important point which is so commonly forgotten when logic is defined as being in some way a study of "psychical processes," or when the reference to the presence of "minds" in which propositions exist, is intended into logical science. We cannot too strongly insist that for logic the question so constantly raised in a multitude of text-books, what processes actually take place when we pass from the assertion of the premises to the assertion of the conclusion, is an irrelevant one, and that the only logical problem raised by inference is whether the assertion of the premises as true *warrants* the further assertion of the conclusion, supposing it to be made. (At the risk of a little digression I cannot help pointing out that the confusion between a logical and a psychological problem is committed whenever we attempt, as is so often done, to make the self-evidence of a principle identical with our psychological inability to believe the contradictory. From the strictly logical point of view, all that is to be said about the two sides of such an ultimate contradiction is that the one is true and the other is false. Whether it is or is not possible, as a matter of psychical fact for me to affirm with equal conviction, both sides of a contradiction, knowing that I am doing so, is a question of empirical psychology which is possibly insoluble, and at any rate seems not to have received from the psychologists the attention it deserves. But the logician, so far as I can see, has no interest as a logician in its solution. For him it would still be the case even though all mankind should actually and consciously affirm both sides of a given contradiction, that one of the affirmations would be true, and the other untrue.) Logic thus seems to become either the whole or an integral part of the science of order, and there remain only two possible ways of distinguishing it from metaphysics. It might be suggested that logical order, the order of implication between truths, is only one species of a wider genus, order in general by the side, for example, of spatial, temporal, and numerical order, and thus that logic is one subordinate branch of the wider science of metaphysics. Such a view, of course, implies that there are a plurality of ultimately independent forms of order irreducible to a single type. Whether this is the case, I must confess myself at present incompetent to decide, though the signal success with which the principles of number have already been deduced from the fundamental definitions and axioms of symbolic logic, and number itself defined, as by Mr. Russell, in terms of the purely logical concept of class-relation, seems to afford some presumption to the contrary. Or it may be held that the difference is purely one of the degree of completeness with which the inquiry into order is pursued. Thus the ordinary symbolic logic of what Schröder has called the "identical calculus," or "calculus of domains," consists of a series of deductions from the fundamental concepts of class and number, identical equality, totality or the "logical 1," zero or the null-class, and the three principles of identity, subsumption, and negation. The moment you cease to accept these data in their totality as the given material for your science, and to inquire into their mutual coherence, by asking for instance whether any one of them could be denied, and yet a body of consistent results deduced from the rest, your inquiry, it might be said, becomes metaphysics. So, again, the discussion of the well-known contradictions which arise when we try to apply these principles in their entirety and without modification to classes of classes instead of classes of individuals, or of the problem raised by Peano and Russell, whether the assertions "Socrates is a man" and "the Greeks are men" affirm the same or a different relation between their subject and predicate (which seems indeed to be the same question differently stated), would generally be allowed to be metaphysical. And the same thing seems to be equally true of the introduction of time-relations into the interpretation of our symbols for predication employed by Boole in his treatment of hypotheticals, and subsequently adopted by his successors as the foundation of the "calculus of equivalent statements."

However we may decide such questions, we seem at least driven by their existence to the recognition of two important conclusions. (1) The relation between logical and metaphysical problems is so close that you cannot in consistency deny the possibility of a science of metaphysics unless you are prepared with the absolute skeptic to go the length of denying the possibility of logic also, and reducing the first principles of inference to the level of formulae which have happened hitherto to prove useful but are, for all we know, just as likely to fail us in future application as not. (Any appeal to the doctrine of chances would be out of place here, as that doctrine is itself based on the very principles at stake.) (2) The existence of fundamental problems of this kind which remained almost or wholly unsuspected until revealed in our own time by the creation of a science of symbolic logic should console us if ever we are tempted to suspect that metaphysics is at any rate a science in which all the main constructive work has already been accomplished by the great thinkers of the past. To me it appears, on the contrary, that the recent enormous developments in the purely formal sciences of logic and mathematics, with the host of fundamental problems they open up, give promise of an approaching era of fresh speculative construction which bids fair to be no less rich in results than any of the great "golden" periods in the past history of our science. Indeed, but that I would avoid the slightest suspicion of a desire to advertise personal friends, I fancy I might even venture to name some of those to whom we may reasonably look for the work to be done.

Of the relation of metaphysics to pure mathematics it would be impertinent for any but a trained mathematician to say very much. I must therefore be content to point out that the same difficulty in drawing boundary lines meets us here as in the case of logic. Not so long ago this difficulty might have been ignored, as it still is by too many writers on the philosophy of science. Until recently mathematics would have been thought to be adequately defined as the science of numerical and quantitative relations, and adequately distinguished from metaphysics by the non-quantitative and non-numerical character of the latter, though it would probably have been admitted that the problem of the definition of quantity and number themselves is a metaphysical one. But in the present state of our knowledge such an account seems doubly unsatisfactory. On the one hand, we have to recognize the existence of branches of mathematics, such as the so-called descriptive geometry, which are neither quantitative nor numerical, and, on the other, quantity as distinct from number appears to play no part in mathematical science, while number itself, thanks to the labors of such men as Cantor and Dedekind, seems, as I have said before, to be known now to be only a special type of order in a series. Thus there appears to be ground for regarding serial order as the fundamental category of mathematics, and we are thrown back once more upon the difficult task of deciding how many ultimately irreducible types of order there may be before we can undertake any precise discrimination between mathematical and metaphysical science. However we may regard the problem, it is at least certain that the recent researches of mathematicians into the meaning of such concepts as continuity and infinity have, besides opening up new metaphysical problems, done much to transfigure the familiar ones, as all readers of Professor Royce must be aware. For instance I imagine all of us here present, even the youngest, were brought up on the Aristotelian doctrine that there is and can be no such thing as an actually existing infinite collection, but which of us would care to defend that time-honored position to-day? Similarly with continuity all of us were probably once on a time instructed that whereas "quantity" is continuous, number is essentially "discrete," and is indeed the typical instance of what we mean by the non-continuous. To-day we know that it is in the number-series that we have our one certain and familiar instance of a perfect continuum. Still a third illustration of the transforming light which is thrown upon old standing metaphysical puzzles by the increasing formal development of mathematics may be found in the difficulties attendant upon the conception of the "infinitely little," once regarded as the logical foundation of the so-called Differential Calculus. With the demonstration, which may be found in Mr. Russell's important work, that "infinitesimal," unlike "infinite," is a purely relative term, and that there are no infinitesimal real numbers, the supposed logical significance of the concept seems simply to disappear. Instances of this kind could easily be multiplied almost indefinitely, but those already cited should be sufficient to show how important are the metaphysical results which may be anticipated from contemporary mathematical research, and how grave a mistake it would be to regard existing metaphysical construction, *e. g.*, that of the Hegelian system, as adequate in principle to the present state of our organized knowledge. In fact, all the materials for a new *Kategorienlehre,* which may be to the knowledge of our day what Hegel's *Logic* was to that of eighty years ago, appear to lie ready to hand when it may please Providence to send us the metaphysician who knows how to avail himself of them. The proof, given since this address was delivered, by E. Zermelo, that every assemblage can be well ordered, is an even more startling illustration of the remarks in the text.

It remains to say something of the relation of metaphysical speculation to the various sciences which make use of empirical premises. On this topic I may be allowed to be all the more brief, as I have quite recently expressed my views at fair length in an extended treatise (*Elements of Metaphysics,* Bks. 3 and 4), and have nothing of consequence to add to what has been there said. The empirical sciences, as previously defined, appear to fall into two main classes, distinguished by a difference which corresponds to that often taken in the past as the criterion by which science is to be separated from philosophy. We may study the facts of temporal sequence either with a view to the actual control of future sequences or with a view to detecting under the sequence some coherent purpose. It is in the former way that we deal with facts in mechanics, for instance, or in chemistry, in the latter that we treat them when we study history for the purpose of gaining insight into national aims and character. We may, if we please, with Professor Royce, distinguish the two attitudes toward fact as the attitude respectively of description and of appreciation or evaluation. Now as regards the descriptive sciences, the position to which, as I believe, metaphysicians are more and more tending is that here metaphysics has, strictly speaking, no right at all to interfere. Just because of the absence from metaphysics itself of all empirical premises, it can be no business of the metaphysician to determine what the course of events will be or to prescribe to the sciences what methods and hypotheses they shall employ in the work of such determination. Within these sciences any and every hypothesis is sufficiently justified, whatever its nature, so long as it enables us more efficiently than any other to perform the actual task of calculation and prediction. And it was owing to neglect of this caution that the *Naturphilosophie* of the early nineteenth century speedily fell into a disrepute fully merited by its ignorant presumption. As regards the physical sciences, the metaphysician has indeed by this time probably learned his lesson. We are not likely to-day to repeat the mistake of supposing that it is for us as metaphysicians to dictate what shall be the physicist's or chemist's definition of matter or mass or elementary substance or energy, or how he shall formulate the laws of motion or of chemical composition. Here, at any rate, we can see that the metaphysician's work is done when his analysis has made it clear that we are dealing with no self-evident truths such as the laws of number, but with inductive, and therefore problematic and provisional results of empirical assumptions as to the course of facts, assumptions made not because of their inherent necessity, but because of their practical utility for the special task of calculation. It is only when such empirical assumptions are treated as self-evident axioms, in fact when mechanical science gives itself out as a mechanistic philosophy, that the metaphysician obtains a right to speak, and then only for the purpose of showing by analysis that the presence of the empirical postulates which is characteristic of the natural sciences of itself excludes their erection into a philosophy of first principles.

What is important in this connection is that we should recognize quite clearly that psychology stands in this respect on precisely the same logical footing as physics or chemistry. It is tempting to suppose that in psychology, at any rate, we are dealing throughout with absolute certainties, realities which "consciousness" apprehends just as they are without any of that artificial selection and construction which, as we are beginning to see, is imposed upon the study of physical nature by the limitations of our purpose of submitting the course of events to calculation and manipulation. And it is a natural consequence of this point of view to infer that since psychology deals directly with realities, it must be taken as the foundation of the metaphysical constructions which aim at understanding the general character of the real as such. The consequence, indeed, disappears at once if the views maintained in this address as to the intimate relation of metaphysics and logic, and the radical expulsion from logic of all discussion of mental processes as such, be admitted. But it is still important to note that the premises from which the conclusion in question was drawn are themselves false. We must never allow ourselves to forget that, as the ever-increasing domination of psychology by the highly artificial methods of observation and experiment introduced by Fechner and Wundt is daily making more apparent, psychology itself, like physics, deals not directly with the concrete realities of individual experience, but with an abstract selected from that experience, or rather a set of artificial symbols only partially corresponding with the realities symbolized, and devised for the special object of submitting the realm of mental sequences to mathematical calculation. We might, in fact, have based this inference upon the single reflection that every psychological "law" is obtained, like physical laws, by the statistical method of elimination of individual peculiarities, and the taking of an average from an extended series of measurements. For this very reason, no psychological law can possibly describe the unique realities of individual experience. We have in psychology, as in the physical sciences, the duty of suspecting *exact* correspondence between the single case and the general "law" to be of itself proof of error somewhere in the course of our computation. These views, which I suppose I learned in the first instance from Mr. F. H. Bradley's paper called *A Defence of Phenomenalism in Psychology*, may now, I think, be taken as finally established beyond doubt by the exhaustive analysis of Professor Münsterberg's *Grundzüge der Psychologie*. They possess the double advantage of freeing the psychologist once for all from any interference by the metaphysician in the prosecution of his proper study, and delivering metaphysics from the danger of having assumptions whose sole justification lies in their utility for the purpose of statistical computation thrust upon it as self-evident principles. For their full discussion I may perhaps be allowed to refer to the first three chapters of the concluding book of my *Elements of Metaphysics*.

When we turn to the sciences which aim at the appreciation or evaluation of empirical fact, the case seems rather different. It may fairly be regarded as incumbent on the metaphysician to consider how far the general conception he has formed of the character of reality can be substantiated and filled in by our empirical knowledge of the actual course of temporal sequence. And thus the way seems to lie open to the construction of what may fairly be called a Philosophy of Nature and History. For instance, a metaphysician who has rightly or wrongly convinced himself that the universe can only be coherently conceived as a society of souls or wills may reasonably go on to ask what views seem best in accord with our knowledge of human character and animal intelligence as to the varying degrees of organized intelligence manifested by the members of such a hierarchy of souls, and the nature and amount of mutual intercourse between them. And again, he may fairly ask what general way of conceiving what we loosely call the inanimate world would at once be true to fundamental metaphysical principles and free from disagreement with the actual state of our physical hypotheses. Only he will need to bear in mind that since conclusions on these points involve appeal to the present results of the inductive sciences, and thus to purely empirical postulates, any views he may adopt must of necessity share in the problematic and provisional character of the empirical sciences themselves, and can have no claim to be regarded as definitely demonstrated in respect of their details. I will here only indicate very briefly two lines of inquiry to which these reflections appear applicable. The growth of evolutionary science, with the new light it has thrown upon the processes by which useful variations may be established without the need for presupposing conscious preëxisting design, naturally gives rise to the question whether such unconscious factors are of themselves sufficient to account for the actual course of development so far as it can be traced, or whether the actual history of the world offers instances of results which, so far as we can see, can only have issued from deliberate design. And thus we seem justified in regarding the problem of the presence of ends in Nature as an intelligible and legitimate one for the philosophy of the future. I would only suggest that such an inquiry must be prosecuted throughout by the same empirical methods, and with the same consciousness of the provisional character of any conclusions we may reach which would be recognized as in place if we were called on to decide whether some peculiar characteristic of an animal group or some singular social practice in a recently discovered tribe does or does not indicate definite purpose on the part of breeders or legislators.

The same remarks, in my opinion, apply to the familiar problems of Natural Theology relative to the existence and activity of such non-human intelligences as are commonly understood by the names "God" or "gods." Hume and Kant, as it seems to me, have definitely shown between them that the old-fashioned attempts to demonstrate from self-evident principles the existence of a supreme personal intelligence as a condition of the very being of truth all involve unavoidable logical paralogisms. I should myself, indeed, be prepared to go further, and to say that the conception of a single personality as the ground of truth and reality can be demonstrated to involve contradiction, but this I know is a question upon which some philosophers for whom I entertain the profoundest respect hold a contrary opinion. The more modest question, however, whether the actual course of human history affords probable ground for believing in the activity of one or more non-human personalities as agents in the development of our species I cannot but think a perfectly proper subject for empirical investigation, if only it be borne in mind that any conclusion upon such a point is inevitably affected by the provisional character of our information as to empirical facts themselves, and can claim in consequence nothing more than a certain grade of probability. With this proviso, I cannot but regard the question as to the existence of a God or of gods as one upon which we may reasonably hope for greater certainty as our knowledge of the empirical facts of the world's history increases. And I should be inclined only to object to any attempt to foreclose examination by forcing a conclusion either in the theistic or in the atheistic sense on alleged grounds of *a priori* metaphysics. In a word, I would maintain not only with Kant that the "physico-theological" argument is specially deserving of our regard, but with Boole that it is with it that Natural Theology must stand or fall.

NOTE ON EXTENSION AND INTENSION OF TERMS

Among the numerous difficulties which beset the teaching of the elements of formal logic to beginners, one of the earliest is that of deciding whether all names shall be considered to have meaning both in extension and intension. As we all know, the problem arises in connection with two classes of names, (1) proper names of individuals, (2) abstract terms. I should like to indicate what seems to me the true solution of the difficulty, though I do not remember to have seen it advocated anywhere in just the form I should prefer.

(1) As to proper names. It seems clear that those who regard the true proper name as a meaningless label are nearer the truth than those who assert with Jevons that a proper name has for its intension all the predicates which can be truly ascribed to the object named. As has often been observed, it is a sufficient proof that, for example, John does not *mean* "a human being of the male sex," to note that he who names his daughter, his dog, or his canoe John, makes no false assertion, though he may commit a solecism. So far the followers of Mill seem to have a satisfactory answer to Jevons, when they say, for example, that he confuses the intension of a term with its accidental or acquired associations. (So, again, we can see that Socrates cannot *mean* "the wisest of the Greek philosophers," by considering that I may perfectly well understand the statement "there goes Socrates" without being aware that Socrates is wise or a Greek or a philosopher.) And if we objected that no proper name actually in use is ever without some associations which in part determine its meaning by restricting its applicability, it would be a valid rejoinder that in pure logic we have to consider not the actual usages of language, but those that

would prevail in an ideal language purged of all elements of irrelevancy. In such an ideal scientific language, it might be said, the proper name would be reduced to the level of a mere mark serviceable for identification, but conveying no implication whatever as to the special nature of the thing identified. Thus it would be indifferent *what* mark we attach to any particular individual, just as in mathematics it is indifferent what alphabetical symbol we appropriate to stand for a given class or number. I think, however, that even in such an ideal scientific language the proper name would have a certain intension. In the first place, the use of proper name seems to inform us that the thing named is not unique, is not the only member of a class. To a monotheist, for instance, the name "God" is no true proper name, nor can he consistently give a proper name to his Deity. It is only where one member of a class has to be distinguished from others that the bestowal of a proper name has a meaning. And, further, to give a thing a proper name seems to imply that the thing is itself not a class. In logic we have, of course, occasion to form the concept of classes which have other classes for their individual members. But the classes which compose such classes of classes could not themselves be identified by means of proper names. Thus the employment of a proper name seems to indicate that the thing named is not the only member of its class, and further that it is not itself a class of individuals. Beyond this it seems to be a mere question of linguistic convention what information the use of a proper name shall convey. Hence it ought to be said, not that the proper name has no intension, but that it represents a limiting case in which intension is at a minimum.

(2) As to abstract terms. Ought we to say, with so many English formal logicians, that an abstract term is always singular and non-intensional? The case for asserting that such terms are all singular, I own, seems unanswerable. For it is clear that if the name of an attribute or relation is equally the name of another attribute or relation, it is ambiguous and thus not properly one term at all. To say, for example, that whiteness means two or more distinct qualities seems to amount to saying that it has no one definite meaning. Of course, it is true that milk is white, paper is white, and snow is white, and yet the color-tones of the three are distinct. But what we assert here is, not that there are different whitenesses, but only that there are different degrees of approximation to a single ideal standard or type of whiteness. It is just because the whiteness we have in view is one and not many that we can intelligibly assert, for example, that newly fallen snow is *whiter* than any paper. All the instances produced by Mill to show that abstract terms may be general seem to me either to involve confusion between difference of kind and difference in degree of approximation to type, or else to depend upon treating as abstract

a term which is really concrete. Thus when we say red, blue, green, are different kinds of color, surely what we mean is different kinds of colored surface. Quà colored, they are not different; I mean just as much and no more when I say "a red thing is colored," or "has color," as when I say "a green thing is colored." If Mill were right, the proposition "red is a color" ought to mean exactly the same as "red is red." Or, to put it in another way, it would become impossible to form in thought any concept of a single class of colored things.

But need we infer because abstract terms are singular that therefore they have no intension and are mere meaningless marks? Commonly as this inference is made, it seems to me clearly mistaken. It seems, in fact, to rest upon the vague and ill-defined principle that an attribute can have no attributes of its own. That it is false is shown, I think, by the simple reflection that scientific definitions are one and all statements as to the meaning of abstract names of attributes and relations. For example, the definition of a circle is a statement as to the meaning of circularity, the legal definition of responsible persons a statement as to the meaning of the abstraction "responsibility," and so on. (We only evade the point if we argue that abstract terms when used as the subjects of propositions are really being employed concretely. For "cruelty is odious," for instance, does not merely mean that cruel acts are odious acts, but that they are odious *because* they are cruel.) In fact, the doctrine that abstract terms have no intension would seem, if thought out, to lead to the view that there are only classes of individuals, but no classes of classes. Thus to say "cruel acts are odious because cruel" implies, not only that I can form the concept of a class of cruel acts, but also that of classes of odious acts of which the class of cruel acts in its turn is a member. And to admit as much as this is to admit that the class of cruel acts, considered as a member of the class of odious acts, shares the common predicate of odiousness with the other classes of acts composing the higher class. Hence the true account of abstract terms seems to me to be that we have in them another limiting case, a case in which the extension and the intension are coincident. Incidentally, by illustrating the ambiguity of the principle that attributes have no attributes of their own, our discussion seems to indicate the advantage of taking the purely extensional view as opposed to the predicative view of the import of propositions as the basis of an elementary treatment of logical doctrine.

THE PRESENT PROBLEMS OF METAPHYSICS

BY ALEXANDER T. ORMOND

[**Alexander Thomas Ormond**, McCosh Professor of Philosophy, Princeton University, since 1897. **b.** 1847, Punxsutawney, Pennsylvania. Mental Science Fellow, Princeton, 1877–78; Post-grad. Bonn and Berlin, 1884–85; Ph.D. Princeton, 1880; A.B. *ibid.* 1877; LL.D. Miami, 1899. Professor of Philosophy and History, University of Minnesota, 1880–83; Professor of Mental Science and Logic, Princeton University, 1883–97. Member American Philosophical Association, American Psychological Association.]

I

THE PRELIMINARY QUESTION

THE living problems of any science arise out of two sources: (1) out of what men may think of it, in view of its nature and claims, and (2) the problems that at any period are vital to it, and in the solution of which it realizes the purpose of its existence. Now if we distinguish the body of the sciences which deal with aspects of the world's phenomena — and here I would include both the psychic and the physical — from metaphysics, which professes to go behind the phenomenon and determine the world in terms of its inner, and, therefore, *ultimate* reality, it may be truly said of the body of the sciences that they are in a position to disregard in a great measure questions that arise out of the first source, inasmuch as the data from which they make their departure are obvious to common observation. Our world is all around us, and its phenomena either press upon us or are patent to our observation. Lying thus within the field of observation, it does not occur to the average mind to question either the legitimacy or the possibility of that effort of reflection which is devoted to their investigation and interpretation. Metaphysics, however, enjoys no such immunity as this, but its claims are liable to be met with skepticism or denial at the outset, and this is due partly to the nature of its initial claims, and partly to the fact that its real data are less open to observation than are those of the sciences. I say partly to the nature of the initial claims of metaphysics, for it is characteristic of metaphysics that it refuses to regard the distinction between phenomena and ground or inner nature, on which the sciences rest, as final, and is committed from the outset to the claim that the real is in its inner nature one and to be interpreted in the light of, or in terms of, its inner unity; whereas, science has so indoctrinated the modern mind with the supposition that only the outer movements of things are open to knowledge, while their inner and real nature must forever remain inaccessible to our powers; I say that the modern mind has been so imbued with this pretension as to have almost completely forgotten the fact that the distinction of phenomenon and ground is one of science's own making. Neither the plain man nor the cultured man, if he happens not to be tinctured with science, finds his world a duality. The things he deals with are the realities, and it is only when his naïve realism begins to break down before the complex demands of his growing life, that the thought occurs to him that his world may be more complex than he has dreamed. It is clear, then, that the distinction of our world into phenomena and ground, on which science so largely rests, is a first product of reflection, and not a fact of observation at all.

If this be the case, it may be possible and even necessary for reflection at some stage to transcend this distinction. At least, there can be no reason except an arbitrary one for taking this first step of reflection to be a finality. And there would be the same justification for a second step that would transcend this dualism, as for the initial step out of which the distinction arose; provided, it should be found that the initial distinction does not supply an adequate basis for a rational interpretation of the world that can be taken as final. Now, it is precisely because the dualistic distinction of the sciences does fail in this regard, that a further demand for a reflective transformation of the data arises. Let us bear in mind that the data of the sciences are not the simple facts of observation, but rather those facts transformed by an act of reflection by virtue of which they become phenomena distinguished from a more fundamental nature on which they depend and which itself is not open to observation. The real data of science are found only when the world of observation has been thus transformed by an act of reflection. If then at some stage in our effort to interpret our world it should become clear that the sciences of phenomena, whatever value their results may possess, are not giving us an interpretation in terms that can be taken as final, and that in order to ground such an interpretation a further transformation of our data becomes necessary, I do not see why any of the sciences should feel that they have cause to demur. In truth, it is out of just such a situation as this that the metaphysical interpretation arises (as I propose very briefly here to show), a situation that supplies a genuine demand in the light of which the effort of metaphysics to understand its world seems to possess as high a claim to legitimacy as that of the sciences of phenomena. Let us take our stand with the plain man or the child, within the world of unmodified observation. The things of observation, in this world, are the realities, and at first we may suppose have undergone little reflective transformation. The first reflective effort to change this world in any way will, no doubt, be an effort to *number* or *count* the things that present themselves to observation, and out of this effort will arise the transformation of the world that results from considering it under the concepts and categories of number. In short, to mathematical reflection of this simple sort, the things of observation will resolve themselves into a plurality of countable things, which the numbering reflection becoming explicit in its ordinal and cardinal moments will translate into a system that will be regarded as a whole made up of the sum of its parts. The very first step, then, in the reflective transformation of things resolves them into a dual system, the world conceived as a cardinal whole that is made up of its ordinal parts, and exactly equal to them. This mathematical conception is moreover purely quantitative; involving the exact and stable equivalence of its parts or units and that of the sum of the parts with the whole. Now it is with this purely quantitative transformation that mathematics and the mathematical sciences begin. We may ask, then, why should there be any other than mathematical science,[1] and what ground can non-mathematical science point to as substantiating its claims? I confess I can see no other final reason than this, that mathematical science does not meet the whole demand we feel obliged to make on our world. If mathematics were asked to vindicate itself, it no doubt would do so by claiming that things present quantitative aspects on which it founds its procedure. In like manner non-mathematical, or, as we may call it, physical or natural science, will seek to substantiate its claims by pointing to certain ultra-quantitative or qualitative aspects of things. It is true that, so far as things are merely *numerable*, they are purely quantitative; but mathematics abstracts from the content and character of its units and aggregates, which may and do change, so that a relation of stable equivalence is not maintained among them. In fact, the basis of these sciences is found in the tendency of things to be always changing and becoming different from what they were before. The problem of these sciences is how to ground a rational scheme of knowledge in connection with a fickle world like that of qualitative change. It is here that reflection finds its problem, and noticing that the tendency of this world of change is for a to pass into b and thus to lose its own identity, the act of reflection that rationalizes the situation is one that connects a and b by relating them to a common ground x of which they stand as successive manifestations or symbols. X thus supplies the thread of identity that binds the two changes a and b into a relation to which the name causation may be applied. And just as quantitative equivalence is the principle of relationship among the parts of the simple mathematical world, so here in the world of the dynamic or natural sciences, the principle of relation is natural causation.[2] We find, then, that the non-mathematical sciences rest on a basis that is constituted by a *second act of reflection;* one that translates our world into a system of phenomena causally inter-related and connected with their underlying grounds.

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[1] I do not raise the question of qualitative mathematics at all. It is clear that the first mathematical reflection will be quantitative.

[2] By natural causation I mean such a relationship between a and b in a phenomenal system as enables a through its connection with its ground to determine b.

We have now reached a point where it will be possible in a few sentences to indicate the rise of the metaphysical reflection and the ground on which it rests. If we consider both the mathematical and the physical ways of looking at things, we will find that they possess this feature in common, — they are purely external, having nothing to say respecting the *inner* and, therefore, *real* nature of the things with which they deal. Or, if we concede the latest claims of some of the physical speculators and agree that the aim of physics is an ultimate physical explanation of reality, it will still be true that the whole standpoint of this explanation will be external. Let me explain briefly what I mean substantially by the term *external* as I use it here. Every interpretation of a world is a function of some knowing consciousness, and consequently of some knowing self. This is too obvious to need proof. A system will be *external* to such a knower just to the extent that the knower finds it dominated and determined by categories that are different from those of its own determination. A world physically interpreted is one that is brought completely under the rubrics of physics and mathematics; whose movements yield themselves completely, therefore, to a mechanical calculus that gives rise to purely descriptive formulæ; *or* to the control of a dynamic principle; that of natural causation, by virtue of which everything is determined without thought of its own, by the impulse of another, which impulse itself is not directly traceable to any thought or purpose. Now, the occasion for the metaphysical reflection arises when this situation that brings us face to face with, nay, makes us part and parcel of, an alien system of things, becomes intolerable, and the knower begins to demand a closer kinship with his world. The knower finds the categories of his own central and characteristic activity in experience. Here he is conscious of being an agent going out in forms of activity for the realization of his world. The determining categories of the activity he is most fully conscious of, are interest, idea, prevision, purpose, and that selective activity which goes to its termination in some achieved end. The metaphysical interpretation arises out of the demand that the world shall be brought into bonds of kinship with the knower. And this is effected by generalizing the categories of consciousness and applying them as principles of interpretation to the world. The act of reflection on which the metaphysical interpretation proceeds is one, then, in which the world of science is further transformed by bringing the inner nature of things out of its isolation and translating the world-movements into process the terms of which are no longer *phenomena and hidden ground*, but rather inception and realization, or, more specifically, *Idea* and *Reality*. And the point to be noted here is the fact that these metaphysical categories are led up to positivity by an act of reflection that has for its guiding aim an interpretation of the world that will be more ultimately satisfactory to the knower than that of the physical or natural sciences; while negatively, it is led up to by the refusal of the knowing consciousness to rest in a world alien to its own nature and in which it is subordinated to the physical and made a mere epiphenomenon.

II

QUESTIONS OF POINT OF VIEW, PRINCIPLE AND METHOD OF METAPHYSICS

It is clear from what has been said that the metaphysical interpretation proceeds on a presupposition radically different from that of mathematical and physical science. The presumption of these sciences is that the world is physical, that the physical categories supply the norms of reality, and that consciousness and the psychic, in general, are subordinate and phenomenal to the physical. On the contrary, metaphysics arises out of a revolt from these presumptions toward the opposite presumption, namely, that *consciousness itself is the great reality*, and that the norms of an ultimate interpretation of things are to be sought in its categories. This is the great transformation that conditions the possibility and value of all metaphysics. It is the Copernican revolution which the mind must pass through, a revolution in which matter and the physical world yields the primacy to mind; a revolution in which consciousness becomes central, its categories and analogies supplying the principles of final world-interpretation. Let us consider then, in the light of this great Copernican revolution, the questions of the *point of view*, *principle*, and *method* of metaphysics. And here the utmost brevity must be observed. If consciousness be the great reality, then its own central activity, that effort by which it realizes its world, will determine for us the *point of view* or departure of which we are in quest. This will be *inner* rather than *outer;* it will be motived by *interest*, will shape itself into interest-directed effort. This effort will be cognitive; dominated by an *idea* which will be an anticipation of the *goal* of the effort. It will, therefore, become *directive*, *selective*, and will stand as the *end* or *aim* of the completed effort. The whole movement will thus take the form, genetically, of a developing *purpose informed by an idea*, or *teleologically*, of a *purpose going on to its fulfillment* in some *aim* which is also its *motive*. Now, metaphysics determines its point of view in the following reasoning: if in consciousness we find the type of the inner nature of things, then the point of view for the interpretation of this inner nature will be to seek by generalizing the standpoint of consciously determined effort and asserting that this is the true point of view from which the *meaning* of the world is to be sought.

Having determined the metaphysical point of view, the next question of vital importance is that of its *principle*. And we may cut matters short here by saying at once that the principle we are seeking is that of *sufficient reason*, and we may say that a reason will be sufficient when it adequately expresses the world-view or concept under which an investigation is being prosecuted. Let us suppose that this world-view is that of simple mathematics, the principle of sufficient reason here will be that of *quantitative equivalence* of parts; or, from the standpoint of the whole, that of *infinite divisibility*. Whereas, if we take the world of the ultra-mathematical science, which is determined by the notion of *phenomena depending on underlying ground*, we will find that the sufficient reason in this sphere takes the form of *adequate cause or condition*. The determining condition or causes of any physical phenomenon supply, from that point of view, the *ratio sufficiens* of its existence. We have seen that the sufficiency of a reason in the above cases has been determined in view of that notion which defines the kind of world the investigation is dealing with. Let us apply this insight to the problem of the principle of metaphysics, and we will soon conclude that no reason can be metaphysically sufficient that does not satisfy the requirements of a world conceived under the notion of *inception* and *realization* ; or, more specifically, *idea* and *reality*. In short, the *reason* of metaphysics will refuse to regard its world as a mechanism that is devoid of thought and intention; that lacks, in short, the motives of internal determination and movement, and will in all cases insist that an explanation or interpretation can be metaphysically adequate only when its ultimate reference is to an idea that is in the process of *purposive* fulfillment. Such an explanation we call *teleological or rational*, rather than merely mechanical, and such a principle is alone adequate to embody the *ratio sufficiens* of metaphysics.

Having determined the point of view and principle of metaphysics, the question of metaphysical *method* will be divested of some of its greatest difficulties. It will be clear to any one who reflects that the very first problem in regard to the method of metaphysics will be that of its starting-point and the kind of results it is to look for. And little can be accomplished here until it has been settled that consciousness is to have the primacy, and that its prerogative is to supply both standpoint and principle of the investigation. We have gone a long way toward mastering our method when we have settled these points: (1) that the metaphysical world is a world of consciousness; (2) that the conscious form of effort rather than the mechanical is the species of activity or movement with which we have to deal; and, (3) that the world it is seeking to interpret is ultimately one of *idea* *and reality* in which the processes take the *purposive* form. In view of this, the important steps of method (and we use the term method here in the most fundamental sense) will be (1) the question of the *form* of metaphysical activity or agency as contrasted with that of the physical sciences. This may be brought out in the contrast of the two terms *finality* and *mere efficiency*, in which by mere efficiency is meant an agency that is presumed to be thoughtless and purposeless, and consequently without *foresight*. All this is embodied in the term *force* or physical energy, and less explicitly in that of *natural causation*. Contrasted with this, *finality* is a term that involves the forward impulse of *idea*, *prevision*, and *purpose*. Anything that is capable of any sort of *foretaste* has in it a principle of prevision, selection, choice, and purpose. The impulse that motives and runs it, that also stands out as the *end* of its fulfillment, is a foretaste, an *Ahnung*, an anticipation, and the whole process or movement, as well as every part of it, will take on this character. (2) The second question of method will be that of the nature of this category of which *finality* is the form. What is its content, pure idea or pure will, or a synthesis that includes both? We have here the three alternatives of *pure rationalism*, *voluntarism*, and a doctrine hard to characterize in a single word; that rests on a *synthesis* of the norms of both rationalism and voluntarism. Without debating these alternatives, I propose here briefly to characterize the *synthetic* concept as supplying what I conceive to be the most satisfactory doctrine. The principle of *pure rationalism* is one of insight but is lacking in practical energy, whereas, that of *voluntarism* supplies practical energy, but is lacking in insight. Pure voluntarism is *blind*, while pure rationalism is *powerless*. But the synthesis of *idea* and *will*, provided we go a step further (as I think we must) and presuppose also a germ of *feeling* as *interest*, supplies both *insight* and *energy*. So that the spring out of which our world is to arise may be described as either the *idea informed with purposive energy*, or *purpose or will informed and guided by the idea*. It makes no difference which form of conception we use. In either case if we include feeling as interest we are able to conceive movements originating in some species of apprehension, taking the dynamic form of purpose, and motived and selected, so to speak, by interest; and in describing such activity we are simply describing these normal movements of consciousness with which our experience makes us most familiar. (3) The third question of method involves the relation or correlation of the metaphysical interpretation with that of the natural or physical science. Two points are fundamental here. In the first place, it must be borne in mind that it is the same world with which the plain man, the man of science, and the metaphysician are concerned. We cannot partition off the external world to the plain man, the atoms and ethers to the man of science, leaving the metaphysician in exclusive and solitary possession of the world of consciousness. It is the same world for all. The metaphysician cannot shift the physical world, with its oceans and icebergs, its vast planetary systems and milky ways, on to the shoulders of the physicist. This is the metaphysician's own recalcitrant world, which will doubtless task all his resources to explain. In the *second* place, though it is the same world that is clamoring for interpretation, it is a world that passes through successive transformations, in order to adapt itself to progressive modes of interpretation. The plain man is called to pass through a species of Copernican revolution that subordinates the phenomenon to its ground, before he can become a man of science. In turn, the man of science must go through the Copernican process, and learn to subordinate his atoms and ethers to consciousness before he can become a metaphysician. And it is this transformation that marks one of the most fundamental steps in the method of metaphysics. The world must experience this transformation, and it must become habitual to the thinker to subordinate the physical to the mental before the metaphysical point of view can be other than foreign to him. If, then, it be the same content with which the sciences and metaphysics are called on to deal, it is clear that we have on our hands another problem on the answer to which the fate of metaphysics vitally depends; the question of the *correlation* of its method with that of the sciences so that it may stand vindicated as the final interpretation of things.

III

QUESTION OF THE CORRELATION OF METAPHYSICS WITH THE SCIENCES

We have reached two conclusions that are vital here: (1) that the metaphysical way of looking at the world involves a transformation of the world of physical science; (2) that it is the same world that lies open to both science and metaphysics. Out of this arises the problem of the *correlation* of the two views; the two interpretations of the world. If science be right in conceiving the world under such categories as quantity and natural causation; if science be right in seeking a mechanical explanation of phenomena (that is, one that excludes prevision, purpose, and aim); and if metaphysics be right in refusing to accept this explanation as final and in insisting that the principle of ultimate interpretation is teleological, that it falls under the categories of prevision, purpose, and aim; then it is clear that the problem of correlation is on our hands. In dealing with this problem, it will be convenient to separate it into two questions: (1) that of the fact; (2) that of its rationale. The fact of the correlation is a thing of common experience. We have but to consider the way in which this Congress of Science has been brought about in order to have an exhibition of the method of correlation. Originating first in the sphere of thought and purpose, the design has been actualized through the operation of mechanical agencies which it has somehow contributed to liberate. On the scale of individual experience we have the classic instance of the arm moving through space in obedience to a hidden will. There can be no question as to the fact and the great difficulty of metaphysics does not arise in the task of generalizing the fact and conceiving the world as a system of thought-purposes working out into forms of the actual through mechanical agencies. This generalization somehow lies at the foundation of all metaphysical faith, and, this being the case, the real task here, aside from the profounder question of the *rationale*, is that of exhibiting the actual points of correlation; those points in the various stages of the sciences from physics to ethics and religion, at which the last category or result of science is found to hold as its immediate implication some first term of the more ultimate construction of metaphysics. The working out of this task is of the utmost importance, inasmuch as it makes clear to both the man of science and the metaphysician the intrinsic necessity of the correlation. It is a task analogous to the Kantian deduction of the categories.

IV

QUESTIONS OF THE ULTIMATE NATURE OF REALITY

We come, then, to the question of the rationale of this correlation, and it is clear here that we are dealing with a phase of the problem of the ultimate nature of reality. For the question of the correlation now is how it is possible that our thoughts should affect things so that they move in response; how mind influences body or the reverse, how, when we will, the arm moves through space. And without going into details of discussion here, let us say at once, that whatever the situation may be for any science, — and it may be that some form of *dualism* is a necessary presupposition of science, — for metaphysics it is clear that no dualism of substances or orders can be regarded as final. The life of metaphysics depends on finding the one for the many; the one that when found will also ground the many. If, then, the phenomenon of *mind and body* presents the appearance of a correspondence of two different and, so far as can be determined, mutually exclusive agencies, the problem of metaphysics is the reduction of these agencies to one species. Here we come upon the issue between materialism and immaterialism. But inasmuch as the notion of metaphysics itself seems to exclude materialism, the vital alternative is that of immaterialism. Again, if psycho-physics presents as its basal category a *parallelism* between two orders of phenomena, psychic and physical, it is the business of metaphysics to seek the explanation of this dualism in some more ultimate and unitary conception. Now, since the very notion of metaphysics again excludes the physical alternative from the category of finality, we are left with the psychic term as the one that, by virtue of the fact that it embodies a form of *conscious* activity, promises to be most fruitful for metaphysics. From one point of view, then, we have reduced our world to immaterialism; from another, to some form or analogue of the psychic. Now it is not necessary here to carry the inquiry further in this direction. For what metaphysics is interested in, specially, is the fact that the world must be reduced to one kind of being and one type of agency. If this be done, it is clear that the dualism of *body and mind* and the *parallel orders* of psycho-physics cannot be regarded as final, but must take their places as phenomena that are relative and reducible to a more fundamental unity. The metaphysician will say that the arm moves through space in response to the will, and that everywhere the correlation between mechanical and teleological agency takes place because in the last analysis *there is only one type of agency;* an agency that finds its initiative in interest, thought, purpose, design, and thus works out its results in the fields of space and mechanical activities.

Furthermore, on the question to which these considerations lead up; that of the ultimate interpretation we are to put on the reality of the world, the issue is not so indeterminate as it might seem from some points of view. Taking it that the very notion of metaphysics excludes the material and the physical as ultimate types of the real, we are left with the notions of the immaterial and the psychic; and while the former is indefinite, it is a fact that in the psychic and especially in the form of it which man realizes in his own experience, he finds an intelligible type and the only one that is available to him for the definition of the immaterial. He has his choice, then, either to regard the world as *absolutely opaque,* showing nothing but its phenomenal dress which ceases to have any meaning; or to apply to the world's inner nature the intelligible types and analogies of his own form of being. That this *is* the alternative that is embodied in the existence of metaphysics is clearly demonstrated by the fact that the metaphysical interpretation embodies itself in the categories of *reason, design, purpose,* and *aim.* Whatever difficulties we may encounter, then, in the *use* and application of the *psychic analogy* in determining the nature of the real, it is clear that its employment is inevitable and indispensable. Let us, then, employ the term *rational* to that characterization of the nature of things which to metaphysics is thus inevitable and indispensable. The world must in the last analysis be *rational* in its constitution, and its agencies and forms of being must be construed as *rational* in their type.

And here we come upon the last question in this field, that of the *ultimate being of the world.* We have already concluded that the *real* is in the last analysis rational. But we have not answered the question whether there shall be one rational or many. Now it has become clear that with metaphysics *unity* is a cardinal interest; that, therefore, the world must be *one* in *thought, purpose, aim.* And it is on this insight that the metaphysical doctrine of the *absolute* rests. There must be *one* being whose thought and purpose are all-inclusive, in order that the world may be one and that it may have meaning as a whole. But the world presents itself as a plurality of finite *existents* which our metaphysics requires us to reduce in the last analysis to the psychic type. What of this plurality of psychic existents? It is on this basis that metaphysics constructs its doctrine of *individuality.* Allowing for latitude of opinion here, the trend of metaphysical reflection sets strongly toward a doctrine of reality that grounds the world in an Absolute whose all-comprehending thought and purpose utters or realizes itself in the plurality of finite individuals that constitutes the world; the degree of reality that shall be ascribed to the plurality of individuals being a point in debate, giving rise to the contemporary form of the issue between idealism and realism. Allowing for minor differences, however, there is among metaphysicians a fair degree of assent to the doctrine that in order to be completely rational the world of individual plurality must be regarded as implying an *Absolute,* which, whether it is to be conceived as an individual or not, is the author and bearer of the thought and design of the world as a whole.

V

QUESTIONS OF METAPHYSICAL KNOWLEDGE AND ULTIMATE CRITERIA OF TRUTH

We have only time to speak very briefly, in conclusion, of two vital problems in metaphysics: (1) that of the nature and limits of metaphysical knowledge; (2) that of the ultimate criteria of truth. In regard to the question of knowledge, we may either *identify thought with reality,* or we may regard thought as *wholly inadequate to represent the real;* in one case we will be *gnostic,* in the other *agnostic.* Now whatever may be urged in favor of the gnostic alternative, it remains true that *our* thought, in order to follow along intelligible lines, must be guided by the categories and analogies of our own experience. This fixes a limit, so that the thought of man is never in a position to grasp the real completely. Again, whatever may be urged in behalf of the agnostic alternative, it is to be borne in mind that our experience does supply us with intelligible types and categories; and that under the impulse of the *infinite* and *absolute,* or the transcendent, to which our thought responds (to put it no stronger), a dialectical activity arises; on the one hand, the application of the experience-analogies to determine the real; on the other, the incessant removal of limits by the impulse of transcendence (as we may call it). Thus arises a *movement of approximation* which while it never completely compasses its goal, yet proceeds along intelligent lines; constitutes the mind's effort to know; and results in an *approximating series of intelligible and relatively adequate conceptions.* Metaphysically, we are ever approximating to ultimate knowledge; though it can never be said that we have attained it. The type of metaphysical knowledge cannot be characterized, therefore, as either gnostic or agnostic.

As to the question of ultimate *criteria,* it is clear that we are here touching one of the living issues of our present-day thought. Shall the judgment of truth, on which certitude must found, exclude practical considerations of value, or shall the consideration of value have weight in the balance of certitude? On this issue we have at the opposite extremes (1) the *pure rationalist* who insists on the rigid exclusion from the epistemological scale of every consideration except that of pure logic. The truth of a thing, he urges, is always a purely logical consideration. On the other hand, we have (2) the *pure pragmatist,* who insists on the "*will to believe*" as a legitimate datum or factor in the determination of certitude. The pragmatic platform has two planks: (1) the *ontological* — we select our world that we call real at the behest of our interests; (2) the *ethical* — in such a world practical interest has the right of way in determining what we are to accept as true as well as what we are to choose as good. It is my purpose in thus outlining the extremes of doctrine to close with a suggestion or two toward less ultra-conclusions. It is a sufficient criticism on the *pure rationalist's* position to point out the fact that his separation of practical and theoretic interests is a pure fiction that is never realized anywhere. The motives of science and the motives of practice are so blended that interest in the conclusion always enters as a factor in the process. A conclusion reached by the pure rationalist's method would be one that would only interest the pure rationalist in so far as he could divest himself of all motives except the bare love of fact for its own sake. The *pure pragmatist* is, I think, still more vulnerable. He must, to start with, be a pure subjective idealist, otherwise he would find his world at many points recalcitrant to his ontology. Furthermore, the mere *will to believe* is arbitrary and involves the suppression of reason. In order that the will to believe may work *real* conviction, the point believed must at least amount to a postulate of the practical reason; it must become somehow evident that the refusal to believe would create a situation that would be theoretically unsound or irrational;

as, for instance, if we assume that the immortality of the soul is a *real postulate* of practical reason, it must be so because the negative of it would involve the irrationality of our world; and therefore a degree of theoretic imperfection or confusion. Personally I believe the lines here converge in such a way that the ideal of truth will always be found to have practical value; and *conversely*, as to practical ideals, that a sound practical postulate will have weight in the theoretic scales. And it is doubtless true, as Professor Royce urges in his presidential address on *The Eternal and The Practical*, that all judgments must find their final warrant at the Court of the Eternal where, so far as we can see, the theoretical and practical coalesce into one.

At the close of the work of this Section and upon the invitation of Dr. Armstrong, a number of distinguished members in attendance joined freely in the discussion, to the great pleasure of the many specialists who were present. Among those participating were Professor Boltzmann of Vienna, Professor Hoeffding of Copenhagen, Professor Calkins of Wellesley, and Professor French of the University of Nebraska, to whom replies were made by the principal speakers, Messrs. Taylor and Ormond.

SHORT PAPERS

A short paper was contributed to the work of the Section by Professor W. P. Montague of Columbia University, on the "Physical Reality of Secondary Qualities." The speaker said that from the beginning of modern philosophy there has existed a strong tendency among all schools of thought — monists of the idealistic or materialistic types, as well as outspoken dualists — to treat the distinction between primary and secondary qualities as coincident, so far as it goes, with the distinction between physical and psychical. Colors, sounds, odors, etc., are regarded as purely subjective or mental in their nature, and as having no true membership in the physical order; while correlatively all special forms and relations have been in their turn extruded from the field of the psychical. Let it be noted that introspection offers little or nothing in support of this view. There is nothing, for example, about the color red that would make it appear more distinctively psychical or subjective than a figure or a motion. The perception of a square or a triangle is not a square or triangular perception; but neither is the perception of red or blue a red or blue perception. Now with the affective or emotional contents of experience the case is quite different.

A feeling of pain is a painful feeling, a consciousness of anger is an angry consciousness. Pains are more and less painful, according as we are more and less aware of them. With feelings and volitions *esse* is indeed *percipi*. Colors and other secondary qualities, however, do not seem thus to increase or diminish in their reality concomitantly with our perceptions of them. Red is red, neither more nor less, regardless of the amount to which we attend to it. And yet it remains true that, notwithstanding this seeming objectivity, the secondary qualities have long been contrasted with the primary, and classed along with the affective and volitional states as purely subjective facts. It has always seemed curious that a view so important as this in its consequences, and so radically at variance, not only with Pre-Cartesian philosophy, but also with our instinctive beliefs, should have won its way to the position of an accepted dogma; and the purpose of this paper was first to examine the grounds upon which this belief rests, and second to show that the problem of the independent reality of the physical world and the problem of the relation of physical and psychical appear in a clearer and more hopeful light when disentangled from the quite different problem of the relation of primary and secondary qualities.

There were two reasons why the older or Pre-Cartesian view of this question should give place to the modern doctrine. First, because of the rediscovery of the idea of mechanism, without which predictive science had been virtually impossible. The second reason for reducing the secondary qualities to a merely subjective status lay in the fact that they are much more dependent than the primary qualities upon the bodily organism of the one who perceives them. In closing Professor Montague said: —

"I wish in closing to point out two consequences of the view which I have been opposing. First, the present paradoxical status of the eternal world; second, the equally paradoxical status of the relation of that world to the world of mind. Berkeley was the first thinker clearly to perceive the unsubstantial nature of a world made up solely of primary qualities. Indeed, in the last analysis, a world of primary qualities, and nothing else, is a world of relations without terms, a geometrical fiction, the objective (or, for that matter, the subjective) existence of which the idealist would be right in denying. In Biology we have abandoned obscurantist methods, and no longer attribute the distinctive vital functions of growth and reproduction to a vital force or vital substance, but solely to the peculiar configuration of the material elements of a cell. Why may we not in psychology with equal propriety attribute the distinctively psychical functions of subjectivity or consciousness, not to the action of a hyper-psychical soul-substance, nor to the presence of a transcendental ego, but simply to that peculiar configuration of sensory elements which constitutes a what we call psychosis?"

SECTION B — PHILOSOPHY OF RELIGION

SECTION B

PHILOSOPHY OF RELIGION

(*Hall* 1, *September* 21, 3 *p. m.*)

CHAIRMAN: PROFESSOR THOMAS C. HALL, Union Theological Seminary, N. Y.
SPEAKERS: PROFESSOR OTTO PFLEIDERER, University of Berlin.
PROFESSOR ERNST TROELTSCH, University of Heidelberg.
SECRETARY: DR. W. P. MONTAGUE, Columbia University.

THE RELATION OF THE PHILOSOPHY OF RELIGION TO THE OTHER SCIENCES

BY PROFESSOR OTTO PFLEIDERER

[**D. Otto Pfleiderer**, Professor of Theology, University of Berlin since 1875. **b.** September 1, 1839, Stetten, Würtemberg. **Grad.** Tübingen, 1857–61. **Post-grad.** *ibid.* 1864–68. City Professor, Heilbronn, 1868–69; Superintendent, Jena, 1869–70; Professor of Theology, Jena, 1870–75. **Author of** *Religion and its Essential Characteristics; Religious Philosophy upon Historical Foundation;* and many other works and papers on Theology.]

IN order to answer this question, we need to consider a preliminary question, namely, whether religion can be regarded as the object of scientific knowledge in the same manner as other processes of the intellectual life of the race, such as law, history, and art. It is well known that this question has not always received an affirmative answer, and indeed it can never be answered in the affirmative so long as the position is maintained that the only religion is that of the Christian Church, whose doctrines and teachings rest upon an immediate divine revelation, and that these must be accepted by men in blind belief. Under the position of an authoritative ecclesiastical faith there can indeed exist a theoretical consideration of the doctrines of faith, as it was the case with the scholastic theology of the Middle Ages, which with great earnestness sought to harmonize faith and knowledge; nevertheless, no one of the present day would give to the scholastic theology the name of science with the modern meaning of the term science. The scholastic theology used great formal acuteness and skill in the work of defining and defending ecclesiastical traditions, still there was lacking that which for us is the essential condition of scientific knowledge, the free examination of tradition according to the laws of human thought and the analogy of the general experience of humanity. The great hindrance to the progress of the knowledge of religion was the accepted position that the truth of the ecclesiastical doctrines was beyond human reason and outside of human examination, since their truth rested upon an immediate divine revelation. Whether this supernatural authority was ascribed to the Church or the Bible makes very little difference, for in either case the assumption of such an authority is a hindrance to the free examination of that which claims to be the divine revealed truth.

But is this assumption really justifiable in the nature of the case? Do the doctrines of the Church rest upon a supernatural divine revelation? So soon as this question was really earnestly considered, and the thinking mind could not always avoid the consideration, then there was revealed the inadequacy of the assumption. Two ways of examination led to a common critical result, the philosophical analysis of the religious consciousness and the historical comparison of various religions. The first to enter upon these ways and at the same time to become the founder of the modern science of religion was the keen Scotch thinker David Hume. Truly the thought of Hume was still a one-sided, disorganizing skepticism; even as his theory of knowledge disturbed the truth of all our previous common-sense opinions and conceptions, so also his philosophy of religion sought to demonstrate that all religion cannot be proved and is full of doubt, and that the origin of religion was neither to be found in divine revelation nor in the reason of man, but in the passions of the heart and in the illusions of imagination. As unsatisfactory as this result was, nevertheless it gave an important advance to the rational study of religion in two directions, in that of religion being an experience of the inner life of the soul and in that of religion being a fact of human history.

Kant added the positive criticism of reason to the negative skepticism of Hume; that is, Kant showed that the human intellect moved independently in the formation of theoretical and practical judgments, and that the various materials of thought, desire, and feelings were regulated by the intellect according to innate original ideas of the true and good and beautiful. Thus as a natural result there came the conception that the doctrines of belief arose not as complete truths, given by divine revelation, but, like every other form of conscious knowledge, these came to us through the activity of our own mind, and that therefore these doctrines cannot be regarded as of absolute authority for all time, but that we are to seek to understand their origin in historical and psychical motives. So far as one looked at the ceremonial forms of positive religion, these motives indeed were found according to Kant in irrational conceptions, but as far as the essence of religion was concerned they were rather found to be rooted in the moral nature of man. This is the consciousness of obligation of the practical reason or of the conscience, which raises man to a faith in the moral government of the world, in immortality and God. With the reduction of religion from all external forms, doctrines, and ceremonies and the finding of the real essence of religion in the human mind and spirit, the way was opened to a knowledge of religion free from all external authority. Those philosophers who came after Kant followed essentially this course, though here and there they may separate in their opinions according to their thought of the psychological function of religion. When Kant had emphasized the close connection between religion and the moral obligation, then came Schleiermacher, who emphasized the feeling of our dependence upon the Eternal, and who sought to find the explanation of all religious thoughts and conceptions in the many relations of the feeling to religious experience. Hegel on the other hand sought the truth of religion in the thought of the absolute spirit as found in the finite spirit. Thus Hegel made religion a sort of popular philosophy.

At present all agree that all sides of the soul-life have part in religion; now one side may be the more prominent, now another, according to the peculiarity of certain religions or the individual temperaments. The philosophy of religion has, in common with scientific psychology, the question of the relation of feeling to the intellect and the will, and as yet there may be many views of this question. Altogether the philosophy of religion is looking for important solutions to many of its problems from the realm of the present scientific psychology. Experiences, such as religious conversions, appear under this point of view as ethical changes in which the aim of a personal life is changed from a carnal and selfish end to that of a spiritual and altruistic purpose. These are extraordinary and seemingly supernatural processes; nevertheless in them there can still be found a certain development of the soul-life according to law. Modern psychology especially has thrown light upon the abnormal conditions of consciousness which have so often been made manifest in the religious experience of all times. That which religious history records concerning inspiration, visions, ecstasy, and revelation, we now classify with the well-known appearances of hypnotism, the induction of conceptions and motives of the will through foreign suggestion or through self-suggestion, of the division of consciousness in different egos, and in the union of several consciousnesses into one common mediumistic fusion of thought and will. The explanation of these experiences may not yet be satisfactory, but nevertheless we do not doubt the possibility of a future explanation from the general laws controlling the life of the soul. The fact that we can through psychological experiments produce such abnormal conditions of consciousness justifies us in taking the position, that certain psychical laws are at the foundation of these conditions which in their kind are as natural and regular in their functions as the physical laws which we observe in physical experiments. These solutions which modern psychology so far has given, and hopes still further to give, are of great importance to the philosophy of religion. They are an indorsement of the general principle which one hundred years ago had been advanced by critical speculation, namely, that in all experiences of the religious life the same principles which control the human mind in all other intellectual and emotional fields shall hold sway. Nothing therefore should hinder us in scientific research from following the well-defined maxims of thought, and unreservedly applying the same methods of scientific analysis in theology as is done generally in the other sciences.

The claim of the Church to infallibility and divine inspiration of its dogmas is weakened under this view of the work of the philosophy of religion. Prophetical inspiration and ecstasy, which usually were thought to be supernatural revelations, are now declared by the present psychology to come under the category of other analogous experiences, such as the action of mental powers which, under definite conditions of individual gifts and on historical occasions, have manifested themselves in extraordinary forms of consciousness. However, these enthusiastic forms of prophetical consciousness cannot be accepted for a higher form of knowledge or even as of divine origin and as an infallible proclamation of the truth; on the contrary, these forms are to be judged as pathological appearances, which may be more harmful than beneficent for the ethical value of the prophetical intuition. At least, it has come to pass that all forms of revelation must come under the examination of a psychological analysis and of an analogical judgment. Hence their traditional nimbus of unique, supernatural, and absolute authority is for all time destroyed.

We are carried to the same result by the comparative study of the history of religions. The study shows us that the Christian Church, with its dogma of the divine inspiration of the Bible, does not stand alone; that before and after Christianity other religions made exactly the same claims for their sacred scriptures. By the pious Brahman the Veda is regarded as infallible and eternal; he believes the hymns of the old seers were not composed by the seers themselves, but were taken from an original copy in heaven. The Buddhist sees in the sayings of his sacred book "Dhammapadam" the exact inheritance of the infallible words of his omniscient teacher Buddha. For the confessor of Ahuramazda the Zendavesta contains the scriptural revelation of the good spirit unto the prophet Zarathustra; according to the rabbis the laws revealed unto Moses on Mount Sinai were even before the creation of the world the object of the observation of God; for the faithful Mohammedan the Koran is the copy of an ever-present original in heaven, the contents of which were dictated word for word to Mohammed by the angel Gabriel. Whoever ponders the similar claims of all these religions for the infallibility of their sacred books, to him it becomes difficult to hold the dogma of the Christian Church concerning the inspiration and infallibility of the Bible as alone true and the similar dogmas of other religions as being false. Rather he will accept the view that in all these examples there are found the same motives of the religious mind, that here is given an expression to the same need common to all seeking for an absolute and abiding basis for their faith.

The study of the comparison of religions has discovered in religions other than that of Christianity many very striking parallels to many narratives and teachings of the Bible. It may be well to recall very briefly some of the important points. Owing to the fact that the Assyrian cuneiform writings have now been deciphered, there has been found a story of the creation which has many characteristics in common with those of the Bible. There is found a story of a flood, which in its very details can be regarded as the forerunner of the story of the flood in the Bible. There have been found Assyrian penitential psalms, which, in consciousness of guilt and in earnestness of prayer for forgiveness, can well be compared with many psalms of the Bible. Recently the Code of the Assyrian King Hammurabi, who reigned two thousand three hundred years before Christ, has been discovered. The similarity of this Code with many of the early Mosaic Laws has called general attention to this fact. In the Persian religion there are found teachings of the Kingdom of God, of the good spirits who surround the throne of God, of the Spirit hostile to God and of an army of his demons, of the judgment of each soul after death, of a heaven with eternal light and of the dark abyss of hell, of the future struggle of the multitudes of good and bad spirits and the victory over the bad through a divine hero and saviour, of the general resurrection of the dead, of the awful destruction of the world and the creation of a new and better world, — teachings which are also found in the later Jewish theology and apocalypse, so that the acceptance of a dependence of Jewish upon corresponding Persian teaching can hardly be avoided. Also Grecian influence is observed in later Jewish literature, in proverbs, in the wisdom of Solomon and the Son of Sirach; especially in the Alexandrian Jewish theology are found Platonic thoughts of an eternal, ideal world, of the heavenly home of the soul, and the Stoic conception of a world-ruling divine Logos.

It is from this source that the Logos to which Philo had already ascribed the meaning of the Son of God and the Bringer of a divine revelation crossed over into Christian theology and became the foundation of the dogma of the Church concerning the person of Christ. Of still greater importance than even all this was the opening of the Indian and especially the Buddhistic religious writings. In these we have, five hundred years before Christianity, the revelation of redemptive religion, resting upon the ethical foundation of the abnegation of self and the withdrawal from the world. In the centre of this religion is Gautama Buddha, the ideal teacher of redeeming truth, whose human life was adorned by the faith of his followers with a crown of wonderful legends; from an abode in heaven, out of mercy to the world, he descended into the world, conceived and born of a virgin mother, greeted and entertained by heavenly spirits, recognized beforehand by a pious seer as the future redeemer of the world; as a youth he manifested a wisdom beyond that of his teachers. Then after the reception of an illuminating revelation, he victoriously overcomes the temptation of the devil, who would cause him to become faithless to his call to redemption. Then he begins to preach of the coming of the Kingdom of Justice, and sends forth his disciples, two by two, as messengers of his gospel to all people. Although he declares that it is not his calling to perform miracles, nevertheless the legends indeed tell how many sick were healed, how with the contents of a small basket hundreds were fed, how possessed of all knowledge he reveals hidden things; how overcoming the limitations of space and time, swaying in the air, being transfigured in a heavenly light, he reveals himself to his disciples just before his death. And at last, in the faith of his followers, having passed from the position of a human teacher to that of an eternal heavenly spirit and lord of the world, he is exalted as the object of prayer and reverence, to many millions of the human race in Southern and Eastern Asia.

It is hardly possible that the knowledge of this parallel from India to the New Testament, and of the Babylonian and Persian parallel to the Old Testament, can be without influence upon the religious thought of Christian people. Although we may be ever so much convinced concerning the essential superiority of our religion over all other religions, nevertheless the dogmatic contrast between absolute truth on the one side and complete falsity on the other can no more be maintained. In place of this view there must enter the view of a relative grade of differences between the higher and lower stages of development. No longer can we see in other religions only mistakes and fiction, but under the husk of their legends many precious kernels of truth must be seen, expressions of inner religious feelings and of noble ethical sentiments. One should therefore accept the position not to object to the same discrimination between husk and kernel in the matter of one's own religion, and to recognize in its inherited traditions and dogmas legendary elements, the explanation of which is to be found in psychical motives and in historical surroundings, even as they are found in the corresponding parts of religions other than the Christian religion. Therefore the historical comparison of religions takes us away from an absolute dogmatic positivism to a relative evolutionary manner of study, placing all religions without exception under the laws of time progression and under the causal connection of the law of cause and effect. The isolation of religion therefore is no more. It is regarded as being a part of other human historical affairs, and must yield to the test of a thorough unhindered research. The value of the Christian religion can never suffer in the view of a reasonable man, when it is not accepted in blind faith, but as the result of discriminating comparison.

As the evolutionary philosophy of religion uses the method of science without exception in the case of all historical religions, so also it does not shrink from taking up the question of the beginning of religion, but believes that here also is found the key in the analytical, critical, and comparative method. And here is found the assistance of the comparative study of languages, ethnology, and paleontology.

The celebrated Sanscrit scholar, Max Müller, sought in the comparative study of mythology to prove the etymological relation of many of the Grecian gods and heroes with those of the mythology of India and to trace the common origin of all these mythical beings and legends in the personification of the movements of the heavenly bodies, the thunder and lightning, the tempest and the rain. All mythical belief in gods of the Indo-Germanic peoples seems to have arisen out of a poetical view and dramatic personification of the powers of nature. Suggestive as this hypothesis is, it is not by any means sufficient to give us a complete explanation of the subject. In fact, others have shown that primitive religion does not altogether consist in mythical conceptions, but mainly in reverential actions, sacrifices, sacraments, vows, and other similar cults, which have very little to do with the atmospherical powers of nature, but rather with the social life of primitive people. And when once the sight was clearly directed to the social meaning of the religious rites, it was then observed that even the earliest legends concerning the gods were connected far more closely with the habits and customs of early society than with the facts of nature. Tylor's celebrated book concerning "Primitive Civilization" is written from this standpoint, an epoch-making book, showing the original close connection of religion with the entire civilization of humanity, with the views of life and death, the social customs, the forms of law, their strivings in art and science; a book with a large amount of information, brought together from observation on all sides. In this channel are found all the researches which to-day are classified under the name of Folklore; seeking to gather the still existing characteristic customs and forms, legends, stories, and sayings, in order to compose these and to discover the survivals of earliest religion, poetry, and civilization of humanity. The gain of this study pursued with so great diligence is not to be underrated. These studies show that all that, which at one time existed as faith in the spirit of humanity, possessed within its very nature the strongest power of continuance, so that in new and strange conditions and in other forms it continued to remain. Under all changes and progress of history there is still found an unbroken connection of constant development.

As important, however, as the possession of a general knowledge of historical forms of development is to the philosophy of religion, nevertheless the possession of this knowledge is not wholly a fulfillment of the purpose of the philosophy of religion. To understand a development means not merely to know how one thing follows as the result of the other, but also to understand the law which lies at the foundation of all empirical changes and at the same time controls the end of the development. If this principle holds good in the understanding of the development in the processes of nature, much more does the principle hold good in understanding the processes of intellectual development of humanity, which have for us not only a theoretical, but at the same time an eminently practical interest. The philosopher of religion sees in religious history not merely the coming together of similar forms, but an advance from the lowest stage of childlike ignorance to an ever purer and richer realization of the idea of religion, a divinely ordained progress for the education of humanity from the slavery of nature to the freedom of the spirit. The question now arises: where do we find the principle and law of this ever-rising development? Where do we find the measure of judgment for the relative value of religious appearances? It is clear that the general principle of the complete development cannot be found in a single fact which is only one of the many manifestations of the general principle, and it is just as clear that the absolute norm of judgment is not found in a single fact always relative, presenting to us the object of judgment and therefore being impossible to stand as the norm of judgment. Therefore the principle of religious development and the norm of its judgment can only be found in the inner being of the spirit of humanity, namely, in the necessary striving of the mind into an harmonious arrangement of all our conceptions, or the idea of the truth, and into the complete order of all our purposes, or the idea of the good. These ideas unite in the highest unity, in the Idea of God. Therefore the consciousness of God is the revelation of the original innate longing of reason after complete unity as a principle of universal harmony and consistence in all our thinking and willing. Hence, in the first place, arises the result that the development of the consciousness of God in the history of religion is always dependent upon the existing conditions of the two united sides, the theoretical perception of the truth and the moral standard of life. In the second place the result arises that the judgment of the value of all appearances in the history of religion depends as to whether and how far these appearances agree with the idea of the true and the good, and correspond with the demands of reason and conscience. That science which is engaged with the idea of the good we name Ethics; that which is engaged with the last principles of the perception of truth, using the expression of Aristotle, we may name Metaphysics, or following Plato — Dialectic. Recognizing then in the idea of God the synthesis of the idea of the true and the good, the philosophy of religion is closely related with both, Ethics and Metaphysics.

At present the relation of religion to morality is an object of much controversy. There are many who hold that morality without religion is not only possible but also very desirable; since they are of the opinion that moral strength is weakened, the will is without freedom, and its motives corrupted on account of religious conceptions. On the other hand, the Church, considering the experience of history, finds that religion has ever proved itself to be the strongest and most necessary aid to morality. In this contest the philosophy of religion occupies the position of a judge who is called upon to adjust the relative rights of the parties. The philosophy of religion brings to light the historical fact that from the very beginnings of human civilization, social life and morality were closely connected with religious conceptions and usages, and indeed always so interchangeable in their influence that the position of social civilization on the one side corresponded with the position of religious civilization on the other, just as the water-level in two communicating pipes. Therefore it follows that it is unjust and not historical to blame religion on account of the defects of a national and temporal morality; for these defects of morality, with the corresponding errors of religion, find a common ground in a low stage of development of the entire civilization of the people of the time and age. Further, it becomes the task of the philosophy of religion to examine whether this correspondence of religion and morality, recognized in history, is also found in the very nature of morality and religion. This question in the main is answered without doubt in the affirmative, for it is clear that the religious feeling of dependence upon one all-ruling power is well adapted not only to make keen the moral consciousness of obligation and to deepen the feeling of responsibility, but also to endow moral courage with power and to strengthen the hope of the solution of moral purposes. The clearer religious faith comprehends the relation of man to God, so much the more will that faith prove itself as a strong motive and a great incentive of the moral life. Such a conception will not make the moral will unfree but truly free, not in the sense of a selfish choice, but in the sense of a love that serves, knowing itself as an instrument of the divine will, who binds us all into a social organism, the kingdom of God. And, on the other hand, the more ideal the moral view of life, the higher and greater its aims, the more it recognizes its great task to care for the welfare not only of the individual but of all, to coöperate in the welfare and development of all forms of society, the more earnestly the moral mind will need a sincere faith that this is God's world, that above all the changes of time an eternal will is on the throne, whose all-wise guidance causes everything to be for the best unto those who love him.

A like middle position of arbitration falls to the philosophy of religion in the matter of the relation of religion to science. The first demand of science is freedom of thought, according to its own logical laws, and its fundamental assumption is the possibility of the knowledge of the world on the basis of the unchangeable laws of all existence and events. With this fundamental demand science places itself in opposition to the formal character of ecclesiastical doctrine so far as the doctrine claims infallible authority resting upon a divine revelation. And the fundamental assumption of the regular law of the course of the world is in opposition to the contents of ecclesiastical doctrine concerning the miraculous interposition in the course of nature and of history. To the superficial observer there appears therefore to exist an irreconcilable conflict between science and religion. Here is the work of the philosophy of religion, to take away the appearance of an irreconcilable opposition between science and religion, in that the philosophy of religion teaches first of all to distinguish between the essence of religion and the ecclesiastical doctrines of a certain religion, and to comprehend the historical origin of these doctrines in the forms of thought of past times. To this purpose the method of psychological analysis and of historical comparison mentioned above is of service. When, then, by this critical process religion is traced to its real essence in the emotional consciousness of God, to which the dogmatic doctrines stand as secondary products and varied symbols, then it remains to show that between the essence of religion and that which science demands and presupposes, there exists not conflict but harmony. When the idea of God is recognized as the synthesis of the ideas of the true and the good, so then must all truth as sought by science, even as the highest good, which the system of ethics places as the purpose of all action — these must be recognized as the revelation of God in his eternal reason and goodness. The laws of our rational thinking then cannot be in conflict with divine revelation in history, and the laws of the natural order of the world can no more stand in conflict with the world-governing Omnipotence; but both, the laws of our thinking and those of the real world, reveal themselves as the harmonious revelations of the creative reason of God, which, according to Plato's fitting word, is the efficient ground of being as well as of knowing. It is therefore not merely a demand of religious belief that there is real truth in our God-consciousness, that there should be an activity and revelation of God himself in the human mind; it is also in the same manner a demand of science considering its last principles, that the world, in order to be known by us as a rational, regulated order, must have for its principle an eternal creative reason. Long ago the old master of thinking, Aristotle, recognized this fact clearly, when he said that order in the world without a principle of order could be as little thinkable as the order of an army without a commanding general.

But while it is true that science, as the ground of the possibility of its knowledge of the truth, must presuppose the same general principle of intellectual knowledge which religion has as the object of its practical belief, then by principle the apprehension is excluded that any possible progress on the part of science in its knowledge of the world can ever destroy religion. We are rather the more justified in the hope that all true knowledge of science will be a help to religion, and will serve as the means of purifying religion from the dross of superstition.

Truly it can easily be shown that a divine government of the world breaking through, and now and then suspending the regular order of nature through miraculous intervention, would not be more majestic, but far more limited and human, than such a government which reveals itself as everywhere and always the same in and through its own ordained laws in the world. And again, that a revelation prescribing secret and incomprehensible doctrines and rites, demanding from humanity a blind faith, would far less be in harmony with the guiding wisdom and love of God, and far less could work for the intellectual liberty and perfection of humanity, than such a revelation which is working in and through the reason and conscience of humanity, and is realizing its purpose in the progressive development of our intellectual and moral capacities and powers. When therefore science raises critical misgivings against the supernatural and irrational doctrines of positive religion, then the real and rightly understood interests of religion are not harmed but rather advanced; for this criticism serves religion in helping it to become free from the unintellectual inheritance of its early days, in helping religion to consider its true intellectual and moral essence, and to bring to a full display all the blessed powers which are concealed within its nature, to press through the narrow walls of an ecclesiasticism out into the full life of humanity, and to work as leaven for the ennoblement of humanity. Not in conflict with science and moral culture, but only in harmony with these, can religion come nearer to the attainment of its ideal, which consists in the worship of God in spirit and in truth. Even though they may not be conscious of their purpose, but nevertheless in fact all honest work of science and all the endeavors of social and ethical humanity have part in the attainment of this ideal.

It is the work of the philosophy of religion to make clear that all work of the thinking and striving spirit of humanity, in its deepest meaning, is a work in the kingdom of God, as service to God, who is truth and goodness. It is the work of the philosophy of religion to explain various misunderstandings, to bring together opposing sides, and so to prepare the way for a more harmonious coöperation of all, and for an always hopeful progress of all on the road to the high aims of a humanity fraternally united in the divine spirit.

MAIN PROBLEMS OF THE PHILOSOPHY OF RELIGION: PSYCHOLOGY AND THEORY OF KNOWLEDGE IN THE SCIENCE OF RELIGION

BY PROFESSOR ERNST TROELTSCH

(*Translated from the German by Dr. J. H. Woods, Harvard University.*)

[**Ernst Troeltsch,** Professor of Systematic Theology, University of Heidelberg, since 1894. **b.** February 17, 1865, Augsburg, Bavaria. Doctor of Theology. Professor University of Bonn, 1892–94. **Author of** *John Gerhard and Melanchthon; Richard Rubbe; The Scientific Attitude and its Demands on Theology; The Absoluteness of Christianity, and of the History of Religion; Political Ethics and Christianity; The Historic Element in Kant's Religious Philosophy.*]

THE philosophy of religion of to-day is philosophy of religion so far only, and in such a sense, as this word means science of religion or philosophy with reference to religion. The science of religion of former days was first dogmatic theology, deriving its dogmas from the Bible and from Church tradition, expounding them apologetically with the metaphysical speculation of the later period of antiquity, and regarding the non-Christian religions as sinful derangements and obscure fragments of the primitive revelation. This lasted sixteen centuries, and is confined to-day to strictly ecclesiastical circles. Next, science of religion became natural theology, which proved the existence of God by the nature of thought and by the constitution of reality, and also the immortality of the soul by the concept of the soul and by moral demands, thus constructing natural or rational dogmas and putting these dogmas into more or less friendly relations with traditional Christianity. This lasted about two centuries, and is to-day of the not strictly ecclesiastical or pietistic circles, which still wish to hold fast to religion. Both kinds of science of religion exist no longer for the strict science. The first was, in reality, supernaturalistic dogmatics, the second was, in reality, a substitution of philosophy for religion. The first was demolished by the criticism of miracles in the eighteenth century, the second by the criticism of knowledge in the nineteenth century, which, in its turn, rests upon Hume and Kant.

The science of religion of to-day keeps in touch with that which without doubt factually exists and is an object of actual experience, *the subjective religious consciousness.* The distrust of ecclesiastical and rationalistic dogmas has made, in the thought of the present, every other treatment impossible. So the spirit of empiricism has here as at other points completely prevailed. But empiricism in this field means psychological analysis. This analysis is pursued by the present to the widest extent: on the one side by anthropologists and archæologists, who investigate the life of the soul in primitive peoples and thus indicate the particular function and condition of religion in these states; on the other side, by the modern experimental psychologists and psychological empiricists, who, by self-observation, and especially by the collection of observations by others and of personal testimony, study religion, and then, from the point of view of the concepts of experimental psychology, examine the main phenomena thus found.

Now, such an empirical psychology of religion has been constructed with considerable success. In this German literature, it is true, has coöperated to a slight degree only. The German theologians have held to the older statements of the psychology of Kant, of Schleiermacher, of Hegel, and of Fries, alone, which, in principle, were on the right path, but which combined the purely psychological with metaphysical and epistemological problems to such a degree that it was impossible to reach a really unprejudiced attitude. German psychologists remain, furthermore, under the spell of psycho-physiology and of quantitative statements of measure, and have, consequently, not liked to advance into this field, which is inaccessible to such statements. More productive than the German psychology for this subject is the French, which has attacked the complex facts far more courageously. Here, however, under the predominance of positivism, there prevails, on the whole, the tendency to regard religion, in its essence, anthropologically or medically and pathologically in connection with bodily conditions. This is the confusion of conditions and origins with the essence of the thing itself, which can be determined only by the thing, and is, by no means, bound exclusively to these conditions. Notwithstanding, the works of Marillier, Murisier, and Flournoy have considerably aided the problem. More impartially than all of these, the English and American psychology has investigated our subject. Here we have a masterpiece in the Gifford Lectures of William James, which collects into a single reservoir similar investigations such as have been carried on by Coe and Starbuck. There is here no tendency to a mechanism of consciousness, or to the dogma of the causal and necessary structure of consciousness. And to just this is due the freshness and impartiality of the analyses which James gives out of his enviable knowledge of characteristic cases. James rightly emphasizes the endlessly different intensity of religious experiences, and the great number of points of view and of judgments which thereby results. He also rightly emphasizes the connection of this different intensity with irreducible typical constitutions of the soul's life, with the optimistic and the melancholy disposition; hence there arise constantly, even within the same religion, essentially different types of religiousness. Limiting himself, then, to the most intense experiences, he decides that the characteristic of religious states is the sense of presence of the divine, which one might perhaps describe in other terms, but which still continues the specifically divine, with the opposed emotional effects of a solemn sense of contrast and of enthusiastic exaltation. He pictures these senses of presence, and illustrates them by visionary and hallucinatory representations of the abstract. With this are connected impulsive and inhibitive conditions for the appearance of these senses of presence and of reality, descriptions of the effects upon the emotional life and action, and, above all, the analysis of the event usually called conversion, in which the religious experience out of subconscious antecedents becomes, in various ways, the centre of the soul's life. All this is description, but it is based upon a mass of examples and explained by general psychological categories which, by the occurrence of the religious event only, receive a thoroughly specific coloring. It is a description after the manner of Kirchhoff's mechanics; permanent and similar types, and, likewise, similar conditions for their relations to the rest of the soul's life are sought out everywhere, without maintaining to have proven at the same time, in this way, an intellectual necessity for the connection. But the characteristic peculiarity of religious phenomena is thus conceived as in no other previous analysis.

All this is still, however, nothing more than psychologic. For the science of religion it accomplishes nothing more than the psychological determination of the peculiarity of the phenomenon, of its environment, its relations and consequences. It is evident that the phenomenon occurs in an indefinite number of varieties; and the chosen point of departure, in unusual and excessive cases, frequently diffuses over religion itself the character of the bizarre and abnormal. Consequently nothing whatever is said about the amount of truth or of reality in these cases. This, by the very principles of such a psychology, is impossible. It analyzes, produces types and categories, points out comparatively constant connections and interactions. But this cannot be the last word for the science of religion. It demands, above all, empirical knowledge of the phenomenon; but it demands this only in order, on the basis of this knowledge, to be able to answer the question of the amount of truth. But this leads to an entirely different problem, that of the *theory of knowledge*, which has its own conditions of solution. It is impossible to stop at a merely empirical psychology. The question is not merely of given facts, but of the amount of knowledge in these facts. But pure empiricism will not succeed in answering this question. The question with regard to the amount of truth is always a question of validity. The question with regard to validity can, however, be decided only by logical and by general, conceptual investigations. Thus we pass over from the ground of empiricism to that of rationalism, and the question is, what the theory of knowledge or rationalism signifies for the science of religion.

Such a synthesis of the rational and irrational, of the psychological and the theory of knowledge, is the main problem raised by the teaching of Kant, and the significance of Kant is that he clearly and once for all raised the problem in this way. He had the same strong mind for the empirical and actual as for the rational and conceptual elements of human knowledge, and constructed science as a balance between the two. (He destroyed forever the *a priori* speculative rationalism of the necessary ideas of thought, and the analytical deductions from them, which undertakes to call reality out of the necessity of thought as such. He restricted regressive rationalism to metaphysical hypotheses and probabilities, the evidence for which rests upon the inevitability of the logical operations which leads to them, which, however, apply general concepts without reference to experience, and therefore become empty, and thus afford no real knowledge.) On the other hand, he proclaimed the formal, immanent rationalism of experience, in attempting to unite Hume's truth with the truth of Leibnitz and of Plato. In this way he succeeded in grasping the great problem of thought by the root, and in putting attempts at solutions on the right basis. So it is not a mere national custom of German philosophizing, if we take our bearings, for the most part, from this greatest of German thinkers, but it is, absolutely, the most fruitful and keenest way of putting the problem. It is true, the solutions which Kant made, and which are closely connected with the classical mechanics of that time, with the undeveloped condition of the psychology of that time, and with the incompleteness of historical thinking then just beginning, have been, meantime, more than once given up again. A simple return to him is therefore impossible. But the problem was put by him in a fundamental way, and his solutions need nothing more than modification and completion.

Now all this is especially true in the case of the science of religion. Here also Kant took the same course, which seemed to me right for the theoretical knowledge of the natural sciences and for anthropology. In practical philosophy also, to which he rightly counts philosophy of religion, he seeks laws of the practical reason analogous to the laws of theoretical reason, axioms of the ethical, æsthetic, and religious consciousness which are already contained *a priori* in the elementary appearances in these fields, and, in application to concrete reality, produce just these activities of the reason. Here also one should grasp reason only as contained in life itself, the *a priori* law itself already effective in the diversity of the appearances should make one's self clear-sighted and so competent for a criticism of the stream of the soul's appearances. Seizing upon itself in the practical reality, the practical reason criticises the psychological complex, rejects as illusion and error that which cannot be comprehended in an *a priori* law, selects that part of the same which needs basis and centre and requires only clearness with regard to itself, clears the way for revelations of a life consciousness of its own legality and becomes capable of the development of critically purified experience.

If this is, in principle, valid, the Kantian thought, in the further detail, is maintained in principle only and as a whole. The elaboration itself will have to be quite different from that of his own. Even by Kant himself, on this very point, the synthesis of empiricism and rationalism is far from being elaborated with the necessary rigor and consistency. And to-day we have a quite differently developed psychology of religion, in contrast with which that presupposed by Kant is bare and thin. Finally, there remain in the whole method of the critical system unsolved problems; by failure to solve these, or by too hasty solution, science of religion, especially, is affected.

To make clear the present condition of the problem, one ought, above all, to indicate the modifications to which the Kantian theory of religion must submit, — must submit, especially, by reason of a more delicate psychology, such as we have, with remarkable richness, in James and the American psychologists connected with him. There are *four* points with regard to this question.

The first is the question of the relation of psychology and theory of knowledge in the very establishment of the laws of the theory of knowledge. Are not the search for and discovery of the laws of the theory of knowledge themselves possible only by way of psychological ascertainment of facts, itself then a psychological undertaking and consequently dependent upon all its conditions? It is the much discussed question of the circle which itself lies at the outset of the critical system. The answer to this is that this circle lies in the very being of all knowledge, and must therefore be resolutely committed. It signifies nothing more than the presupposition of all thought, the trust in a reason which establishes itself only by making use of itself. The unmistakable elements of the logical assert themselves as logical in distinction from the psychological, and from this point on reason must be trusted in all its confusions and entanglements to recognize itself within the psychological. It is the courage of thought, as Hegel says, which may presuppose that the self-knowledge of reason may trust itself, presuppose that reason is contained within the psychological; or it is the ethical and teleological presupposition of all thought, as Lotze says, which believes in knowledge and the validity of its laws for the sake of a connected meaning for reality, and which, therefore, trusts to recognize itself out of the psychological mass. The establishment, therefore, of the laws of the theory of knowledge is not itself a psychological analysis, but a knowledge of self by the logical by virtue of which it extricates itself out of the psychological mass. Theory of knowledge, like every rationalism, includes, it is true, very real presuppositions with regard to the significant, rational, and teleologically connective character of reality, and without this presupposition it is untenable; in it lies its root. It is insight of former days, the importance of which, however, must constantly be emphasized anew, that discusses the validity of the rational as opposed to the merely empirical. But still more important than this thesis are several *inferences* which are given with it.

The establishment of the laws of consciousness, in which we produce experience, is a selection of the laws out of experience itself, a knowledge of itself by the reason contained in the very experience by way of the analysis which extracts it. It is then an endless task, completed by constantly renewed attacks, and always only approximately solvable. The complete separation of the merely psychological and actual and of the logical and necessary will never be completely accomplished, but will always be open to doubt; one can only attempt always to limit more vigorously the field of what is doubtful. And with this something further is connected.

The inexhaustible production of life becomes constantly, in the latent amount of reason, richer than the analysis discerns, or, in other words, the laws which are brought into the light of logic will always be less the amount of reason not brought into consciousness, and conscious logic will always be obliged to correct itself and enrich itself out of the unartificial logical operations arising in contact with the object. So a finished system of *a priori* principles, but this system will always be in growth, will be obliged unceasingly to correct itself, and to contain open spaces.

Finally, and above all, in case of this separation, there remains within the psychologically conditioned appearance, a residuum, which is either not conceived, but is later reduced to law and thereby a conceived phenomenon, or which never can be so, and is therefore illusion and error. If the psychological and the theoretical for knowledge are to be separated, then that can occur, not merely to show that both must always be together, and form real experience only when together, but there must also be a rejection of that which is merely psychological and not rational since it is illusion and error. The distinction between the apparent and the real was the point of departure which made the whole theory necessary, and, accordingly, the merely psychological must remain appearance and error side by side with that which is psychological and, at the same time, theoretical for knowledge. There always remains in consciousness a residuum of the inconceivable, that is, inconceivable since it is illusion and error. This amounts to saying that reality is never fully rational, but is engaged in a struggle between the rational and anti-rational. The anti-rational or irrational, in the sense of psychological illusion and error, belongs also to the real, and strives against the rational. The true and rational reality to be attained by thought is always in conjunction with the untrue reality, the psychological, that containing illusion and error.

All this signifies that the rationalism of the theory of knowledge must be conditional, partly owing to the corrective and enriching fecundation by primitive and naïve thought, partly owing to never quite separable admixture of illusion and error. So, long ago, the system of categorical forms, as Kant constructed it for theoretical and practical reason, began to change, and can never again acquire the rigidity which Kant's rationalism intended to give it forevermore. And thus the critical system's rational reality of law produced by reason always contains below itself and beside itself the merely psychological reality of the factual, to which also illusion and error belong, — a reality which can never be rationalized, but only set aside. This, too, is also true for the philosophy of religion: the rational reduction of the psychological facts of religion to the general laws of consciousness which prevail among them is a task constantly to be resumed anew by the study of reality, and follows the movements of primitive religion in order to find there first the rational basis; the reduction is, however, always approximate, can comprehend the main points only, and must leave much open, the rational ground for which is not or not yet evident; finally it has unceasingly to reckon with the irrational as illusion and error, which attaches to the rational, and yet is not explainable by it. The two realities, which the critical system must recognize at its very foundation, continue in strife with each other, and this strife as the strife of divine truth with human illusion is for the science of religion of still more importance.

The second correction of the Kantian teaching is only a further consequence from this state of things. If the attitude of psychology and theory of knowledge requires a strict separation, it requires it only for the purpose of more correct relation. The laws of the theory of knowledge are separated from the merely psychological actuality, but still can be produced only out of it. Thus, as a matter of fact, psychological analysis is always the presupposition for the correct conception of all these laws. Psychology is the entrance gate to theory of knowledge. This is true for theoretical logic as well as for the practical logic of the moral, the æsthetical, and the religious. But just at this point the present, on the basis of its psychological investigation, presses far beyond the original form of the Kantian teaching. This is not the place to describe this, more closely, with reference to the first of the subjects just mentioned. But it is important to insist that this is especially true with respect to the Kantian doctrine of religion. The Kantian doctrine of religion is founded on the moral and religious psychology of Deism, which had made the connection, frequent in experience, of moral feelings with religious emotion the sole basis of the philosophy of religion, and had, in the manner of the psychology of the eighteenth century, immediately changed this connection into intellectual reflections, in accord with which the moral law demands its originator and guarantee. Kant accepted this psychology of religion without proof and built upon it his main law of the religious consciousness, in accordance with which a synthetic judgment *a priori* is operative in religion (arising in the moral experience of freedom), which requires that the world be regarded as subject to the purposes of freedom. It is, however, extremely one-sided, to give religion its place just between the elements, and a rather violent translation of the religious constitution into reflection. The error of this psychology of religion had been discovered and corrected already by Schleiermacher. But Schleiermacher, for his part too, also failed to deny himself an altogether too sudden metaphysical interpretation of the religious *a priori* which he had demonstrated, since he not only described the *a priori* judgment of things, from the point of view of absolute dependence upon God, as a vague feeling, but raised this feeling, by reason of the supposed lack of difference, in it, between thought and will, reason and being, to a world-principle, and interpreted the idea of God contained in this feeling in the terms of his Spinozism, the lack of difference between God and Nature within the Absolute. A real theory of knowledge of religion must keep itself much more independent of all metaphysical presuppositions and inferences, and must admit that the essence of the religious *a priori* is extorted from a thoroughly impartial psychological analysis. And this is always the place where works, such as those of James, come into play. Religion as a special category or form of psychical constitution, the result of a more or less vague presence of the divine in the soul, the feeling of presence and reality with reference to the superhuman or infinite, that is without any doubt a much more correct point of departure for the analysis of the rational *a priori* of religion, and it remains to make this new psychology fruitful for the theory of knowledge of religion. That will be one of the chief tasks of the future.

The third change relates to the distinction of the empirical and intelligible Ego, which Kant connected closely, almost indissolubly with his main epistemological thought of the formal rationalisms immanent in experience. Kant rationalized the whole outer and inner experience, by means of *a priori* laws, into a totality, conforming to law, appearing in intuitive forms of space and time, causally and necessarily rigidly connected. The freedom autonomously determining itself out of the logical idea, and contrasting itself with the psychological stream, produces out of the confused psycholican reality this scientific formation of the true reality. The product of thought, however, swallows its own maker. For the same acts of freedom, which autonomously produced the formation of the reality of law, remain themselves in the temporal sequence of psychical events, and, therefore, themselves, with that formation, lapse into the sequence which is under mechanical law. The intelligible Ego creates the world of law, and finds itself therein, with its activity, as empirical Ego, that is, as product of the great world-mechanism and of its causal sequence. It is an intolerable, violent contradiction, and it is no solution of this contradiction to refer the empirical Ego to appearance, and the intelligible Ego to actuality existing in itself, if the operations of the intelligible Ego, also a constituent part of what takes place in the soul, occur in time and so relapse irrecoverably into phenomenality and its mechanism. All the ingenuity of modern interpretation of Kant has not succeeded in making this circle more tolerable, all shifting of one and the same thing to different points of view has only enriched scientific terminology with masterpieces of parenthetical caution, but not removed the objection that two different points of view do not, as a matter of fact, exist side by side, but conflict within the same object.

This circle is especially intolerable for the psychology of religion and its application to the theory of knowledge. The psychology of religion certainly shows us that the deeper feeling of all religion is not a product of the mechanical sequence, but an effect of the supersensuous itself as it is felt there; it believes that it arises in the intelligible Ego by way of some kind of connection with the supersensuous world. This, however, becomes completely impossible for the Kantian theory of the empirical Ego, and all distinctions of a double point of view in no wise change the fact that these points of view are mutually absolutely exclusive. Here we have the results of psychology which the expression of religious emotion confirms, in that religion can be causally reduced to nothing else, totally opposed to the consequences of such a theory of knowledge. Kant had himself often enough practically felt this, and spoke then of freedom as an experience of communion with the supersensuous as a possible but unprovable affair, while all that, in case of a strict adherence to the phenomenality of time and of the theory of the empirical Ego, which is a consequence of it, is completely impossible. Nothing can be of any assistance here except a decisive renunciation of those epistemological positions which contradict the results of teaching. This is not the place to describe this, more closely, with reference to the first of the subjects just mentioned. But it is important to insist that this is especially true with respect to the Kantian doctrine of religion. The Kantian doctrine of religion is founded on the moral and religious psychology of Deism, which had made the connection, frequent in experience, of moral feelings with religious emotion the sole basis of the philosophy of religion, and had, in the manner of the psychology of the eighteenth century, immediately changed this connection into intellectual reflections, in accord with which the moral law demands its originator and guarantee. Kant accepted this psychology of religion without proof and built upon it his main law of the religious consciousness, in accordance with which a synthetic judgment *a priori* is operative in religion (arising in the moral experience of freedom), which requires that the world be regarded as subject to the purposes of freedom. It is, however, extremely one-sided, to give religion its place just between the elements, and a rather violent translation of the religious constitution into reflection. The error of this psychology of religion had been discovered and corrected already by Schleiermacher. But Schleiermacher, for his part too, also failed to deny himself an altogether too sudden metaphysical interpretation of the religious *a priori* which he had demonstrated, since he not only described the *a priori* judgment of things, from the point of view of absolute dependence upon God, as a vague feeling, but raised this feeling, by reason of the supposed lack of difference, in it, between thought and will, reason and being, to a world-principle, and interpreted the idea of God contained in this feeling in the terms of his Spinozism, the lack of difference between God and Nature within the Absolute. A real theory of knowledge of religion must keep itself much more independent of all metaphysical presuppositions and inferences, and must admit that the essence of the religious *a priori* is extorted from a thoroughly impartial psychological analysis. And this is always the place where works, such as those of James, come into play. Religion as a special category or form of psychical constitution, the result of a more or less vague presence of the divine in the soul, the feeling of presence and reality with reference to the superhuman or infinite, that is without any doubt a much more correct point of departure for the analysis of the rational *a priori* of religion, and it remains to make this new psychology fruitful for the theory of knowledge of religion. That will be one of the chief tasks of the future.

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The fourth problem arises, when we examine the rational law of the religious nature or of the having of religion which lies in the being and organization of the reason. The having of religion may be demonstrated as a law of the normal consciousness from the immanent feeling of necessity and obligation which properly belongs to religion, and from its organic place in the economy of consciousness, which receives its concentration and its relation to an objective world-reason only from religion. But precisely because religion is reduced to this, it is clear that this is only a reduction which abstracts from the empirical actuality just as the categories of pure reason do. This abstraction, then, should under no circumstances itself be regarded as the real religion. It is only the rational *a priori* of the psychical appearances, but not the replacement of appearances by the truth free from confusion. The psychical reality in which alone the truth is effective should never be forgotten out of regard for the truth. This is, however, the fact in the Kantian theory of religion in *two* directions.

It is always noticeable that the *a priori* of the practical reason is treated by Kant quite differently from the theoretical. In case of the latter the main idea of the synthesis, immanent in experience, of rationalism and empiricism, is retained, and the *a priori* of the pure forms of intuition and of the pure categories is nothing without the contents of concrete reality which become shaped in it. It may be very difficult actually to grasp the coöperation of the *a priori* and the empirical in the single case, and Kant's theory of the categories may have to be entirely reshaped and approximated to *a priori* hypotheses requiring verification, but the principle itself is always the disposition of the real and genuine problem of all knowledge. In case of the practical *a priori* Kant did, it is true, firmly emphasize the formal character of the ethical, æsthetical, and religious law, but, in doing this, does not lose quite out of sight the psychical reality. They appear not as empty forms which attain to their reality only when filled with the concrete ethical tasks, the artistic creations, and the religious states, but as abstract truths of reason, which have to take the place of the intricacies of usual consciousness. At this point one has always been right in feeling a relapse on the part of Kant into the abstract, analytical, conceptual, rationalism, and for this very reason Kant's statements about these things are of great sublimity and rigor of principle, but scanty in content. It is more important in case also of this *a priori* of the practical reason to keep in mind that it is a purely formal *a priori* and in reality must constantly be in relation with the psychical content, in order to give this content the firm core of the real and the principle of the critical regulation of self. So the *a priori* of morals is not to be represented abstractly merely by itself, but it is to be conceived in its relation to all the tasks which we feel as obligatory, and it extends itself from that point outwards over the total expanse of the activity of reason. Likewise the *a priori* of art is not to be denoted in the abstract idea of the unity of freedom and necessity, but to be shown in the whole expanse which is present to the soul as artistic form or conception. Thus, in especial degree, religion is not to be reduced to the belief of reason in a moral world-order, and simply contrasted with all supposed religion of any other kind, but the religious *a priori* should only serve in order to establish the essential in the empirical appearance, but without stripping off this appearance altogether, and from this point of the essential to correct the intricacies and narrowness, the errors and false combinations of the psychical situation. Kant, by his original thought of the *a priori*, was urged in different ways to such a view, and construed epistemologically the empirical psychological religion as imaginary illustrations of the *a priori*. But that is occasional only and does not dominate Kant's real view of religion. This is and still remains only a translation of the usual moral and theological rationalism from the formula of Locke and Wolff into the formula of the critical philosophy.

The same revision occurs in quite a different direction. If religion is an *a priori* of reason, it is, once for all, established together with reason, and all religion is everywhere and always religious in the same proposition as it is in any way realized. Schleiermacher expressly stated this in his development of the Kantian theory, and, in so far as the practical reason is always penetrated with freedom, and consequently religion itself is established with the act of moral freedom, this was also asserted by Kant himself. Such an assertion, however, contradicts every psychological observation whatsoever. It is true such observation can prove that religious emotions adjust themselves easily to all activities of reason, but it must sharply distinguish what is nothing more than the religiousness of vague feeling of supersensual regulations, which usually are joined with art and morals, from real and characteristic religiousness, in which, each single time, a purely personal relation of presence to the supersensuous takes place. But this whole problem signifies nothing else than the actualizing of the religious *a priori*, which actualizing always occurs in quite specific and, in spite of all difference, essentially similar psychical experiences and states. This problem of the actualizing of the religious *a priori* and of its connection with concrete individual psychical phenomena, Kant completely overlooked in his abstract concept of religion, or rather, deliberately ignored, because, as he wrote to Jacobi, he saw all the dangers of mysticism lurking in it. This fear was justified; for, as a matter of fact, all the specific occurrences of mysticism, from conversion, prayer, and contemplation to enthusiasm, vision, and ecstasy, do lurk in it. But without this mysticism there is no real religion, and the psychology of religion shows most clearly how the real pulse of religion beats in the mystical experiences. A religion without it is only a preliminary step, or a reverberation of real and actual religion. Moreover, the states are easily conceived in a theory of knowledge, if one sees in them the actualizing of the religious *a priori*, the production of actual religion in the fusion of the rational law with the concrete individual psychical fact. The mysticism recognized as essential by the psychology of religion must find its place in the theory of knowledge, and it finds it as the psychological actualizing of the religious *a priori*, in which alone that interlacing of the necessary, the rational, the conformable to law, and the factual occurs, which characterizes real religion. The dangers of such a mysticism, which are recognized a thousandfold in experience, cannot be dispelled altogether by the displacement of mysticism, for that would mean to displace religion itself. It would be the same, if one should try to avoid the dangers of illusion and error, by keeping to the pure categories alone, and ceasing to employ them in the actual thinking of experience. Rather, they can be dispelled only in that the actualizing of the rational *a priori* is recognized in the mystical occurrences, and thus the intricacies and one-sidedness of the mere psychological stream of religiousness be avoided. The psychological reality of religion must always remember the rational substance of religion, and always bring religion as central in the system of consciousness into fruitful and adjusted contact with the total life of the reason. Thus the psychological reality corrects and purifies itself out of its own *a priori*, without, however, destroying itself; or rather, the actual religion in the psychical category of the mystical occurrences will subside to a more or less degree. Thus we have the irrational prevailing here in its third form, which like the two others was contained in the very outset of the critical system, in the form of the once-occurring, factual, and individual, which, of course, has a rational basis or a rational element in itself, but is besides a pure fact and reality. Just this is the excellence of the rationalism immanent in experience (the critical system), that it makes room for this feature beside the general and conceptual rationality. It did not make room for it to the extent really required, and it especially left no space for it in its abstract philosophy of religion. This space must again be opened by the theory of the actualizing of the religious *a priori*, and there again lies another improvement of the critical system under the influence of modern psychology.

If we summarize all this, we have a quantity of concessions by the formal epistemological rationalism to the irrationality of the psychological facts and a repeated breaking down of the over-rigorous Kantian rationalism. Contrariwise, however, the pure psychological investigation is also compelled to withdraw from the unlimited quantity and the absolute irrationality of the multifarious (and of the confusion of appearance and truth) to a rational criterium, which can be found in the rational *a priori* of the reason only, and in the organic position of this *a priori* in the system of consciousness in general. By this rationalism alone may the true validity of religion be founded, and by this alone the uncultivated psychical life may be critically regulated. Religion will be conceived in its concrete vitality and not mutilated; it will constantly be brought out of the jumble of its distortions, blendings, one-sidedness, narrowness, and exuberance back again to its original content, and to its organic relations to the totality of the life of reason, to the scientific moral and artistic accomplishments. That is everything that science can do for it, but is not this service great enough and indispensable enough to justify the work of such a science? We do not stop with nothing more than "varieties of religious experience" which is the result of James's method; but neither do we stop with nothing more than a rational idea of religion, which overpowers experience, as was still so in the case of Kant. But we must learn how intimately to combine the empirical and psychological with the critical and normative. The ideas of Hume and of Leibnitz must once more be brought into relation with the continuations of Kant's work, and the combination of the Anglo-Saxon sense for reality with the German spirit of speculation is still the task for the new century as well as for the century past.

SHORT PAPERS

A short paper was contributed to this Section by Professor Alexander T. Ormond, of Princeton University, on "Some Roots and Factors of Religion." The speaker said that religion, like everything else human, has its rise in man's experience. It has also doubtless had a history that will present the outlines of a development, if but the course of that development can be traced. "But in the case of religion our theory of development will be largely qualified by our judgment as to its origin; while, regarding origin itself, we have to depend on hypotheses constructed from our more or less imperfect acquaintance with the races, and especially the savage races, of the present. The primitive pre-religious man is a construction from present data, and will always remain more or less hypothetical. This will partially explain, and at the same time partially excuse, what we will agree is the unsatisfactory character of the anthropological theories as accounts of the origin of religion. But there are other reasons for this partial failure that are less excusable. One of these is the rather singular failure of the leading anthropologists, in dealing with the origin of religion, to distinguish between *fundamental* and merely tributary causes. For instance, if we suppose that man has in some way come into possession of a germ of religiousness, many things will become genuine tributaries to its development that when urged as explanations of the germ itself would be obviously futile. There must be a cause for the pretty general failure to note this distinction which is vital to religious theory, and I am convinced that the principal cause is a certain lack of psychological insight and of philosophical grasp in dealing with the problem of the first data and primary roots of religion in man's nature.

"In the first place, it is needful in dealing with the religion of the hypothetical man that we should have some idea of what constitutes religion in the actual man. Now, back of all the outward manifestations of religion, will stand the religious consciousness of the man and the community, and it will be this that will determine the idea of religion in its most essential form. The developed idea of religion, therefore, arising out of this germinal impression, would take the form of a sense (we may now call it concept) of relatedness to some being *akin* to man himself, and yet transcending him in some real though undetermined respects. Anything short of this would, I think, leave religion in some respects unaccounted for; while anything more would perhaps exclude some genuine manifestations of religion.

"If the idea of religion arises out of an *impression*, then it will not be possible to deny to it an intellectual root. I make this statement with some diffidence, because if I do not misinterpret them, some recent psychologists have practically denied the intellectual root in their doctrine that religion can have no original intellectual content. If I am not further misled, however, these writers would admit that a content is achieved by the symbolic use of experience. This is perhaps all I need argue for here; since our epistemology is teaching us that the distinction between symbolism and perception is only that between the direct and the indirect; while here it is clear that its use in developing the significance of the religious impression would have all the directness and, therefore, all the cogency of an immediate inference.

"Let us now restore the intellectual and emotional elements of religion to their place in a synthesis; we will then have a concrete religious experience out of which may be analyzed at least two fundamental factors. The first of these is what we may call the *personal factor* in religion. We are treading in the foot-steps of the anthropologists when we find among the most undeveloped savages a tendency to personify the objects of their worship. When it comes to the question of determining the rôle that this personalizing tendency has actually played in the development of religion, the anthropologists divide into two camps, one of these, led by Max Müller, regarding it as a symbolic interpretation put upon the impression of some great natural or cosmic object or phenomenon; while others, including Herbert Spencer and Mr. Tylor, prefer to seek the originals of religion in ancestral dream-images and ghostly apparitions. These writers thus start with completely anthropomorphic terms, and their problem is to de-anthropomorphize the elements to the extent necessary to constitute them data of religion. The second factor standing over against the personal, as its opposite, is that of transcendence. By transcendence I mean that deifying, infinitating process that is ever working contra to the anthropomorphic influence in the sphere of religious conceptions. The School of Spencer regard this as the only legitimate tendency in religion. We do not argue this point here, but agree that it is as legitimate and real a factor as that of personality. The root of this factor, if our diagnosis of the idea of religion be correct, is to be sought in the original impression of religion, and it no doubt has its origin in man's feeling-reaction from that impression. We have pointed to submission as one of the religious emotions. Now submission rests on some deeper feeling-attitude, which some have translated into the feeling or sense of dependence. This, however, is not adequate, since men have the sense of social dependence on finite beings, and we have it with reference to the floor we are standing on. Rather, it seems to me, we must translate it into the stronger and more unconditional feeling of helplessness. One real ground of our religious consciousness is the sense or feeling of helplessness toward God; the sense that we have no standing in being as against the Deity. This radical feeling utters itself in every note of the religious scale, from the lowest superstitious terror to the highest mystical self-annihilation.

"These two factors, the forces of personalization and transcendence, are inseparable. They constitute the terms of a dialectic within the religious consciousness by virtue of which in one phase our religious conceptions are becoming ever more adequate and satisfying, while from another point of view their insufficiency grows more and more apparent. And, on the broader field of religious history, they embody themselves in a law of tendency, which Spencer has only half-expressed, by virtue of which the objects of religion are on one hand becoming ever more intelligible; on the other, ever more transcendent of our conceptions."

A short paper was read by Professor F. C. French, Professor of Philosophy in the University of Nebraska, on "The Bearing of Certain Aspects of the Newer Psychology on the Philosophy of Religion." The speaker said in part:

"The relation of science to religion has received, to be sure, much study, but to most minds hitherto this has meant the relation of only the physical sciences to religion. The older psychology was largely speculative and metaphysical in character. There were, of course, some who employed the empirical method in psychology, but they were so far from comprehending the full scope of mental phenomena that, at best, their work gave the promise of a science rather than a science itself.

It is not the fact that the newer psychology takes account of the physiological conditions of mental life; it is not the fact that the subject is now pursued in laboratories with instruments of precision, that gives it its full standing as a science: it is much more the fact that the psychology of to-day has found a place in the natural system of mental things for those strange and relatively unusual phenomena of consciousness which to the scientifically minded seemed totally unreal and to the superstitious manifestations of the supernatural. . . .

"In showing that the abnormal can be explained in terms of the normal, psychology does now for the phenomena of mind what the physical sciences have long done for the phenomena of nature. . . .

"Psychology as a science postulates the reign of natural law in the subjective sphere just as rigorously as physics postulates the reign of law in the objective sphere. . . .

"It is not in the unusual and the abnormal that the reflective mind is to see God. It is not through gaps in nature that we are to get glimpses of the supernatural. Rather is it in the very nature of nature, rational, harmonious, law-conforming, subject to scientific interpretation, that we have the best evidence that the world is made mind-wise, that it is the work of an intelligent mind, that there is a rational spirit at the core of the universe.

"For science the transcendent does not enter into the perceptual realm external or internal. It is, indeed, hard for the religious mind to admit this fact in all its fullness. Until it does, however, religion must always stand more or less in fear of science. Once give up the perceptual, in all its bearings, to science, and religion will find that it has lost a weak support only to gain a stronger one. Ultimately, I believe, we shall find that the full acceptance of science in the mental domain as well as in the physical will strengthen the rational grounds of theistic belief."

SECTION C — LOGIC

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(*Hall* 6, *September* 22, 10 *a. m.*)

Chairman: Professor George M. Duncan, Yale University.
Speakers: Professor William A. Hammond, Cornell University.
Professor Frederick J. E. Woodbridge, Columbia University.
Secretary: Dr. W. H. Sheldon, Columbia University.

The Chairman of this Section, Professor George M. Duncan, Professor of Logic and Mathematics at Yale University, in introducing the speakers spoke briefly of the scope and importance of the subject assigned to the Section; expressed, on behalf of those in attendance, regret at the inability of Professor Wilhelm Windelband to be present and take part in the work of the Section, as had been expected; congratulated the Section on the papers to be presented and the speakers who were to present them; and announced the final programme of the Section.

THE RELATIONS OF LOGIC TO OTHER DISCIPLINES

BY PROFESSOR WILLIAM A. HAMMOND

[**William Alexander Hammond,** Assistant Professor of Ancient and Medieval Philosophy and Æsthetics, Cornell University. **b.** May 20, 1861, New Athens, Ohio. A.B. Harvard, 1885; Ph.D. Leipzig, 1891. Lecturer on Classics, King's College, Windsor, N. S., 1885–88; Secretary of the University Faculty, Cornell; Member American Psychological Association, American Philosophical Association. **Author of** *The Characters of Theophrastus*, translated with Introduction; *Aristotle's Psychology*, translated with Introduction.]

In 1787, in the preface to the second edition of the *Kr. d. r. V.*, Kant wrote the following words: "That logic, from the earliest times, has followed that secure method" (namely, the secure method of a science witnessed by the unanimity of its workers and the stability of its results) "may be seen from the fact that since Aristotle it has not had to retrace a single step, unless we choose to consider as improvements the removal of some unnecessary subtleties, or the clearer definition of its matter, both of which refer to the elegance rather than to the solidity of the science. It is remarkable, also, that to the present day, it has not been able to make one step in advance, so that to all appearances it may be considered as completed and perfect. If some modern philosophers thought to enlarge it, by introducing *psychological* chapters on the different faculties of knowledge (faculty of imagination, wit, etc.), or *metaphysical* chapters on the origin of knowledge or different degrees of certainty according to the difference of objects (idealism, skepticism, etc.), or, lastly, *anthropological* chapters on prejudices, their causes and remedies, this could only arise from their ignorance of the peculiar nature of logical science. We do not enlarge, but we only disfigure the sciences, if we allow their respective limits to be confounded; and the limits of logic are definitely fixed by the fact that it is a science which has nothing to do but fully to exhibit and strictly to prove the formal rules of all thought (whether it be *a priori* or empirical, whatever be its origin or its object, and whatever be the impediments, accidental or natural, which it has to encounter in the human mind)." —[Translated by Max Müller.] Scarcely more than half a century after the publication of this statement of Kant's, John Stuart Mill (Introduction to *System of Logic*) wrote: "There is as great diversity among authors in the modes which they have adopted of defining logic, as in their treatment of the details of it. This is what might naturally be expected on any subject on which writers have availed themselves of the same language as a means of delivering different ideas. . . . This diversity is not so much an evil to be complained of, as an inevitable, and in some degree a proper result of the imperfect state of those sciences" (that is, of logic, jurisprudence, and ethics). "It is not to be expected that there should be agreement about the definition of anything, until there is agreement about the thing itself." This remarkable disparity of opinion is due partly to the changes in the treatment of logic from Kant to Mill, and partly to the fact that both statements are extreme. That the science of logic was "completed and perfect" in the time of Kant could only with any degree of accuracy be said of the treatment of syllogistic proof or the deductive logic of Aristotle. That the diversity was so great as pictured by Mill is not historically exact, but could be said only of the new epistemological and psychological treatment of logic and not of the traditional formal logic. The confusion in logic is no doubt largely due to disagreement in the delimitation of its proper territory and to the consequent variety of opinions as to its relations to other disciplines. The rise of inductive logic, coincident with the rise and growth of physical science and empiricism, forced the consideration of the question as to the relation of formal thought to reality, and the consequent entanglement of logic in a triple alliance of logic, psychology, and metaphysics. How logic can maintain friendly relations with both of these and yet avoid endangering its territorial integrity has not been made clear by logicians or psychologists or metaphysicians, and that, too, in spite of persistent attempts justly to settle the issue as to their respective spheres of influence. Until modern logic definitely settles the question of its aims and legitimate problems, it is difficult to see how any agreement can be reached as to its relation to the other disciplines. The situation as it confronts one in the discussion of the relations of logic to allied subjects may be analyzed as follows:

1. The relation of logic as science to logic as art.
2. The relation of logic to psychology.
3. The relation of logic to metaphysics.

The development of nineteenth century logic has made an answer to the last two of the foregoing problems exceedingly difficult. Indeed, one may say that the evolution of modern epistemology has had a centrifugal influence on logic, and instead of growth towards unity of conception we have a chaos of diverse and discordant theories. The apple of discord has been the theory of knowledge. A score of years ago when Adamson wrote his admirable article in the *Encyclopædia Britannica* (article "Logic," 1882), he found the conditions much the same as I now find them. "Looking to the chaotic state of logical text-books at the present time, one would be inclined to say that there does not exist anywhere a recognized currently received body of speculations to which the title logic can be unambiguously assigned, and that we must therefore resign the hope of attaining by any empirical consideration of the received doctrine a precise determination of the nature and limits of logical theory." I do not, however, take quite so despondent a view of the logical chaos as the late Professor Adamson; rather, I believe with Professor Stratton (*Psy. Rev.* vol. III) that something is to be gained for unity and consistency by more exact delimitation of the subject-matter of the philosophical disciplines and their interrelations, which precision, if secured, would assist in bringing into clear relief the real problems of the several departments of inquiry, and facilitate the proper classification of the disciplines themselves.

The attempt to delimit the spheres of the disciplines, to state their interrelations and classify them, was made early in the history of philosophy, at the very beginning of the development of logic as a science by Aristotle. In Plato's philosophy, logic is not separated from epistemology and metaphysics. The key to his metaphysics is given essentially in his theory of the reality of the concept, which offers an interesting analogy to the position of logic in modern idealism. Before Plato there was no formulation of logical theory, and in his dialogues it is only contained in solution. The nearest approach to any formulation is to be found in an applied logic set forth in the precepts and rules of the rhetoricians and sophists. Properly speaking, Aristotle made the first attempt to define the subject of logic and to determine its relations to the other sciences. In a certain sense logic for Aristotle is not a science at all. For science is concerned with some *ens*, some branch of reality, while logic is concerned with the methodology of knowing, with the formal processes of thought whereby an *ens* or a reality is ascertained and appropriated to knowledge. In the sense of a method whereby all scientific knowledge is secured, logic is a propædeutic to the sciences. In the idealism of the Eleatics and Plato, thought and being are ultimately identical, and the laws of thought are the laws of being. In Aristotle's conception, while the processes of thought furnish a knowledge of reality or being, their formal operation constitutes the technique of investigation, and their systematic explanation and description constitute logic. Logic and metaphysics are distinguished as the science of being and the doctrine of the thought-processes whereby being is known. Logic is the doctrine of the organon of science, and when applied is the organon of science. The logic of Aristotle is not a purely formal logic. He is not interested in the merely schematic character of the thought-processes, but in their function as mediators of apodictic truth. He begins with the assumption that in the conjunction and disjunction of correctly formed judgments the conjunction or disjunction of reality is mirrored. Aristotle does not here examine into the powers of the mind as a whole; that is done, though fragmentarily, in the *De Anima* and *Parva Naturalia*, where the mental powers are regarded as phases of the processes of nature without reference to normation; but in his logic he inquires only into those forms and laws of thinking which mediate proof. Scientific proof, in his conception, is furnished in the form of the syllogism, whose component elements are terms and propositions. In the little tract *On Interpretation* (*i. e.* on the judgment as *interpreter* of thought), if it is genuine, the proposition is considered in its logical bearing. The treatise on the *Categories*, which discusses the nature of the most general terms, forms a connecting link between logic and metaphysics. The categories are the most general concepts or universal modes under which we have knowledge of the world. They are not simply logical relations; they are existential forms, being not only the modes under which thought regards being, but the modes under which being exists. Aristotle's theory of the methodology of science is intimately connected with his view of knowledge. Scientific knowledge in his opinion refers to the essence of things; for example, to those universal aspects of reality which are given in particulars, but which remain self-identical amidst the variation and passing of particulars. The universal, however, is known only through and after particulars. There is no such thing as innate knowledge or Platonic reminiscence. Knowledge, if not entirely empirical, has its basis in empirical reality. Causes are known only through effects. The universals have no existence apart from things, although they exist *realiter* in things. Empirical knowledge of particulars must, therefore, precede in time the conceptual or scientific knowledge of universals. In the evolution of scientific knowledge in the individual mind, the body of particulars or of sense-experience is to its conceptual transformation as potentiality is to actuality, matter to form, the completed end of the former being realized in the latter. Only in the sense of this power to transform and conceptualize, does the mind have knowledge within itself. The genetic content is experiential; the developed concept, judgment, or inference is *in form* noëtic. Knowledge is, therefore, not a mere "precipitate of experience," nor is Aristotle a complete empiricist. The conceptual form of knowledge is not immediately given in things experienced, but is a product of noëtic discrimination and combination. Of a sensible object as such there is no concept; the object of a concept is the generic essence of a thing; and the concept itself is the thought of this generic essence. The individual is generalized; every concept does or can embrace several individuals. It is an "aggregate of distinguishing marks," and is expressed in a definition. The concept as such is neither true nor false. Truth first arises in the form of a judgment or proposition, wherein a subject is coupled with a predicate, and something is said about something.

A judgment is true when the thought (whose inward process is the judgment and the expression in vocal symbols is the proposition) regards as conjoined or divided that which is conjoined or divided in actuality; in other words, when the thought is congruous with the real. While Aristotle does not ignore induction as a scientific method, (how could he when he regards the self-subsistent individual as the only real?) yet he says that, as a method, it labors under the defect of being only proximate; a complete induction from *all* particulars is not possible, and therefore cannot furnish demonstration. Only the deductive process proceeding syllogistically from the universal (or essential truth) to the particular is scientifically cogent or apodictic. Consequently Aristotle developed the science of logic mainly as a syllogistic technique or instrument of demonstration. From this brief sketch of Aristotle's logical views it will be seen that the epistemological and metaphysical relations of logic which involve its greatest difficulty and cause the greatest diversity in its modern exponents, were present in undeveloped form to the mind of the first logician. It would require a mighty optimism to suppose that this difficulty and diversity, which has increased rather than diminished in the progress of historical philosophy, should suddenly be made to vanish by some magic of re-statement of subject-matter, or theoretical delimitation of the discipline. As Fichte said of philosophy, "The sort of a philosophy that a man has, depends on the kind of man he is;" so one might almost say of logic, "The sort of logic that a man has, depends on the kind of philosopher he is." If the blight of discord is ever removed from epistemology, we may expect agreement as to the relations of logic to metaphysics. Meanwhile logic has the great body of scientific results deposited in the physical sciences on which to build and test, with some assurance, its doctrine of methodology; and as philosophy moves forward persistently to the final solution of its problems, logic may justly expect to be a beneficiary in its established theories.

After Aristotle's death logic lapsed into a formalism more and more removed from any vital connection with reality and oblivious to the profound epistemological and methodological questions that Aristotle had at least raised. In the Middle Ages it became a highly developed exercise in inference applied to the traditional dogmas of theology and science as premises, with mainly apologetic or polemical functions. Its chief importance is found in its application to the problem of realism and nominalism, the question as to the nature of universals. At the height of scholasticism realism gained its victory by syllogistically showing the congruity of its premises with certain fundamental dogmas of the Church, especially with the dogma of the unity and reality of the Godhead. The heretical conclusion involved in nominalism is equivalent (the accepted dogma of the Church being axiomatic) to *reductio ad absurdum.* A use of logic such as this, tending to conserve rather than to increase the body of knowledge, was bound to meet with attack on the awakening of post-renaissance interest in the physical world, and the acquirement of a body of truth to which the scholastic formal logic had no relation. The anti-scholastic movement in logic was inaugurated by Francis Bacon, who sought in his *Novum Organum* to give science a real content through the application of induction to experience and the discovery of universal truths from particular instances. The syllogism is rejected as a scientific instrument, because it does not lead *to* principles, but proceeds only *from* principles, and is therefore not useful for discovery. It permits at most only refinements on knowledge already possessed, but cannot be regarded as creative or productive. The Baconian theory of induction regarded the accumulation of facts and the derivation of general principles and laws from them as the true and fruitful method of science. In England this empirical view of logic has been altogether dominant, and the most illustrious English exponents of logical theory, Herschel, Whewell, and Mill, have stood on that ground. Since the introduction of German idealism in the last half century a new logic has grown up whose chief business is with the theory of knowledge.

Kant's departure in logic is based on an epistemological examination of the nature of judgment, and on the answer to his own question, "How are synthetic judgments *a priori* possible?" The *a priori* elements in knowledge make knowledge of the real nature of things impossible. Human knowledge extends to the phenomenal world, which is seen under the *a priori* forms of the understanding. Logic for Kant is the science of the formal and necessary laws of thought, apart from any reference to objects. Pure or universal logic aims to understand the forms of thought without regard to metaphysical or psychological relations, and this position of Kant is the historical beginning of the subjective formal logic.

In the metaphysical logic of Hegel, which rests on a panlogistic basis, being and thought, form and content, are identical. Logical necessity is the measure and criterion of objective reality. The body of reality is developed through the dialectic self-movement of the idea. In such an idealistic monism, formal and real logic are by the metaphysical postulate coincident.

Schleiermacher in his dialectic regards logic from the standpoint of epistemological realism, in which the real deliverances of the senses are conceptually transformed by the spontaneous activity of reason. This spirit of realism is similar to that of Aristotle, in which the one-sided *a priori* view of knowledge is controverted. Space and time are forms of the existence of things, and not merely *a priori* forms of knowing. Logic he divides into dialectic and technical logic. The former regards the idea of knowledge as such; the formal or technical regards knowledge in the process of becoming or the idea of knowledge in motion. The forms of this process are induction and deduction. The Hegelian theory of the generation of knowledge out of the processes of pure thought is emphatically rejected.

Lotze, who is undoubtedly one of the most influential and fruitful writers on logic in the last century, attempts to bring logic into closer relations with contemporary science, and is an antagonist of one-sided formal logics. For him logic falls into the three parts of (1) pure logic or the logic of thought; (2) applied logic or the logic of investigation; (3) the logic of knowledge or methodology; and this classification of the matter and problems of logic has had an important influence on subsequent treatises on the discipline. His logic is formal, as he describes it himself, in the sense of setting forth the modes of the operation of thought and its logical structure; it is real in the sense that these forms are dependent on the nature of things and not something independently given in the mind. While he aims to maintain the distinct separation of logic and metaphysics, he says (in the discussion of the relations between formal and real logical meaning) the question of meaning naturally raises a metaphysical problem: "Ich thue besser der Metaphysik die weitere Erörterung dieses wichtigen Punktes zu überlassen." (*Log.* 2d ed. p. 571.) How could it be otherwise when his whole view of the relations and validity of knowledge is inseparable from his realism or teleological idealism, as he himself characterizes his own standpoint?

Drobisch, a follower of Herbart, is one of the most thoroughgoing formalists in modern logical theory. He attempts to maintain strictly the distinction between thought and knowledge. Logic is the science of thought. He holds that there may be formal truth, for example, logically valid truth, which is materially false. Logic, in other words, is purely formal; material truth is matter for metaphysics or science. Drobisch holds, therefore, that the falsity of the judgment expressed in the premise from which a formally correct syllogism may be deduced, is not subject-matter for logic. The sphere of logic is limited to the region of inference and forms of procedure, his view of the nature and function of logic being determined largely by the bias of his mathematical standpoint. The congruity of thought with itself, judgments, conclusions, analyses, etc., is the sole logical truth, as against Trendelenburg, who took the Aristotelian position that logical truth is the "agreement of thought with the object of thought."

Sigwart looks at logic mainly from the standpoint of the technology of science, in which, however, he discovers the implications of a teleological metaphysic. Between the processes of consciousness and external changes he finds a causal relation and not parallelism. Inasmuch as thought sometimes misses its aim, as is shown by the fact that error and dispute exist, there is need of a discipline whose purpose is to show us how to attain and establish truth and avoid error. This is the practical aim of logic, as distinguished from the psychological treatment of thought, where the distinction between true and false has no more place than the distinction between good and bad. Logic presupposes the impulse to discover truth, and it therefore sets forth the criteria of true thinking, and endeavors to describe those normative operations whose aim is validity of judgment. Consequently logic falls into the two parts of (1) critical, (2) technical, the former having meaning only in reference to the latter; the main value of logic is to be sought in its function as art. "Methodology, therefore, which is generally made to take a subordinate place, should be regarded as the special, final, and chief aim of our science." (*Logic,* vol. I, p. 21, Eng. Tr.) As an art, logic undertakes to determine under what conditions and prescriptions judgments are valid, but does not undertake to pass upon the validity of the content of given judgments. Its prescriptions have regard only to formal correctness and not to the material truth of results. Logic is, therefore, a formal discipline. Its business is with the due procedure of thought, and it attempts to show no more than how we may advance in the reasoning process in such way that each step is valid and necessary. If logic were to tell us *what* to think or give us the content of thought, it would be commensurate with the whole of science. Sigwart, however, does not mean by formal thought independence of content, for it is not possible to disregard the particular manner in which the materials and content of thought are delivered through sensation and formed into ideas. Further, logic having for its chief business the methodology of science, the development of knowledge from empirical data, it ought to include a theory of knowledge, but it should not so far depart from its subjective limits as to include within its province the discussion of metaphysical implications or a theory of being. For this reason, Sigwart relegates to a postscript his discussion of teleology, but he gives an elaborate treatment of epistemology extending through vol. I and develops his account of methodology in vol. II. The question regarding the relation between necessity, the element in which logical thought moves, and freedom, the postulate of the will, carries one beyond the confines of logic and is, in his opinion, the profoundest problem of metaphysics, whose function is to deal with the ultimate relation between "subject and object, the world and the individual, and this is not only basal for logic and all science, but is the crown and end of them all."

Wundt's psychological and methodological treatment of logic stands midway between the purely formal treatises on the one hand, and the metaphysical treatises on the other hand. The general standpoint of Wundt is similar to that of Sigwart, in that he discovers the function of logic in the exposition of the formation and methods of scientific knowledge; for example, in epistemology and methodology. Logic must conform to the conditions under which scientific inquiry is actually carried on; the forms of thought, therefore, cannot be separate from or indifferent to the content of knowledge; for it is a fundamental principle of science that its particular methods are determined by the nature of its particular subject-matter. Scientific logic must reject the theory that identifies thought and being (Hegel) and the theory of parallelism between thought and reality (Schleiermacher, Trendelenburg, and Ueberweg), in which the ultimate identity of the two is only concealed. Both of these theories base logic on a metaphysics, which makes it necessary to construe the real in terms of thought, and logic, so divorced from empirical reality, is powerless to explain the methods of scientific procedure. One cannot, however, avoid the acceptance of thought as a competent organ for the interpretation of reality, unless one abandons all question of validity and accepts agnosticism or skepticism. This interpretative power of thought or congruity with reality is translated by metaphysical logic into identity. Metaphysical logic concerns itself fundamentally with the content of knowledge, not with its evidential or formal logical aspects, but with being and the laws of being. It is the business of metaphysics to construct its notions and theories of reality out of the deliverances of the special sciences and inferences derived therefrom. The aim of metaphysics is the development of a world-view free from internal contradictions, a view that shall unite all particular and plural knowledges into a whole. Logic stands in more intimate relation to the special sciences, for here the relations are reciprocal and immediate; for example, from actual scientific procedure logic abstracts its general laws and results, and these in turn it delivers to the sciences as their formulated methodology. In the history of science the winning of knowledge precedes the formulation of the rules employed, that is, precedes any scientific methodology. Logic, as methodology, is not an *a priori* construction, but has its genesis in the growth of science itself and in the discovery of those tests and criteria of truth which are found to possess an actual heuristic or evidential value. It is not practicable to separate epistemology and logic, for such concepts as causality, analogy, validity, etc., are fundamental in logical method, and yet they belong to the territory of epistemology, are epistemological in nature, as one may indeed say of all the general laws of thought. A formal logic that is merely propædeutic, a logic that aims to free itself from the quarrels of epistemology, is scientifically useless. Its norms are valueless, in so far as they can only teach the arrangement of knowledge already possessed, and teach nothing as to how to secure it or test its real validity. While formal logic aims to put itself outside of philosophy, metaphysical logic would usurp the place of philosophy. Formal logic is inadequate, because it neither shows how the laws of thought originate, why they are valid, nor in what sense they are applicable to concrete investigation. Wundt, therefore, develops a logic which one may call epistemological methodological, and which stands between the extremes of formal logic and metaphysical logic. The laws of logic must be derived from the processes of psychic experience and the procedure of the sciences. "Logic therefore needs," as he says, "epistemology for its foundation and the doctrine of methods for its completion."

Lipps takes the view outright that logic is a branch of psychology; Husserl in his latest book goes to the other extreme of a purely formal and technical logic, and devotes almost his entire first volume to the complete sundering of psychology and logic.

Bradley bases his logic on the theory of the judgment. The logical judgment is entirely different from the psychological. The logical judgment is a qualification of reality by means of an idea. The predicate is an adjective or attribute which in the judgment is ascribed to reality. The aim of truth is to qualify reality by general notions. But inasmuch as reality is individual and self-existent, whereas truth is universal, truth and reality are not coincident. Bradley's metaphysical solution of the disparity between thought and reality is put forward in his theory of the unitary Absolute, whose concrete content is the totality of experience. But as thought is not the whole of experience, judgments cannot compass the whole of reality. Bosanquet objects to this, and maintains that reality must not be regarded as an ideal construction. The real world is the world to which our concepts and judgments refer. In the former we have a world of isolated individuals of definite content; in the latter, we have a world of definitely systematized and organized content. Under the title of the Morphology of Knowledge Bosanquet considers the evolution of judgment and inference in their varied forms. "Logic starts from the individual mind, as that within which we have the actual facts of intelligence, which we are attempting to interpret into a system" (*Logic*, vol. I, p. 247). The real world for every individual is *his* world. "The work of intellectually constituting that totality which we call the real world is the work of knowledge. The work of analyzing the process of this constitution or determination is the work of logic, which might be described . . . as the reflection of knowledge upon itself" (*Logic*, vol. I, p. 3). "The relation of logic to truth consists in examining the characteristics by which the various phases of the one intellectual function are fitted for their place in the intellectual totality which constitutes knowledge" (*ibid.*). The real world is the intelligible world; reality is something to which we attain by a constructive process. We have here a type of logic which is essentially a metaphysic. Indeed, Bosanquet says in the course of his first volume: "I entertain no doubt that in content logic is one with metaphysics, and differs, if at all, simply in mode of treatment — in tracing the evolution of knowledge in the light of its value and import, instead of attempting to summarize its value and import apart from the details of its evolution" (*Logic*, vol. I, 247).

Dewey (*Studies in Logical Theory*, p. 5) describes the essential function of logic as the inquiry into the relations of thought as such to reality as such. Although such an inquiry may involve the investigation of psychological processes and of the concrete methods of science and verification, a description and analysis of the forms of thought, conception, judgment, and inference, yet its concern with these is subordinate to its main concern, namely, the relation of "thought at large to reality at large." Logic is not reflection on thought, either on its nature as such or on its forms, but on its relations to the real. In Dewey's philosophy, logical theory is a description of thought as a mode of adaptation to its own conditions, and validity is judged in terms of the efficiency of thought in the solution of its own problems and difficulties. The problem of logic is more than epistemological. Wherever there is striving there are obstacles; and wherever there is thinking there is a "material-in-question." Dewey's logic is a theory of reflective experience regarded functionally, or a pragmatic view of the discipline. This logic of experience aims to evaluate the significance of social research, psychology, fine and industrial art, and religious aspiration in the form of scientific statement, and to accomplish for social values in general what the physical sciences have done for the physical world. In Dewey's teleological pragmatic logic the judgment is essentially instrumental, the whole of thinking is functional, and the meaning of things is identical with valid meaning (*Studies in Logical Theory*, cf. pp. 48, 82, 128). The real world is not a self-existent world outside of knowledge, but simply the totality of experience; and experience is a complex of strains, tensions, checks, and attitudes. The function of logic is the redintegration of this experience. "Thinking is adaptation *to* an end *through* the adjustment of particular objective contents" (*ibid.* p. 81). Logic here becomes a large part, if not the whole, of a metaphysics of experience; its nature and function are entirely determined by the theory of reality.

In this brief and fragmentary *résumé* are exhibited certain characteristic movements in the development of logical theory, the construction put upon its subject-matter and its relation to other disciplines. The *résumé* has had in view only the making of the diversity of opinion on these questions historically salient. There are three distinct types of logic noticed here: (1) formal, whose concern is merely with the structural aspect of inferential thought, and its validity in terms of internal congruity; (2) metaphysical logic whose concern is with the functional aspect of thought, its validity in terms of objective reference, and its relation to reality; (3) epistemological and methodological logic, whose concern is with the genesis, nature, and laws of logical thinking as forms of scientific knowledge, and with their technological application to the sciences as methodology. I am not at present concerned with a criticism of these various viewpoints, excepting in so far as they affect the problem of the interrelationship of logic and the allied disciplines.

For my present purpose I reject the extreme metaphysical and formal positions, and assume that logic is a discipline whose business is to describe and systematize the formal processes of inferential thought and to apply them as practical principles to the body of real knowledge.

I wish now to take up *seriatim* the several questions touching the various relations of logic enumerated above, and first of all the question of the relation of logic as science to logic as art.

I. *Logic as science and logic as art.*

It seems true that the founder of logic, Aristotle, regarded logic not as a science, but rather as propædeutic to science, and not as an end in itself, but rather technically and heuristically as an instrument. In other words, logic was conceived by him rather in its application or as an art, than as a science, and so it continued to be regarded until the close of the Middle Ages, being characterized indeed as the *ars artium;* for even the *logica docens* of the Scholastics was merely the formulation of that body of precepts which are of practical service in the syllogistic arrangement of premises, and the Port Royal Logic aims to furnish *l'art de penser*. This technical aspect of the science has clung to it down to the present day, and is no doubt a legitimate description of a part of its function. But no one would now say that logic *is* an art; rather it is a body of theory which may be technically applied. Mill, in his examination of Sir William Hamilton's Philosophy (p. 391), says of logic that it "is the art of thinking, which means of correct thinking, and the science of the conditions of correct thinking," and indeed, he goes so far as to say (*System of Logic*, Introd. § 7): "The extension of logic as a science is determined by its necessities as an art." Strictly speaking, logic as a science is purely theoretical, for the function of science as such is merely to know. It is an organized system of knowledge, namely, an organized system of the principles and conditions of correct thinking. But because correct thinking is an art, it does not follow that a knowledge of the methods and conditions of correct thinking is art, which would be a glaring case of μετάβασις εἰς ἄλλο γένος. The art-bearings of the science are given in the normative character of its subject-matter. As a science logic is descriptive and explanatory, that is, it describes and formulates the norms of valid thought, although as science it is not normative, save in the sense that the principles formulated in it may be normatively or regulatively applied, in which case they become precepts. What is principle in science becomes precept in application, and it is only when technically applied that principles assume a mandatory character. Validity is not created by logic. Logic merely investigates and states the conditions and criteria of validity, being in this reference a science of evidence. In the very fact, however, that logic is normative in the sense of describing and explaining the norms of correct thinking, its practical or applied character is given. Its principles as known are science; its principles as applied are art. There is, therefore, no reason to sunder these two things or to call logic an art merely or a science merely; for it is both when regarded from different viewpoints, although one must insist on the fact that the rules for practical guidance are, so far as the science is concerned, quite *ab extra*. Logic, ethics, and æsthetics are all commonly (and rightly) called normative disciplines: they are all concerned with values and standards; logic with validity and evidence, or values for cognition; ethics with motives and moral quality in conduct, or values for volition; æsthetics with the standards of beauty, or values for appreciation and feeling. Yet none of them is or can be merely normative, or indeed as science normative at all; if that were so, they would not be bodies of organized knowledge, but bodies of rules. They might be well-arranged codes of legislation on conduct, fine art, and evidence, but not sciences. Strictly regarded, it is the descriptive and explanatory aspect of logic that constitutes its scientific character, while it is the specific normative aspect that constitutes its logical character. Values, whether ethical or logical, without an examination and formulation of their ground, relations, origin, and interconnection, would be merely rules of thumb, popular phrases, or pastoral precepts. The actual methodology of the sciences or applied logic is logic as art.

II. *Relation of logic to psychology.*

The differentiation of logic and psychology in such way as to be of practical value in the discussion of the disciplines has always been a difficult matter. John Stuart Mill was disposed to merge logic in psychology, and Hobhouse, his latest notable apologete, draws no fixed distinction between psychology and logic, merely saying that they have different centres of interest, and that their provinces overlap. Lipps, in his *Grundzüge der Logik* (p. 2), goes the length of saying that "Logic is a psychological discipline, as certainly as knowledge occurs only in the Psyche, and thought, which is developed in knowledge, is a psychical event." Now, if we were to take such extreme ground as this, then ethics, æsthetics, and pure mathematics would become at once branches of psychology and not coördinate disciplines with it, for volitions, the feelings of appreciation, and the reasoning of pure mathematics are psychical events. Such a theory plainly carries us too far and would involve us in confusion. That the demarcation between the two disciplines is not a chasmic cleavage, but a line, and that, too, an historically shifting line, is apparent from the foregoing historical *résumé*.

The four main phases of logical theory include: (1) the concept (although some logicians begin with the judgment as temporally prior in the evolution of language), (2) judgment, (3) inference, (4) the methodology of the sciences. The entire concern of logic is, indeed, with psychical processes, but with psychical processes regarded from a specific standpoint, a standpoint different from that of psychology. In the first place psychology in a certain sense is much wider than logic, being concerned with the whole of psychosis as such, including the feelings and will and the entire structure of cognition, whereas logic is concerned with the particular cognitive processes enumerated above (concept, judgment, inference), and that, too, merely from the point of view of validity and the grounds of validity. In another sense psychology is narrower than logic, being concerned purely with the description and explanation of a particular field of phenomena, whereas logic is concerned with the procedure of all the sciences and is practically related to them as their formulated method. The compass and aims of the two disciplines are different; for while psychology is in different references both wider and narrower than logic, it is also different in the problems it sets itself, its aim being to describe and explain the phenomena of mind in the spirit of empirical science, whereas the aim of logic is only to explain and establish the laws of evidence and standards of validity. Logic is, therefore, selective and particular in the treatment of mental phenomena, whereas psychology is universal, that is, it covers the entire range of mental processes as a phenomenalistic science; logic dealing with definite elements as a normative science. By this it is not meant that the territory of judgment and inference should be delivered from the psychologist into the care of the logician; through such a division of labor both disciplines would suffer. The two disciplines handle to some extent the same subjects, so far as names are concerned; but the essence of the logical problem is not touched by psychology, and should not be mixed up with it, to the confusion and detriment of both disciplines. The field of psychology, as we have said, is the whole of psychical phenomena; the aim of individual psychology in the investigation of its field is: (1) to give a genetic account of cognition, feeling, and will, or whatever be the elements into which consciousness is analyzed; (2) to explain their interconnections causally; (3) as a chemistry of mental life to analyze its complexes into their simplest elements; (4) to explain the totality structurally (or functionally) out of the elements; (5) to carry on its investigation and set forth its results as a purely empirical science; (6) psychology makes no attempt to evaluate the processes of mind either in terms of false and true, or good and bad. From this description of the field and function of psychology, based on the expressions of its modern exponents, it will be found impossible to shelter logic under it as a subordinate discipline. If one were to enlarge the scope of psychology to mean rational psychology, in the sense which Professor Howison advocates (*Psychological Review*, vol. III, p. 652), such a subordination might be possible, but it would entail the loss of all that the new psychology has gained by the sharper delimitation of its sphere and problems, and would carry us back to the position of Mill, who appears to identify psychology with philosophy at large and with metaphysics.

In contradistinction to the aims of psychology as described in the foregoing, the sphere and problems of logic may be summarily characterized as follows: (1) All concepts and judgments are psychological complexes and processes and may be genetically and structurally described; that is the business of psychology. They also have a meaning value, or objective reference, that is, they may be correct or incorrect, congruous or incongruous with reality. The meaning, aspect of thought, or its content as truth is the business of logic. This subject-matter is got by regarding a single aspect in the total psychological complex. (2) Its aim is not to describe factual thought or the whole of thought, or the natural processes of thought, but only certain ideals of thinking, namely, the norms of correct thinking. Its object is not a datum, but an ideal. (3) While psychology is concerned with the natural history of reasoning, logic is concerned with the warrants of inferential reasoning. In the terminology of Hamilton it is the nomology of discursive thought. To use an often employed analogy, psychology is the physics of thought, logic an ethics of thought. (4) Logic implies an epistemology or theory of cognition in so far as epistemology discusses the concept and judgment and their relations to the real world, and here is to be found its closest connection with psychology. A purely formal logic, which is concerned merely with the internal order of knowledge and does not undertake to show how the laws of thought originate, why they hold good as the measures of evidence, or in what way they are applicable to concrete reality, would be as barren as scholasticism.

(5) While logic thus goes back to epistemology for its bases and for the theoretical determination of the interrelation of knowledge and truth, it goes forward in its application to the practical service of the sciences as their methodology. A part of its subject-matter is therefore the actual procedure of the sciences, which it attempts to organize into systematic statements as principles and formulæ. This body of rules given implicitly or explicitly in the workings and structure of the special sciences, consisting in classification, analysis, experiment, induction, deduction, nomenclature, etc., logic regards as a concrete deposit of inferential experience. It abstracts these principles from the content and method of the sciences, describes and explains them, erects them into a systematic methodology, and so creates the practical branch of real logic. Formal logic, therefore, according to the foregoing account, would embrace the questions of the internal congruity and self-consistency of thought and the schematic arrangement of judgments to insure formally valid conclusions; real logic would embrace the epistemological questions of how knowledge is related to reality, and how it is built up out of experience, on the one hand, and the methodological procedure of science, on the other. The importance of mathematical logic seems to be mainly in the facilitation of logical expression through symbols. It is rather with the machinery of the science than with its content and real problem that the logical algorithm or calculus is concerned. In these condensed paragraphs sufficient has been said, I think, to show that logic and psychology should be regarded as coördinate disciplines; for their aims and subject-matter differ too widely to subordinate the former under the latter without confusion to both.

I wish now to add a brief note on the relation of logic to another discipline.

III. *Relation of logic to metaphysics.*

As currently expounded, logic either abuts immediately on the territory of metaphysics at certain points or is entirely absorbed in it as an integral part of the metaphysical subject-matter. I regard the former view as not only the more tenable theoretically, but as practically advantageous for working purposes, and necessary for an intelligible classification of the philosophical disciplines. The business of metaphysics, as I understand it, is with the nature of reality; logic is concerned with the nature of validity, or with the relations of the elements of thought within themselves (self-consistency) and with the relations of thought to its object (real truth), but not with the nature of the objective world or reality as such. Further, metaphysics is concerned with the unification of the totality of knowledge in the form of a scientific cosmology; logic is concerned merely with the inferential and methodological processes whereby this result is reached. The former is a science of content; the latter is a science of procedure and relations. Now, inasmuch as procedure and relations apply to some reality and differ with different forms of reality, logic necessitates in its implications a theory of being, but such implications are in no wise to be identified with its subject-matter or with its own proper problems. Their consideration falls within the sphere of metaphysics or a broadly conceived epistemology, whose business it is to solve the ultimate questions of subject and object, thought and thing, mind and matter, that are implied and pointed to rather than formulated by logic. Inasmuch as the logical judgment says something about something, the scientific impulse drives us to investigate what the latter something ultimately is; but this is not necessary for logic, nor is it one of logic's legitimate problems, any more than it is the proper business of the physicist to investigate the mental implications of his scientific judgments and hypotheses or the ultimate nature of the theorizing and perceiving mind, or of causality to his world of matter and motion, although a general scientific interest may drive him to seek a solution of these ultimate metaphysical problems. Scientifically the end of logic and of every discipline is in itself; it is a territorial unity, and its government is administered with a unitary aim. Logic is purely a science of evidential values, not a science of content (in the meaning of particular reality, as in the special sciences, or of ultimate reality, as in metaphysics); its sole aim and purpose, as I conceive it, is to formulate the laws and grounds of evidence, the principles of method, and the conditions and forms of inferential thinking. When it has done this, it has, as a single science, done its whole work. When one looks at the present tendencies of logical theory, one is inclined to believe that the discipline is in danger of becoming an "*Allerleiwissenschaft*," whose vast undefined territory is the land of "*Weissnichtwo*." The strict delimitation of the field and problems of science is demanded in the interest of a serviceable division of scientific labor and in the interest of an intelligible classification of the accumulated products of research.

THE FIELD OF LOGIC

BY FREDERICK J. E. WOODBRIDGE

[**Frederick J. E. Woodbridge,** Johnsonian Professor of Philosophy in Columbia University, New York, N. Y., since 1902. b. Windsor, Ontario, Canada, March 26, 1867. A.B. Amherst College, 1889; Union Theological Seminary, 1892; A.M. 1898, LL.D. 1903, Amherst College. Post-grad. Berlin University. Instructor in Philosophy, University of Minnesota, 1894–95; Professor of Philosophy and head of department, 1895–1902. Member of American Association for the Advancement of Science, American Philosophical Association, American Pyschological Association. **Editor** of the *Journal of Philosophy, Psychology and Scientific Methods.*]

CURRENT tendencies in logical theory make a determination of the field of logic fundamental to any statement of the general problems of the science. In view of this fact, I propose in this paper to attempt such a determination by a general discussion of the relation of logic to mathematics, psychology, and biology, especially noting in connection with biology the tendency known as pragmatism. In conclusion, I shall indicate what the resulting general problems appear to be.

I

There may appear, at first, little to distinguish mathematics in its most abstract, formal, and symbolic type from logic. Indeed, mathematics as the universal method of all knowledge has been the ideal of many philosophers, and its right to be such has been claimed of late with renewed force. The recent notable advances in the science have done much to make this claim plausible. A logician, a non-mathematical one, might be tempted to say that, in so far as mathematics is the method of thought in general, it has ceased to be mathematics; but, I suppose, one ought not to quarrel too much with a definition, but should let mathematics mean knowledge simply, if the mathematicians wish it. I shall not, therefore, enter the controversy regarding the proper limits of mathematical inquiry. I wish to note, however, a tendency in the identification of logic and mathematics which seems to me to be inconsistent with the real significance of knowledge. I refer to the exaltation of the freedom of thought in the construction of conceptions, definitions, and hypotheses.

The assertion that mathematics is a "pure" science is often taken to mean that it is in no way dependent on experience in the construction of its basal concepts. The space with which geometry deals may be Euclidean or not, as we please; it may be the real space of experience or not; the properties of it and the conclusions reached about it may hold in the real world or they may not; for the mind is free to construct its conception and definition of space in accordance with its own aims. Whether geometry is to be ultimately a science of this type must be left, I suppose, for the mathematicians to decide. A logician may suggest, however, that the propriety of calling all these conceptions "space" is not as clear as it ought to be. Still further, there seems to underlie all arbitrary spaces, as their foundation, a good deal of the solid material of empirical knowledge, gained by human beings through contact with an environing world, the environing character of which seems to be quite independent of the freedom of their thought. However that may be, it is evident, I think, that the generalization of the principle involved in this idea of the freedom of thought in framing its conception of space, would, if extended to logic, give us a science of knowledge which would have no necessary relation to the real things of experience, although these are the things with which all concrete knowledge is most evidently concerned. It would inform us about the conclusions which necessarily follow from accepted conceptions, but it could not inform us in any way about the real truth of these conclusions. It would, thus, always leave a gap between our knowledge and its objects which logic itself would be quite impotent to close. Truth would thus become an entirely extra-logical matter. So far as the science of knowledge is concerned, it would be an accident if knowledge fitted the world to which it refers. Such a conception of the science of knowledge is not the property of a few mathematicians exclusively, although they have, perhaps, done more than others to give it its present revived vitality. It is the classic doctrine that logic is the science of thought as thought, meaning thereby thought in independence of any specific object whatever.

In regard to this doctrine, I would not even admit that such a science of knowledge is possible. You cannot, by a process of generalization or free construction, rid thought of connection with objects; and there is no such thing as a general content or as content-in-general. Generalization simply reduces the richness of content and, consequently, of implication. It deals with concrete subject-matter as much and as directly as if the content were individual and specialized. "Things equal to the same thing are equal to each other," is a truth, not about thought, but about things. The conclusions about a fourth dimension follow, not from the fact that we have thought of one, but from the conception about it which we have framed. Neither generalization nor free construction can reveal the operations of thought in transcendental independence.

It may be urged, however, that nothing of this sort was ever claimed. The bondage of thought to content must be admitted, but generalization and free construction, just because they give us the power to vary conditions as we please, give us thinking in a relative independence of content, and thus show us how thought operates irrespective of, although not independent of, its content. The binomial theorem operates irrespective of the values substituted for its symbols. But I can find no gain in this restatement of the position. It is true, in a sense, that we may determine the way thought operates irrespective of any specific content by the processes of generalization and free construction; but it is important to know in what sense. Can we claim that such irrespective operation means that we have discovered certain logical constants, which now stand out as the distinctive tools of thought? Or does it rather mean that this process of varying the content of thought as we please reveals certain real constants, certain ultimate characters of reality, which no amount of generalization or free construction can possibly alter? The second alternative seems to me to be the correct one. Whether it is or not may be left here undecided. What I wish to emphasize is the fact that the decision is one of the things of vital interest for logic, and properly belongs in that science. Clearly, we can never know the significance of ultimate constants for our thinking until we know what their real character is. To determine that character we must most certainly pass out of the realm of generalization and free construction; logic must become other than simply mathematical or symbolic.

There is another sense in which the determination of the operations of thought irrespective of its specific content is interpreted in connection with the exaltation of generalization and free construction. Knowledge, it is said, is solely a matter of implication, and logic, therefore, is the science of implication simply. If this is so, it would appear possible to develop the whole doctrine of implication by the use of symbols, and thus free the doctrine from dependence on the question as to how far these symbols are themselves related to the real things of the world. If, for instance, a implies b, then, if a is true, b is true, and this quite irrespective of the real truth of a or b. It is to be urged, however, in opposition to this view, that knowledge is concerned ultimately only with the real truth of a and b, and that the implication is of no significance whatever apart from this truth. There is no virtue in the mere implication. Still further, the supposition that there can be a doctrine of implication, simply, seems to be based on a misconception. For even so-called formal implication gets its significance only on the supposed truth of the terms with which it deals. We suppose that a *does* imply b, and that a *is* true. In other words, we can state this law of implication only as we first have valid instances of it given in specific, concrete cases. The law is a generalization and nothing more. The formal statement gives only an apparent freedom from experience. Moreover, there is no reason for saying that *a* implies *b* unless it does so either really or by supposition. If *a* really implies *b*, then the implication is clearly not a matter of thinking it; and to suppose the implication is to feign a reality, the implications of which are equally free from the processes by which they are thought. Ultimately, therefore, logic must take account of real implications. We cannot avoid this through the use of a symbolism which virtually implies them. Implication can have a logical character only because it has first a metaphysical one.

The supposition underlying the conception of logic I have been examining is, itself, open to doubt and seriously questioned. That supposition was the so-called freedom of thought. The argument has already shown that there is certainly a very definite limit to this freedom, even when logic is conceived in a very abstract and formal way. The processes of knowledge are bound up with their contents, and have their character largely determined thereby. When, moreover, we view knowledge in its genesis, when we take into consideration the contributions which psychology and biology have made to our general view of what knowledge is, we seem forced to conclude that the conceptions which we frame are very far from being our own free creations. They have, on the contrary, been laboriously worked out through the same processes of successful adaptation which have resulted in other products. Knowledge has grown up in connection with the unfolding processes of reality, and has, by no means, freely played over its surface. That is why even the most abstract of all mathematics is yet grounded in the evolution of human experience.

In the remaining parts of this paper, I shall discuss further the claims of psychology and biology. The conclusion I would draw here is that the field of logic cannot be restricted to a realm where the operations of thought are supposed to move freely, independent or irrespective of their contents and the objects of a real world; and that mathematics, instead of giving us any support for the supposition that it can, carries us, by the processes of symbolization and formal implication, to recognize that logic must ultimately find its field where implications are real, independent of the processes by which they are thought, and irrespective of the conceptions we choose to frame.

II

The processes involved in the acquisition and systematization of knowledge may, undoubtedly, be regarded as mental processes and fall thus within the province of psychology. It may be claimed, therefore, that every logical process is also a psychological one. The important question is, however, is it nothing more? Do its logical and psychological characters simply coincide? Or, to put the question in still another form, as a psychological process simply, does it also serve as a logical one? The answers to these questions can be determined only by first noting what psychology can say about it as a mental process.

In the first place, psychology can analyze it, and so determine its elements and their connections. It can thus distinguish it from all other mental processes by pointing out its unique elements or their unique and characteristic connection. No one will deny that a judgment is different from an emotion, or that an act of reasoning is different from a volition; and no one will claim that these differences are entirely beyond the psychologist's power to ascertain accurately and precisely. Still further, it appears possible for him to determine with the same accuracy and precision the distinction in content and connection between processes which are true and those which are false. For, as mental processes, it is natural to suppose that they contain distinct differences of character which are ascertainable. The states of mind called belief, certainty, conviction, correctness, truth, are thus, doubtless, all distinguishable as mental states. It may be admitted, therefore, that there can be a thoroughgoing psychology of logical processes.

Yet it is quite evident to me that the characterization of a mental process as logical is not a psychological characterization. In fact, I think it may be claimed that the characterization of any mental process in a specific way, say as an emotion, is extra-psychological. Judgments and inferences are, in short, not judgments and inferences because they admit of psychological analysis and explanation, any more than space is space because the perception of it can be worked out by genetic psychology. In other words, knowledge is first *knowledge*, and only later a set of processes for psychological analysis. That is why, as it seems to me, all psychological logicians, from Locke to our own day, have signally failed in dealing with the problem of knowledge. The attempt to construct knowledge out of mental states, the relations between ideas, and the relation of ideas to things, has been, as I read the history, decidedly without profit. Confusion and divergent opinion have resulted instead of agreement and confidence. On precisely the same psychological foundation, we have such divergent views of knowledge as idealism, phenomenalism, and agnosticism, with many other strange mixtures of logic, psychology, and metaphysics. The lesson of these perplexing theories seems to be that logic, as logic, must be divorced from psychology.

It is also of importance to note, in this connection, that the determination of a process as mental and as thus falling within the domain of psychology strictly, has by no means been worked out to the general satisfaction of psychologists themselves. Recent literature abounds in elaborate discussion of the distinction between what is a mental fact and what not, with a prevailing tendency to draw the remarkable conclusion that all facts are somehow mental or experienced facts. The situation would be worse for psychology than it is, if that vigorous science had not learned from other sciences the valuable knack of isolating concrete problems and attacking them directly, without the burden of previous logical or metaphysical speculation. Thus knowledge, which is the peculiar province of logic, is increased, while we wait for the acceptable definition of a mental fact. But definitions, be it remembered, are themselves logical matters. Indeed, some psychologists have gone so far as to claim that the distinction of a fact as mental is a purely logical distinction. This is significant as indicating that the time has not yet come for the identification of logic and psychology.

In refreshingly sharp contrast to the vagueness and uncertainty which beset the definition of a mental fact are the palpable concreteness and definiteness of knowledge itself. Every science, even history and philosophy, are instances of it. What constitutes a knowledge ought to be as definite and precise a question as could be asked. That logic has made no more progress than it has in the answer to it appears to be due to the fact that it has not sufficiently grasped the significance of its own simplicity. Knowledge has been the important business of thinking man, and he ought to be able to tell what he does in order to know, as readily as he tells what he does in order to build a house. And that is why the Aristotelian logic has held its own so long. In that logic, "the master of them that know" simply rehearsed the way he had systematized his own stores of knowledge. Naturally we, so far as we have followed his methods, have had practically nothing to add. In our efforts to improve on him, we have too often left the right way and followed the impossible method inaugurated by Locke. Had we examined with greater persistence our own methods of making science, we should have profited more. The introduction of psychology, instead of helping the situation, only confuses it.

Let it be granted, however, in spite of the vagueness of what is meant by a mental fact, that logical processes are also mental processes. This fact has, as I have already suggested, an important bearing on their genesis, and sets very definite limits to the freedom of thought in creating. It is not, however, as mental processes that they have the value of knowledge. A mental process which is knowledge purports to be connected with something other than itself, something which may not be a mental process at all. This connection should be investigated, but the investigation of it belongs, not to psychology, but to logic.

I am well aware that this conclusion runs counter to some metaphysical doctrines, and especially to idealism in all its forms, with the epistemologies based thereon. It is, of course, impossible here to defend my position by an elaborate analysis of these metaphysical systems. But I will say this. I am in entire agreement with idealism in its claim that questions of knowledge and of the nature of reality cannot ultimately be separated, because we can know reality only *as* we know it. But the general question as to how we know reality can still be raised. By this I do not mean the question, how is it possible for us to have knowledge at all, or how it is possible for reality to be known at all, but how, as a matter of fact, we actually do know it? That we really do know it, I would most emphatically claim. Still further, I would claim that what we know about it is determined, not by the fact that we can know in general, but by the way reality, as distinct from our knowledge, has determined. These ways appear to me to be ascertainable, and form, thus, undoubtedly, a section of metaphysics. But the metaphysics will naturally be realistic rather than idealistic.

III

Just as logical processes may be regarded as, at the same time, psychological processes, so they may be regarded, with equal right, as vital processes, coming thus under the categories of evolution. The tendency so to regard them is very marked at the present day, especially in France and in this country. In France, the movement has perhaps received the clearer definition. In America the union of logic and biology is complicated — and at times even lost sight of — by emphasis on the idea of evolution generally. It is not my intention to trace the history of this movement, but I should like to call attention to its historic motive in order to get it in a clear light.

That the theory of evolution, even Darwinism itself, has radically transformed our historical, scientific, and philosophical methods, is quite evident. Add to this the influence of the Hegelian philosophy, with its own doctrine of development, and one finds the causes of the rather striking unanimity which is discoverable in many ways between Hegelian idealists, on the one hand, and philosophers of evolution of Spencer's type, on the other. Although two men would, perhaps, not appear more radically different at first sight than Hegel and Spencer, I am inclined to believe that we shall come to recognize more and more in them an identity of philosophical conception. The pragmatism of the day is a striking confirmation of this opinion, for it is often the expression of Hegelian ideas in Darwinian and Spencerian terminology. The claims of idealism and of evolutionary science and philosophy have thus sought reconciliation. Logic has been, naturally, the last of the sciences to yield to evolutionary and genetic treatment. It could not escape long, especially when the idea of evolution had been so successful in its handling of ethics. If morality can be brought under the categories of evolution, why not thinking also? In answer to that question we have the theory that thinking is an adaptation, judgment is instrumental. But I would not leave the impression that this is true of pragmatism alone, or that it has been developed only through pragmatic tendencies. It is naturally the result also of the extension of biological philosophy. In the biological conception of logic, we have, then, an interesting coincidence in the results of tendencies differing widely in their genesis.

It would be hazardous to deny, without any qualifications, the importance of genetic considerations. Indeed, the fact that evolution in the hands of a thinker like Huxley, for instance, should make consciousness and thinking apparently useless epiphenomena in a developing world, has seemed like a most contradictory evolutionary philosophy. It was difficult to make consciousness a real function in development so long as it was regarded as only cognitive in character. Evolutionary philosophy, coupled with physics, had built up a sort of closed system with which consciousness could not interfere, but which it could know, and know with all the assurance of a traditional logic. If, however, we were to be consistent evolutionists, we could not abide by such a remarkable result. The whole process of thinking must be brought within evolution, so that knowledge, even the knowledge of the evolutionary hypothesis itself, must appear as an instance of adaptation. In order to do this, however, consciousness must not be conceived as only cognitive. Judgment, the core of logical processes, must be regarded as an instrument and as a mode of adaptation.

The desire for completeness and consistency in an evolutionary philosophy is not the only thing which makes the denial of genetic considerations hazardous. Strictly biological considerations furnish reasons of equal weight for caution. For instance, one will hardly deny that the whole sensory apparatus is a striking instance of adaptation. Our perceptions of the world would thus appear to be determined by this adaptation, to be instances of adjustment. They might conceivably have been different, and in the case of many other creatures, the perceptions of the world are undoubtedly different. All our logical processes, referring ultimately as they do to our perceptions, would thus appear finally to depend on the adaptation exhibited in the development of our sensory apparatus. So-called laws of thought would seem to be but abstract statements or formulations of the results of this adjustment. It would be absurd to suppose that a man thinks in a sense radically different from that in which he digests, or a flower blossoms, or that two and two are four in a sense radically different from that in which a flower has a given number of petals. Thinking, like digesting and blossoming, is an effect, a product, possibly a structure.

I am not at all interested in denying the force of these considerations. They have, to my mind, the greatest importance, and due weight has, as yet, not been given to them. To one at all committed to a unitary and evolutionary view of the world, it must indeed seem strange if thinking itself should not be the result of evolution, or that, in thinking, parts of the world had not become adjusted in a new way. But while I am ready to admit this, I am by no means ready to admit some of the conclusions for logic and metaphysics which are often drawn from the admission. Just because thought, as a product of evolution, is functional and judgment instrumental, it by no means follows that logic is but a branch of biology, or that knowledge of the world is but a temporary adjustment, which, as knowledge, might have been radically different. In these conclusions, often drawn with Protagorean assurance, two considerations of crucial importance seem to be overlooked, first, that adaptation is itself metaphysical in character, and secondly, that while knowledge may be functional and judgment instrumental, the character of the functioning has the character of knowledge, which sets it off sharply from all other functions.

It seems strange to me that the admission that knowledge is a matter of adaptation, and thus a relative matter, should, in these days, be regarded as in any way destroying the claims of knowledge to metaphysical certainty. Yet, somehow, the opinion widely prevails that the doctrine of relativity necessarily involves the surrender of anything like absolute truth. "All our knowledge is relative, and, therefore, only partial, incomplete, and but practically trustworthy," is a statement repeatedly made. The fact that, if our development had been different, our knowledge would have been different, is taken to involve the conclusion that our knowledge cannot possibly disclose the real constitution of things, that it is essentially conditional, that it is only a mental device for getting results, that any other system of knowledge which would get results equally well would be equally true; in short, that there can be no such thing as metaphysical or epistemological truth. These conclusions do indeed seem strange, and especially strange on the basis of evolution. For while the evolutionary process might, conceivably, have been different, its results are, in any case, the results of the process. They are not arbitrary. We might have digested without stomachs, but the fact that we use stomachs in this important process ought not to free us from metaphysical respect for the organ. As M. Rey suggests, in the *Revue Philosophique* for June, 1904, a creature without the sense of smell would have no geometry, but that does not make geometry essentially hypothetical, a mere mental construction; for *we* have geometry because of the working out of nature's laws. Indeed, instead of issuing in a relativistic metaphysics of knowledge, the doctrine of relativity should issue in the recognition of the finality of knowledge in every case of ascertainably complete adaptation. In other words, adaptation is itself metaphysical in character. Adjustment is always adjustment between things, and yields only what it does yield. The things or elements get into the state which is their adjustment, and this adjustment purports to be their actual and unequivocal ordering in relation to one another. Different conditions might have produced a different ordering, but, again, this ordering would be equally actual and unequivocal, equally the *one* ordering to issue from them. To suppose or admit that the course of events might have been and might be different is not at all to suppose or admit that it was or is different; it is, rather, to suppose and admit that we have real knowledge of what that course really was and is. This seems to be very obvious.

Yet the evolutionist often thinks that he is not a metaphysician, even when he brings all his conceptions systematically under the conception of evolution. This must be due to some temporary lack of clearness. If evolution is not a metaphysical doctrine when extended to apply to all science, all morality, all logic, in short, all things, then it is quite meaningless for evolutionists to pronounce a metaphysical sentence on logical processes. But if evolution is a metaphysics, then its sentence is metaphysical, and in every case of adjustment or adaptation we have a revelation of the nature of reality in a definite and unequivocal form. This conclusion applies to logical processes as well as to others. The recognition that they are vital processes can, therefore, have little significance for these processes in their distinctive character as logical. They are like all other vital processes in that they are vital and subject to evolution. They are unlike all others in that thought is unlike digestion or breathing. To regard logical processes as vital processes does not in any way, therefore, invalidate them as logical processes or make it superfluous to consider their claim to give us real knowledge of a real world. Indeed, it makes such a consideration more necessary and important.

A second consideration overlooked by the Protagorean tendencies of the day is that judgment, even if it is instrumental, purports to give us knowledge, that is, it claims to reveal what is independent of the judging process. Perhaps I ought not to say that this consideration is overlooked, but rather that it is denied significance. It is even denied to be essential to judgment. It is claimed that, instead of revealing anything independent of the judging process, judgment is just the adjustment and no more. It is a reorganization of experience, an attempt at control. All this looks to me like a misstatement of the facts. Judgment *claims* to be no such thing. It does not function as such a thing. When I make any judgment, even the simplest, I may make it as the result of tension, because of a demand for reorganization, in order to secure control of experience; but the judgment *means* for me something quite different. It means decidedly and unequivocally that in reality, apart from the judging process, things exist and operate just as the judgment declares. If it is claimed that this meaning is illusory, I eagerly desire to know on what solid ground its illusoriness can be established. When the conclusion was reached that gravitation varies directly as the mass and inversely as the square of the distance, it was doubtless reached in an evolutionary and pragmatic way; but it claimed to disclose a fact which prevailed before the conclusion was reached, and in spite of the conclusion. Knowledge has been born of the travail of living, but it has been born as knowledge.

When the knowledge character of judgment is insisted on, it seems almost incredible that any one would think of denying or overlooking it. Indeed, current discussions are far from clear on the subject. Pragmatists are constantly denying that they hold the conclusions that their critics almost unanimously draw. There is, therefore, a good deal of confusion of thought yet to be dispelled. Yet there seems to be current a pronounced determination to banish the epistemological problem from logic. This is, to my mind, suspicious, even when epistemology is defined in a way which most epistemologists would not approve. It is suspicious just because we must always ask eventually that most epistemological and metaphysical question: "Is knowledge true?" To answer, it is true when it functions in a way to satisfy the needs which generated its activity, is, no doubt, correct, but it is by no means adequate. The same answer can be made to the inquiry after the efficiency of any vital process whatever, and is, therefore, not distinctive. We have still to inquire into the specific character of the needs which originate judgments and of the consequent satisfaction. Just here is where the uniqueness of the logical problem is disclosed. With conscious beings, the success of the things they do has become increasingly dependent on their ability to discover what takes place in independence of the knowing process. That is the need which generates judgment. The satisfaction is, of course, the attainment of the discovery. Now to make the judgment itself and not the consequent action the instrumental factor seems to me to misstate the facts of the case. Nothing is clearer than that there is no necessity for knowledge to issue in adjustment. And it is clear to me that increased control of experience, while resulting from knowledge, does not give to it its character. Omniscience could idly view the transformations of reality and yet remain omniscient. Knowledge works, but it is not, therefore, knowledge.

These considerations have peculiar force when applied to that branch of knowledge which is knowledge itself. Is the biological account of knowledge correct? That question we must evidently ask, especially when we are urged to accept the account. Can we, to put the question in its most general form, accept as an adequate account of the logical process a theory which is bound up with some other specific department of human knowledge? It seems to me that we cannot. Here we must be epistemologists and metaphysicians, or give up the problem entirely. This by no means involves the attempt to conceive pure thought set over against pure reality — the kind of epistemology and metaphysics justly ridiculed by the pragmatist — for knowledge, as already stated, is given to us in concrete instances. How knowledge in general is possible is, therefore, as useless and meaningless a question as how reality in general is possible. The knowledge is given as a fact of life, and what we have to determine is not its non-logical antecedents or its practical consequences, but its constitution as knowledge and its validity. It may be admitted that the question of validity is settled pragmatically. No knowledge is true unless it yields results which can be verified, unless it *can* issue in increased control of experience. But I insist again that that fact is not sufficient for an account of what knowledge claims to be. It claims to issue in control because it is true in independence of the control. And it is just this assurance that is needed to distinguish knowledge from what is not knowledge. It is the necessity of exhibiting this assurance which makes it impossible to subordinate logical problems, and forces us at last to questions of epistemology and metaphysics.

As I am interested here primarily in determining the field of logic, it is somewhat outside my province to consider the details of logical theory. Yet the point just raised is of so much importance in connection with the main question that I venture the following general considerations. This is, perhaps, the more necessary because the pragmatic doctrine finds in the concession made regarding the test of validity one of its strongest defenses.

Of course a judgment is not true simply because it is a judgment. It may be false. The only way to settle its validity is to discover whether experience actually provides what the judgment promises, that is, whether the conclusions drawn from it really enable us to control experience. No mere speculation will yield the desired result, no matter with how much formal validity the conclusions may be drawn. That merely formal validity is not the essential thing, I have pointed out in discussing the relation of logic to mathematics. The test of truth is pragmatic. It is apparent, therefore, that the formal validity does not determine the actual validity. What is this but the statement that the process of judgment is not itself the determining factor in its real validity? It is, in short, only valid judgments that can really give us control of experience. The implications taken up in the judgment must, therefore, be real implications which, as such, have nothing to do with the judging process, and which, most certainly, are not brought about by it. And what is this but the claim that judgment as such is never instrumental? In other words, a judgment which effected its own content would only by the merest accident function as valid knowledge. We have valid knowledge, then, only when the implications of the judgment are found to be independent of the judging process. We have knowledge only at the risk of error. The pragmatic test of validity, instead of proving the instrumental character of judgment, would thus appear to prove just the reverse.

Valid knowledge has, therefore, for its content a system of real, not judged or hypothetical implications. The central problem of logic which results from this fact is not how a knowledge of real implications is then possible, but what are the ascertainable types of real implications. But, it may be urged, we need some criterion to determine what a real implication is. I venture to reply that we need none, if by such is meant anything else than the facts with which we are dealing. I need no other criterion than the circle to determine whether its diameters are really equal. And, in general, I need no other criterion than the facts dealt with to determine whether they really imply what I judge them to imply. Logic appears to me to be really as simple as this. Yet there can be profound problems involved in the working out of this simple procedure. There is the problem already stated of the most general types of real implication, or, in other words, the time-honored doctrine of categories. Whether there are categories or basal types of existence seems to me to be ascertainable. When ascertained, it is also possible to discover the types of inference or implication which they afford. This is by no means the whole of logic, but it appears to me to be its central problem.

These considerations will, I hope, throw light on the statement that while knowledge works, it is not therefore knowledge. It works because its content existed before its discovery by the knowledge process, and because its content was not effected or brought about by that process. Judgment was the instrument of its discovery, not the instrument which fashioned it. While, therefore, willing to admit that logical processes are vital processes, I am not willing to admit that the problem of logic is radically changed thereby in its formulation or solution, for the vital processes in question have the unique character of knowledge, the content of which is what it claims to be, a system of real implications which existed prior to its discovery.

In the psychological and biological tendencies in logic, there is, however, I think, a distinct gain for logical theory. The insistence that logical processes are both mental and vital has done much to take them out of the transcendental aloofness from reality in which they have often been placed, especially since Kant. So long as thought and object were so separated that they could never be brought together, and so long as logical processes were conceived wholly in terms of ideas set over against objects, there was no hope of escape from the realm of pure hypothesis and conjecture. Locke's axiom that "the mind, in all its thoughts and reasonings, hath no other immediate object but its own ideas," an axiom which Kant did so much to sanctify, and which has been the basal principle of the greater part of modern logic and metaphysics, is most certainly subversive of logical theory. The transition from ideas to anything else is rendered impossible by it. Now it is just this axiom which the biological tendencies in logic have done so much to destroy. They have insisted, with the greatest right, that logical processes are not set over against their content as idea against object, as appearance against reality, but are processes of reality itself. Just as reality can and does function in a physical or a physiological way, so also it functions in a logical way. The state we call knowledge becomes, thus, as much a part of the system of things as the state we call chemical combination. The problem how thought can know anything becomes, therefore, as irrelevant as the problem how elements can combine at all. The recognition of this is a great gain, and the promise of it most fruitful for both logic and metaphysics.

But, as I have tried to point out, all this surrendering of pure thought as opposed to pure reality, does not at all necessitate our regarding judgment as a process which makes reality different from what it was before. Of course there is one difference, namely, the logical one; for reality prior to logical processes is unknown. As a result of these processes it becomes known. These processes are, therefore, responsible for a known as distinct from an unknown reality. But what is the transformation which reality undergoes in becoming known? When it becomes known that water seeks its own level, what change has taken place in the water? It would appear that we must answer, none. The water which seeks its own level has not been transformed into ideas or even into a human experience. It appears to remain, as water, precisely what it was before. The transformation which takes place, takes place in the one who knows, a transformation from ignorance to knowledge. Psychology and biology can afford us the natural history of this transformation, but they cannot inform us in the least as to why it should have its specific character. That is given and not deduced. The attempts to deduce it have, without exception, been futile. That is why we are forced to take it as ultimate in the same way we take as ultimate the specific character of any definite transformation. To my mind, there is needed a fuller and more cordial recognition of this fact. The conditions under which we, as individuals, know are certainly discoverable, just as much as the conditions under which we breathe or digest. And what happens to things when we know them is also as discoverable as what happens to them when we breathe them or digest them.

But here the idealist may interpose that we can never know what happens to things when we know them, because we can never know them before they become known. I suppose I ought to wrestle with this objection. It is an obvious one, but, to my mind, it is without force. The objection, if pursued, can carry us only in a circle. The problem of knowledge is still on our hands, and every logician of whatever school, the offerer of this objection also, has, nevertheless, attempted to show what the transformation is that thought works, for all admit that it works some. Are we, therefore, engaged in a hopeless task? Or have we failed to grasp the significance of our problem? I think the latter. We fail to recognize that, in one way or other, we do solve the problem, and that our attempts to solve it show quite clearly that the objection under consideration is without force. Take, for instance, any concrete case of knowledge, the water seeking its own level, again. Follow the process of knowledge to the fullest extent, we never find a single problem which is not solvable by reference to the concrete things with which we are dealing, nor a single solution which is not forced upon us by these things rather than by the fact that we deal with them. The transformation wrought is thus discovered, in the progress of knowledge itself, to be wrought solely in the inquiring individual, and wrought by repeated contact with the things with which he deals. In other words, all knowledge discloses the fact that its content is not created by itself, but by the things with which it is concerned.

It is quite possible, therefore, that knowledge should be what we call transcendent and yet not involve us in a transcendental logic. That we should be able to know without altering the things we know is no more and no less remarkable and mysterious than that we should be able to digest by altering the things we digest. In other words, the fact that digestion alters the things is no reason that knowledge should alter them, even if we admit that logical processes are vital and subject to evolution. Indeed, if evolution teaches us anything on this point, it is that knowledge processes are real just as they exist, as real as growth and digestion, and must have their character described in accordance with what they are. The recognition that knowledge can be transcendent and yet its processes vital seems to throw light on the difficulty evolution has encountered in accounting for consciousness and knowledge. All the reactions of the individual seem to be expressible in terms of chemistry and physics without calling in consciousness as an operating factor. What is this but the recognition of its transcendence, especially when the conditions of conscious activity are quite likely expressible in chemical and physical terms? While, therefore, biological considerations result in the great gain of giving concrete reality to the processes of knowledge, the gain is lost, if knowledge itself is denied the transcendence which it so evidently discloses.

IV

The argument advanced in this discussion has had the aim of emphasizing the fact that in knowledge we have actually given, as content, reality as it is in independence of the act of knowing, that the real world is self-existent, independent of the judgments we make about it. This fact has been emphasized in order to confine the field of logic to the field of knowledge as thus understood. In the course of the argument, I have occasionally indicated what some of the resulting problems of logic are. These I wish now to state in a somewhat more systematic way.

The basal problem of logic becomes, undoubtedly, the metaphysics of knowledge, the determination of the nature of knowledge and its relation to reality. It is quite evident that this is just the problem which the current tendencies criticised have sought, not to solve, but to avoid or set aside. Their motives for so doing have been mainly the difficulties which have arisen from the Kantian philosophy in its development into transcendentalism, and the desire to extend the category of evolution to embrace the whole of reality, knowledge included. I confess to feeling the force of these motives as strongly as any advocate of the criticised opinions. But I do not see my way clear to satisfying them by denying or explaining away the evident character of knowledge itself. It appears far better to admit that a metaphysics of knowledge is as yet hopeless, rather than so to transform knowledge as to get rid of the problem; for we must ultimately ask after the truth of the transformation. But I am far from believing that a metaphysics of knowledge is hopeless. The biological tendencies themselves seem to furnish us with much material for at least the beginnings of one. Reality known is to be set over against reality unknown or independent of knowledge, not as image to original, idea to thing, phenomena to noumena, appearance to reality; but reality as known is a new stage in the development of reality itself. It is not an external mind which knows reality by means of its own ideas, but reality itself becomes known through its own expanding and readjusting processes. So far I am in entire agreement with the tendencies I have criticised. But what change is effected by this expansion and readjustment? I can find no other answer than this simple one: the change to knowledge. And by this I mean to assert unequivocally that the addition of knowledge to a reality hitherto without it is simply an addition to it and not a transformation of it. Such a view may appear to make knowledge a wholly useless addition, but I see no inherent necessity in such a conclusion. Nor do I see any inherent necessity of supposing that knowledge must be a useful addition. Yet I would not be so foolish as to deny the usefulness of knowledge. We have, of course, the most palpable evidences of its use. As we examine them, I think we find, without exception, that knowledge is useful just in proportion as we find that reality is not transformed by being known. If it really were transformed in that process, could anything else than confusion result from the multitude of knowing individuals?

To me, therefore, the metaphysics of the situation resolves itself into the realistic position that a developing reality develops, under ascertainable conditions, into a known reality without undergoing any other transformation, and that this new stage marks an advance in the efficiency of reality in its adaptations. My confidence steadily grows that this whole process can be scientifically worked out. It is impossible here to justify my confidence in detail, and I must leave the matter with the following suggestion. The point from which knowledge starts and to which it ultimately returns is always some portion of reality where there is consciousness, the things, namely, which, we are wont to say, are in consciousness. These things are not ideas representing other things outside of consciousness, but real things, which, by being in consciousness, have the capacity of representing *each other*, of standing for or implying each other. Knowledge is not the creation of these implications, but their successful systematization. It will be found, I think, that this general statement is true of every concrete case of knowledge which we possess. Its detailed working out would be a metaphysics of knowledge, an epistemology.

Since knowledge is the successful systematization of the implications which are disclosed in things by virtue of consciousness, a second logical problem of fundamental importance is the determination of the most general types of implication with the categories which underlie them. The execution of this problem would naturally involve, as subsidiary, the greater part of formal and symbolic logic. Indeed, vital doctrines of the syllogism, of definition, of formal inference, of the calculus of classes and propositions, of the logic of relations, appear to be bound up ultimately with a doctrine of categories; for it is only a recognition of basal types of existence with their implications that can save these doctrines from mere formalism. These types of existence or categories are not to be regarded as free creations or as the contributions of the mind to experience. There is no deduction of them possible. They must be discovered in the actual progress of knowledge itself, and I see no reason to suppose that their number is necessarily fixed, or that we should necessarily be in possession of all of them. It is requisite, however, that in every case categories should be incapable of reduction to each other.

A doctrine of categories seems to me to be of the greatest importance in the systematization of knowledge, for no problem of relation is even stateable correctly before the type of existence to which its terms belong has been first determined. I submit one illustration to reinforce this general statement, namely, the relation of mind to body. If mind and body belong to the same type of existence, we have one set of problems on our hands; but if they do not, we have an entirely different set. Yet volumes of discussion written on this subject have abounded in confusion, simply because they have regarded mind and body as belonging to radically different types of existence and yet related in terms of the type to which one of them belongs. The doctrine of parallelism is, perhaps, the epitome of this confusion.

The doctrine of categories will involve not only the greater part of formal and symbolic logic, but will undoubtedly carry the logician into the doctrine of method. Here it is to be hoped that recent tendencies will result in effectively breaking down the artificial distinctions which have prevailed between deduction and induction. Differences in method do not result from differences in points of departure, or between the universal and the particular, but from the categories, again, which give the method direction and aim, and result in different types of synthesis. In this direction, the logician may hope for an approximately correct classification of the various departments of knowledge. Such a classification is, perhaps, the ideal of logical theory.

SECTION D — METHODOLOGY OF SCIENCE

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(*Hall* 6, *September* 22, 3 *p. m.*)

CHAIRMAN: PROFESSOR JAMES E. CREIGHTON, Cornell University.
SPEAKERS: PROFESSOR WILHELM OSTWALD, University of Leipzig.
PROFESSOR BENNO ERDMANN, University of Bonn.
SECRETARY: DR. R. B. PERRY, Harvard University.

ON THE THEORY OF SCIENCE

BY WILHELM OSTWALD

(*Translated from the German by Dr. R. M. Yerkes, Harvard University*)

[**Wilhelm Ostwald**, Professor of Physical Chemistry, University of Leipzig, since 1887. **b.** September 2, 1853, Riga, Russia. **Grad.** Candidate Chemistry, 1877; Master Chemistry, 1878; Doctor Chemistry, Dorpat. Dr. Hon. Halle and Cambridge; Privy Councilor; Assistant, Dorpat, 1875–81; Regular Professor, Riga, 1881–87. Member various learned and scientific societies. **Author of** *Manual of General Chemistry; Electro Chemistry; Foundation of Inorganic Chemistry; Lectures on Philosophy of Nature; Artist's Letters; Essays and Lectures;* and many other noted works and papers on Chemistry and Philosophy.]

ONE of the few points on which the philosophy of to-day is united is the knowledge that the only thing completely certain and undoubted for each one is the content of his own consciousness; and here the certainty is to be ascribed not to the content of consciousness in general, but only to the momentary content.

This momentary content we divide into two large groups, which we refer to the inner and outer world. If we call any kind of content of consciousness an experience, then we ascribe to the outer world such experiences as arise without the activity of our will and cannot be called forth by its activity alone. Such experiences never arise without the activity of certain parts of our body, which we call sense organs. In other words, the outer world is that which reaches our consciousness through the senses.

On the other hand, we ascribe to our inner world all experiences which arise without the immediate assistance of a sense organ. Here, first of all, belong all experiences which we call remembering and thinking. An exact and complete differentiation of the two territories is not intended here, for our purpose does not demand that this task be undertaken. For this purpose the general orientation in which every one recognizes familiar facts of his consciousness is sufficient.

Each experience has the characteristic of uniqueness. None of us doubts that the expression of the poet "Everything is only repeated in life" is really just the opposite of the truth, and that in fact nothing is repeated in life. But to express such a judgment we must be in position to compare different experiences with each other, and this possibility rests upon a fundamental phenomenon of our consciousness, memory. Memory alone enables us to put various experiences in relation to each other, so that the question as to their likeness or difference can be asked.

We find the simpler relations here in the inner experiences. A certain thought, such as twice two is four, I can bring up in my consciousness as often as I wish, and in addition to the content of the thought I experience the further consciousness that I have already had this thought before, that it is familiar to me.

A similar but somewhat more complex phenomenon appears in the experiences in which the outer world takes part. After I have eaten an apple, I can repeat the experience in two ways. First, as an inner experience, I can remember that I have eaten the apple and by an effort of my will I can re-create in myself, although with diminished strength and intensity, a part of the former experience — the part which belonged to my inner world. Another part, the sense impression which belonged to that experience, I cannot re-create by an effort of my will, but I must again eat an apple in order to have a similar experience of this sort. This is a complete repetition of the experience to which the external world also contributes. Such a repetition does not depend altogether on my own powers, for it is necessary that I have an apple, that is, that certain conditions which are independent of me and belong to the outer world be fulfilled.

Whether the outer world takes part in the repetition of an experience or not has no influence upon the possibility of the content of consciousness which we call memory. From this it follows that this content depends upon the inner experience alone, and that we remember an external event only by means of its inner constituents. The mere repetition of corresponding sense impressions is not sufficient for this, for we can see the same person repeatedly without recognizing him, if the inner accompanying phenomena were so insignificant, as a result of lack of interest, that their repetition does not produce the content of consciousness known as memory. If we see him quite frequently, the frequent repetition of the external impression finally causes the memory of the corresponding inner experience.

From this it results that for the "memory"-reaction a certain intensity of the inner experience is necessary. This threshold can be attained either at once or by continued repetition. The repetitions are the more effective the more rapidly they follow each other. From this we may conclude that the memory-value of an experience, or its capacity for calling forth the "memory"-reaction by repetition, decreases with the lapse of time. Further, we must consider the fact mentioned above, that an experience is never exactly repeated, and that therefore the "memory"-reaction occurs even where there is only resemblance or partial agreement in place of complete agreement. Here, too, there are different degrees; memory takes place more easily the more perfectly the two experiences agree, and *vice versa*.

If we look at these phenomena from the physiological side, we may say we have two kinds of apparatus or organs, one of which does not depend upon our will, whereas the other does. The former are the sense organs, the latter constitutes the organ of thought. Only the activities of the latter constitute our experiences or the content of our consciousness.

The activities of the former may call forth the corresponding processes of the latter, but this is not always necessary. Our sense organs can be influenced without our "noticing" it, that is, without the thinking apparatus being involved. An especially important reaction of the thinking apparatus is memory, that is, the consciousness that an experience which we have just had possesses more or less agreement with former experiences. With reference to the organ of thought, it is the expression of the general physiological fact that every process influences the organ in such a way that it has a different relation to the repetition of this process, from the first time, and moreover that the repetition is rendered easier. This influence decreases with time.

It is chiefly upon these phenomena that experience rests. Experience results from the fact that all events consist of a complete series of simultaneous and successive components. When a connection between some of those parts has become familiar to us by the repetition of similar occurrences (for instance, the succession of day and night), we do not feel such an occurrence as something completely new, but as something partially familiar, and the single parts or phases of it do not surprise us, but rather we anticipate their coming or expect them. From expectation to prediction is only a short step, and so experience enables us to prophesy the future from the past and present.

Now this is also the road to science; for science is nothing but systematized experience, that is, experience reduced to its simplest and clearest forms. Its purposes to predict from a part of a phenomenon which is known another part which is not yet known. Here it may be a question of spatial as well as of temporal phenomena. Thus the scientific zoölogist knows how to "determine," that is, to tell, from the skull of an animal, the nature of the other parts of the animal to which the skull belongs; likewise the astronomer is able to indicate the future situation of a planet from a few observations of its present situation; and the more exact the first observations were, the more distant the future for which he can predict. All such scientific predictions are limited, therefore, with reference to their number and their accuracy. If the skull shown to the zoölogist is that of a chicken, then he will probably be able to indicate the general characteristics of chickens, and also perhaps whether the chicken had a top-knot or not; but not its color, and only uncertainly its age and its size. Both facts, the possibility of prediction and its limitation in content and amount, are an expression for the two fundamental facts, that among our experiences there is similarity, but not complete agreement.

The foregoing considerations deserve to be discussed and extended in several directions. First, the objection will be made that a chicken or a planet is not an experience; we call them rather by the most general name of thing. But our knowledge of the chicken begins with the experiencing of certain visual impressions, to which are added, perhaps, certain impressions of hearing and touch. The sight impressions (to discuss these first) by no means completely agree. We see the chicken large or small, according to the distance; and according to its position and movement its outline is very different. As we have seen, however, these differences are continually grading into one other and do not reach beyond certain limits; we neglect to observe them and rest contented with the fact that certain other peculiarities (legs, wings, eyes, bill, comb, etc.) remain and do not change. The constant properties we group together as a thing, and the changing ones we call the states of this thing. Among the changing properties, we distinguish further those which depend upon us (for example, the distance) and those upon which we have no immediate influence (for instance, the position or motion): the first is called the subjective changeable part of our experience, while the second is called the objective mutability of the thing.

This omission of both the subjectively and objectively changeable portion of the experience in connection with the retention of the constant portion and the gathering together of the latter into a unity is one of the most important operations which we perform with our experiences. We call it the process of abstraction, and its product, the permanent unity, we call a concept. Plainly this procedure contains arbitrary as well as necessary factors. Arbitrary or accidental is the circumstance that quite different phases of a given experience come to consciousness according to our attention, the amount of practice we have had, indeed according to our whole intellectual nature. We may overlook constant factors and attend to changeable ones. The objective factors, however, become necessary as soon as we have noticed them; after we have seen that the chicken is black, it is not in our power to see it red. Accordingly, in general, our knowledge of that which agrees must be less than it actually could be, since we have not been able to observe every agreement, and our concept is always poorer in constituents at any given time than it might be. To seek out such elements of concepts as have been overlooked, and to prove that they are necessary factors of the corresponding experiences, is one of the never-ending tasks of science. The other case, namely, that elements have been received in the concept which do not prove to be constant, also happens, and leads to another task. One can then leave that element out of the concept, if further experiences show that the other elements are found in them, or one can form a new concept which contains the former elements, leaving out those that have been recognized as unessential. For a long time the white color belonged to the concept swan. When the Dutch black swans became known, it was possible either to drop the element white from the concept swan (as actually happened), or to make a new concept for the bird which is similar to the swan but black. Which choice is made in a given case is largely arbitrary, and is determined by considerations of expediency.

Into the formation of concepts, therefore, two factors are operative, an objective empirical factor, and a subjective or purposive factor. The fitness of a concept is seen in relation to its purpose, which we shall now consider.

The purpose of a concept is its use for prediction. The old logic set up the syllogism as the type of thought-activity, and its simplest example is the well-known

> All men are mortal,
> Caius is a man,
> Therefore Caius is mortal.

In general, the scheme runs

> To the concept M belongs the element B,
> C belongs under the concept M,
> Therefore the element B is found in C.

One can say that this method of reasoning is in regular use even to this day. It must be added, however, that this use is of a quite different nature from that of the ancients. Whereas formerly the setting up of the first proposition or the major premise was considered the most important thing, and the establishment of the second proposition or minor premise was thought to be a rather trifling matter, now the relation is reversed. The major premise contains the description of a concept, the minor makes the assertion that a certain thing belongs under this concept. What right exists for such an assertion? The most palpable reply would be, since all the elements of the concept M (including B) are found in C, C belongs under the concept M. Such a conclusion would indeed be binding, but at the same time quite worthless, for it only repeats the minor premise. Actually the method of reasoning is essentially different, for the minor premise is not obtained by showing that all the elements of the concept M are found in C, but only some of them. The conclusion is not necessary, but only probable, and the whole process of reasoning runs: Certain elements are frequently found together, therefore they are united in the concept M. Certain of these elements are recognized in the thing C, therefore probably the other elements of the concept M will be found in C.

The old logic, also, was familiar with this kind of conclusion. It was branded, however, as the worst of all, by the name of incomplete induction, since the absolute certainty demanded of the syllogism did not belong to its results. One must admit, however, that the whole of modern science makes use of no other form of reasoning than incomplete induction, for it alone admits of a prediction, that is, an indication of relations which have not been immediately observed.

How does science get along with the defective certainty of this process of reasoning? The answer is, that the probability of the conclusion can run through all degrees from mere conjecture to the maximum probability, which is practically indistinguishable from certainty. The probability is the greater the more frequently an incomplete induction of this kind has proven correct in later experience. Accordingly we have at our command a number of expressions which in their simplest and most general form have the appearance: If an element A is met within a thing, then the element B is also found in it (in spatial or temporal relationship).

If the relation is temporal, this general statement is known by some such name as the law of causality. If it is spatial, one talks of the idea (in the Platonic sense), or the type of the thing, of substance, etc.

From the considerations here presented we get an easy answer to many questions which are frequently discussed in very different senses. First, the question concerning the general validity of the law of causality. All attempts to prove such a validity have failed, and there has remained only the indication that without this law we should feel an unbearable uncertainty in reference to the world. From this, however, we see very plainly that here it is merely a question of expediency. From the continuous flux of our experiences we hunt out those groups which can always be found again, in order to be able to conclude that if the element A is given, the element B will be present. We do not find this relationship as "given," but we put it into our experiences, in that we consider the parts which correspond to the relationship as belonging together.

The very same thing may be said of spatial complexes. Such factors as are always, or at any rate often, found together are taken by us as "belonging together," and out of them a concept is formed which embraces these factors. A question as to the why has here, as with the temporal complexes, no definite meaning. There are countless things that happen together once to which we pay no attention because they happen only once or but seldom. The knowledge of the fact that such a single concurrence exists amounts to nothing, since from the presence of one factor it does not lead to a conclusion as to the presence of another, and therefore does not make possible prediction. Of all the possible, and even actual combinations, only those interest us which are repeated, and this arbitrary but expedient selection produces the impression that the world consists only of combinations that can be repeated; that, in other words, the law of causality or of the type is a general one. However general or limited application these laws have, is more a question of our skill in finding the constant combinations among those that are present than a question of objective natural fact.

Thus we see the development and pursuit of all sciences going on in such a way that on the one hand more and more constant combinations are discovered, and on the other hand more inclusive relations of this kind are found out, by means of which elements are united with each other which before no one had even tried to bring together. So sciences are increasing both in the sense of an increasing complication and in an increasing unification.

If we consider from this standpoint the development and procedure of the various sciences, we find a rational division of the sum total of science in the question as to the scope and multiplicity of the combinations or groups treated of in them. These two properties are in a certain sense antithetical. The simpler a complex is, that is, the fewer elements brought together in it, the more frequently it is met with, and *vice versa*. One can therefore arrange all the sciences in such a way that one begins with the least multiplicity and the greatest scope, and ends with the greatest multiplicity and the least scope. The first science will be the most general, and will therefore contain the most general and therefore the most barren concepts; the last will contain the most specific and therefore the richest.

What are these limiting concepts? The most general is the concept of *thing*, that is, any piece of experience, seized arbitrarily from the flux of our experiences, which can be repeated. The most specific and richest is the concept of *human intercourse*. Between the science of things and the science of human intercourse, all the other sciences are found arranged in regular gradation. If one follows out the scheme the following outline results:

$$\left.\begin{array}{l}\text{1. Theory of order.}\\ \text{2. Theory of numbers, or arithmetic.}\\ \text{3. Theory of time.}\\ \text{4. Theory of space, or geometry.}\end{array}\right\}\text{Mathematics.}$$

5. Mechanics. } Energetics.
6. Physics.
7. Chemistry.
8. Physiology. } Biology.
9. Psychology.
10. Sociology.

This table is arbitrary in so far as the grades assumed can be increased or diminished according to need. For example, mechanics and physics could be taken together; or between physics and chemistry, physical chemistry could be inserted. Likewise between physiology and psychology, anthropology might find a place; or the first five sciences might be united under mathematics. How one makes these divisions is entirely a practical question, which will be answered at any time in accordance with the purposes of division; and dispute concerning the matter is almost useless.

I should like, however, to call attention to the three great groups of mathematics, energetics, and biology (in the wider sense). They represent the decisive regulative thought which humanity has evolved, contributed up to this time, toward the scientific mastery of its experiences. Arrangement is the fundamental thought of mathematics. From mechanics to chemistry the concept of energy is the most important; and for the last three sciences it is the concept of life. Mathematics, energetics, and biology, therefore, embrace the totality of the sciences.

Before we enter upon the closer consideration of these sciences, it will be well to anticipate another objection which can be raised on the basis of the following fact. Besides the sciences named (and those which lie between them) there are many others, as geology, history, medicine, philology, which we find difficulty in arranging in the above scheme, which must, however, be taken into consideration in some way or other. They are often characterized by the fact that they stand in relation with several of the sciences named, but even more by the following circumstance. Their task is not, as is true of the pure sciences above named, the discovery of general relationships, but they relate rather to existing complex objects whose origin, scope, extent, etc., in short, whose temporal and spatial relationships they have to discover or to "explain." For this purpose they make use of relations which are placed at their disposal by the first-named pure sciences. These sciences, therefore, had better be called applied sciences. However, in this connection we should not think only or even chiefly of technical applications; rather the expression is used to indicate that the reciprocal relations of the parts of an object are to be called to mind by the application of the general rules found in pure science.

While in such a task the abstraction process of pure science is not applicable (for the omission of certain parts and the concentration upon others which is characteristic of these is excluded by the nature of the task), yet in a given case usually the necessity of bringing in various pure sciences for the purpose of explanation is evident.

Astronomy is one of these applied sciences. Primarily it rests upon mechanics, and in its instrumental portion, upon optics; in its present development on the spectroscopic side, however, it borrows considerably of chemistry. In like manner history is applied sociology and psychology. Medicine makes use of all the sciences before mentioned, up to psychology, etc. .

It is important to get clearly in mind the nature of these sciences, since, on account of their compound nature, they resist arrangement amongst the pure sciences, while, on account of their practical significance, they still demand consideration. The latter fact gives them also a sort of arbitrary or accidental character, since their development is largely conditioned by the special needs of the time. Their number, speaking in general, is very large, since each pure science may be turned into an applied science in various ways; and since in addition we have combinations of two, three, or more sciences. Moreover, the method of procedure in the applied sciences is fundamentally different from that in the pure sciences. In the first it is a question of the greatest possible analysis of a single given complex into its scientifically comprehensible parts; while pure science, on the other hand, considers many complexes together in order to separate out from them their common element, but expressly disclaims the complete analysis of a single complex.

In scientific work, as it appears in practice, pure and applied science are by no means sharply separated. On the one hand the auxiliaries of investigations, such as apparatus, books, etc., demand of the pure investigator knowledge and application in applied science; and, on the other hand, the applied scientist is frequently unable to accomplish his task unless he himself becomes for the time being a pure investigator and ascertains or discovers the missing general relationships which he needs for his task. A separation and differentiation of the two forms of science was necessary, however, since the method and the aim of each present essential differences.

In order to consider the method of procedure of pure science more carefully, let us turn back to the table on pages 339, 340, and attend to the single sciences separately. The theory of arrangement was mentioned first, although this place is usually assigned to mathematics. However, mathematics has to do with the concepts of number and magnitude as fundamentals, while the theory of arrangement does not make use of these. Here the fundamental concept is rather the thing or object of which nothing more is demanded or considered than that it is a fragment of our experience which can be isolated and will remain so. It must not be an arbitrary combination; such a thing would have only momentary duration, and the task of science, to learn the unknown from the given, could not find application. Rather must this element have such a nature that it can be characterized and recognized again, that is, it must already have a conceptual nature. Therefore only parts of our experience which can be repeated (which alone can be objects of science) can be characterized as things or objects. But in saying this we have said all that was demanded of them. In other respects they may be just as different as is conceivable.

If the question is asked, What can be said scientifically about indefinite things of this sort? it is especially the relations of arrangement and association which yield an answer. If we call any definite combination of such things a group, we can arrange such a group in different ways, that is, we can determine for each thing the relation in which it is to stand to the neighboring thing. From every such arrangement result not only the relationships indicated, but a great number of new ones, and it appears that when the first relationships are given the others always follow in like manner. This, however, is the type of the scientific proposition or natural law (page 335). From the presence of certain relations of arrangement we can deduce the presence of others which we have not yet demonstrated.

To illustrate this fact by an example, let us think of the things arranged in a simple row, while we choose one thing as a first member and associate another with it as following it; with the latter another is associated, etc. Thereby the position of each thing in the row is determined only in relation to the immediately preceding thing. Nevertheless, the position of every member in the whole row, and therefore its relation to every other member, is determined by this. This is seen in a number of special laws. If we differentiate former and latter members we can formulate the proposition, among others, if B is a later member with reference to A, and C with reference to B, then C is also a later member with reference to A.

The correctness and validity of this proposition seems to us beyond all doubt. But this is only a result of the fact that we are able to demonstrate it very easily in countless single cases, and have so demonstrated it. We know only cases which correspond to the proposition, and have never experienced a contradictory case. To call such a proposition, however, a necessity of thinking, does not appear to me correct. For the expression necessity of thinking can only rest upon the fact that every time the proposition is thought, that is, every time one remembers its demonstration, its confirmation always arises. But every sort of false proposition is also thinkable. An undeniable proof of this is the fact that so much which is false is actually thought. But to base the proof for the correctness of a proposition upon the impossibility of thinking its opposite is an impossible undertaking, because every sort of nonsense can be thought: where the proof was thought to have been given, there has always been a confusion of thought and intuition, proof or inspection.

With this one proposition of course the theory of order is not exhausted, for here it is not a question of the development of this theory, but of an example of the nature of the problems of science. Of the further questions we shall briefly discuss the problem of association.

If we have two groups A and B given, one can associate with every member of A one of B; that is, we determine that certain operations which can be carried on with the members of A are also to be carried on with those of B. Now we can begin by simply carrying out the association, member for member. Then we shall have one of three results: A will be exhausted while there are still members of B left, or B will be exhausted first, or finally A and B will be exhausted at the same time. In the first case we call A poorer than B; in the second B poorer than A; in the third both quantities are alike.

Here for the first time we come upon the scientific concept of equality, which calls for discussion. There can be no question of a complete identity of the two groups which have been denominated equal, for we have made the assumption that the members of both groups can be of any nature whatever. They can then be as different as possible, considered singly, but they are alike as groups. However I may arrange the members of A, I can make a similar arrangement of the members of B, since every member of A has one of B associated with it; and with reference to the property of arrangement there is no difference to be observed between A and B. If, however, A is poorer or richer than B, this possibility ceases, for then one of the groups has members to which none of the members in the other group corresponds; so that the operations carried out with these members cannot be carried out with those of the other group.

Equality in the scientific sense, therefore, means equivalence, or the possibility of substitution in quite definite operations or for quite definite relations. Beyond this the things which are called like may show any differences whatever. The general scientific process of abstraction is again easily seen in this special case.

On the basis of the definitions just given, we can establish further propositions. If group A equals B, and B equals C, then A also equals C. The proof of this is that we can relate every member of A to a corresponding member of B and by hypothesis no member will be left. Then C is arranged with reference to B, and here also no member is left. By this process every member of A, through the connecting link of a member of B, is associated with a member of C, and this association is preserved even if we cut out the group B. Therefore A and C are equal. The same process of reasoning can be carried out for any number of groups.

Likewise it can be demonstrated that if A is poorer than B and B poorer than C, then A is also poorer than C. For in the association of B with A some members of B are left over by hypothesis, and likewise some members of C are left over if one associates C with B. Therefore in the association of C with A, not only those members are left over which could not be associated with B, but also those members of C which extend beyond B. This proposition can be extended to any number of groups, and permits the arrangement of a number of different groups in a simple series by beginning with the poorest and choosing each following so that it is richer than the preceding but poorer than the following. From the proposition just established, it follows that every group is so arranged with reference to all other groups that it is richer than all the preceding and poorer than all the following.[1]

In this derivation of scientific proposition or laws of the simplest kinds, the process of derivation and the nature of the result becomes particularly clear. We arrive at such a proposition by performing an operation and expressing the result of it. This expression enables us to avoid the repetition of the operation in the future, since in accordance with the law we can indicate the result immediately. Thus an abbreviation and therefore a facilitation of the problem is attained which is the more considerable the larger the number of operations saved.

If we have a number of equal groups, we know by the process of association that all of the operations with reference to arrangement which we can perform with one of them can be performed with all the others. It is sufficient, therefore, to determine the properties of arrangement of one of these groups in order to know forthwith the properties of all the others. This is an extremely important proposition, which is continually employed for the most various purposes. All speaking, writing, and reading rests upon the association of thoughts with sounds and symbols, and by arranging the signs in accordance with our thoughts we bring it to pass that our hearers or readers think like thoughts in like order. In a similar fashion we make use of various systems of formulæ in the different sciences, especially in the simpler sciences; and these formulæ we correlate with phenomena and use in place of the phenomena themselves, and can therefore derive from them certain characteristics of phenomena without being compelled to use the latter. The force of this process appears very strikingly in astronomy where, by the use of definite formulæ associated with the different heavenly bodies, we can foretell the future positions of these bodies with a high degree of approximation.

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[1] Equal groups cannot be distinguished here, and therefore represent only a group.

From the theory of order we come to the theory of number or arithmetic by the systematic arrangement or development of an operation just indicated (page 343). We can arrange any number of groups in such a way that a richer always follows a poorer. But the complex obtained in this manner is always accidental with reference to the number and the richness of its members. A regular and complete structure of all possible groups is evidently obtained only if we start from a group of one member or from a simple thing, and by the addition of one member at a time make further groups out of those that we have. Thus we obtain different groups arranged according to an increasing richness, and since we have advanced one member at a time, that is, made the smallest step which is possible, we are certain that we have left out no possible group which is poorer than the richest to which the operation has been carried.

This whole process is familiar; it gives the series of the positive whole numbers, that is, the cardinal numbers. It is to be noted that the concept of quantity has not yet been considered; what we have gained is the concept of number. The single things or members in this number are quite arbitrary, and especially they do not need to be alike in any manner. Every number forms a group-type, and arithmetic or the science of numbers has the task of investigating the properties of these different types with reference to their division and combination. If this is done in general form, without attention to the special amount of the number, the corresponding science is called algebra. On the other hand, by the application of formal rules of formation, the number system has had one extension after another beyond the territory of its original validity. Thus counting backward led to zero and to the negative numbers; the inversion of involution to the imaginary numbers. For the group-type of the positive whole numbers is the simplest but by no means the only possible one, and for the purpose of representing other manifolds than those which are met with in experience, these new types have proved themselves very useful.

At the same time the number series gives us an extremely useful type of arrangement. In the process of arising it is already ordered, and we make use of it for the purpose of arranging other groups. Thus, we are accustomed to furnish the pages in a book, the seats in a theatre, and countless other groups which we wish to make use of in any kind of order with the signs of the number series, and thereby we make the tacit assumption that the use of that corresponding group shall take place in the same order as the natural numbers follow each other. The ordinal numbers arising therefrom do not represent quantities, nor do they represent the only possible type of arrangement, but they are again the simplest of all. We come to the concept of magnitude only in the theory of time and space. The theory of time has not been developed as a special science; on the contrary, what we have to say about time first appears in mechanics. Meantime we can present the fundamental concepts, which arise in this connection, with reference to such well-known characteristics of time that the lack of a special science of time is no disadvantage.

The first and most important characteristic of time (and of space, too) is that it is a continuous manifold; that is, every portion of time chosen can be divided at any place whatever. In the number series this is not the case; it can be divided only between the single numbers. The series one to ten has only nine places of division and no more. A minute, or a second, on the other hand, has an unlimited number of places of division. In other words, there is nothing in the lapse of any time which hinders us from separating or distinguishing in thought at any given instant the time which has elapsed till then from the following time. It is just the same with space, except that time is a simple manifold and space a threefold, continuous manifold.

Nevertheless, when we measure them, we are accustomed to indicate times and spaces with numbers. If we first examine, for example, the process of measuring a length, it consists in our applying to the distance to be measured a length conceived as unchangeable, the unit of measure, until we have passed over the distance. The number of these applications gives us the measure or magnitude of the distance. The result is that by the indication of arbitrarily chosen points upon the continuous distance, we place upon it an artificial discontinuity which enables us to associate it with the discontinuous number series.

A still further assumption, however, belongs to the concept of measuring, namely, that the parts of the distance cut off by the unit used as a measure be equal, and it is taken for granted that this requirement will be fulfilled to whatever place the unit of measure is shifted. As may be seen, this is a definition of equality carried further than the former, for one cannot actually replace a part of the distance by another in order to convince one's self that it has not changed. Just as little can one assert or prove that the unit of measure in changing its place in space remains of the same length; we can only say that such distances as are determined by the unit of measure in different places are declared or defined as equal. Actually, for our eye, the unit of measure becomes smaller in perspective the farther away from it we find ourselves.

From this example we see again the great contribution which arbitrariness or free choice has made to all our structure of science. We could develop a geometry in which distances which seem subjectively equal to our eye are called equal, and upon this assumption we would be able to develop a self-consistent system or science. Such a geometry, however, would have an extremely complex and impractical structure for objective purposes (as, for example, land measurement), and so we strive to develop a science as free as possible from subjective factors. Historically, we have before us a process of this sort in the astronomy of Ptolemy and that of Copernicus. The former corresponded to the subjective appearances in the assumption that all heavenly bodies revolved around the earth, but proved to be very complicated when confronted with the task of mastering these movements with figures. The latter gave up the subjective standpoint of the observer, who looked upon himself as the centre, and attained a tremendous simplification by placing the centre of revolution in the sun.

A few words are to be said here about the application of arithmetic and algebra to geometry. It is well known that under definite assumptions (coördinates), geometrical figures can be represented by means of algebraic formulæ, so that the geometrical properties of the figure can be deduced from the arithmetical properties of the formulæ, and *vice versa.* The question must be asked how such a close and univocal relationship is possible between things of such different nature. The answer is, that here is an especially clear case of association. The manifold of numbers is much greater than that of surface or space, for while the latter are determined by two or three independent measurements, one can have any number of independent number series working together. Therefore the manifold of numbers is arbitrarily limited to two or three independent series, and in so far determines their mutual relations (by means of the laws of cosine) that there results a manifold, corresponding to the spatial, which can be completely associated with the spatial manifold. Then we have two manifolds of the same manifold character, and all characteristics of arrangement and size of the one find their likeness in the other.

This again characterizes an extremely important scientific procedure which consists, namely, in constructing a formal manifold for the content of experience of a certain field, to which one attributes the same manifold character which the former possesses. Every science reaches by this means a sort of formal language of corresponding completeness, which depends upon how accurately the manifold character of the object is recognized and how judiciously the formulæ have been chosen. While in arithmetic and algebra this task has been performed fairly well (though by no means absolutely perfectly), the chemical formulæ, for instance, express only a relatively small part of the manifold to be represented; and in biology as far as sociology, scarcely the first attempts have been made in the accomplishment of this task.

Language especially serves as such a universal manifold to represent the manifolds of experience. As a result of its development from a time of less culture, it has by no means sufficient regularity and completeness to accomplish its purpose adequately and conveniently. Rather, it is just as unsystematic as the events in the lives of single peoples have been, and the necessity of expressing the endlessly different particulars of daily life has only allowed it to develop so that the correspondence between word and concept is kept rather indefinite and changeable, according to need within somewhat wide limits. Thus all work in those sciences which must make vital use of these means, as especially psychology and sociology, or philosophy in general, is made extremely difficult by the ceaseless struggle with the indefiniteness and ambiguity of language. An improvement of this condition can be effected only by introducing signs in place of words for the representation of concepts, as the progress of science allows it, and equipping these signs with the manifold which from experience belongs to the concept.

An intermediate position in this respect is taken by the sciences which were indicated above as parts of energetics. In this realm there is added to the concepts order, number, size, space, and time, a new concept, that of energy, which finds application to every single phenomenon in this whole field, just as do those more general concepts. This is due to the fact that a certain quantity, which is known to us most familiarly as mechanical work, on account of its qualitative transformability and quantitative constancy, can be shown to be a constituent of every physical phenomenon, that is, every phenomenon which belongs to the field of mechanics, physics, and chemistry. In other words, one can perfectly characterize every physical event by indicating what amounts and kinds of energy have been present in it and into what energies they have been transformed. Accordingly, it is logical to designate the so-called physical phenomena as energetical.

That such a conception is possible is now generally admitted. On the other hand, its expediency is frequently questioned, and there is at present so much the more reason for this because a thorough presentation of the physical sciences in the energetical sense has not yet been made. If one applies to this question the criterion of the scientific system given above, the completeness of the correspondence between the representing manifold and that to be represented, there is no doubt that all previous systematizations in the form of hypotheses which have been tried in these sciences are defective in this respect. Formerly, for the purpose of representing experiences, manifolds whose character corresponded to the character of the manifold to be represented only in certain salient points without consideration of any rigid agreement, indeed, even without definite question as to such an agreement, have been employed.

The energetical conception admits of that definiteness of representation which the condition of science demands and renders possible. For each special manifold character of the field a special kind of energy presents itself: science has long distinguished mechanical, electric, thermal, chemical, etc., energies. All of these different kinds hold together by the law of transformation with the maintenance of the quantitative amount, and in so far are united. On the other hand, it has been possible to fix upon the corresponding energetical expression for every empirically discovered manifold. As a future system of united energetics, we have then a table of possible manifolds of which energy is capable. In this we must keep in mind the fact that, in accordance with the law of the conservation, energy is a necessarily positive quantity which also is furnished with the property of unlimited possibility of addition; therefore, every particular kind of energy must have this character.

The very small manifold which seems to lack this condition is much widened by the fact that every kind of energy can be separated into two factors, which are only subject to the limitation that their product, the energy, fulfills the conditions mentioned while they themselves are much freer. For example, one factor of a kind of energy can become negative as well as positive; it is only necessary that at the same time the other factor should become negative, viz., positive.

Thus it seems possible to make a table of all possible forms of energy, by attributing all thinkable manifold characteristics to the factors of the energy and then combining them by pairs and cutting out those products which do not fulfill the above-mentioned conditions. For a number of years I have tried from time to time to carry out this programme, but I have not yet got far enough to justify publication of the results obtained.

If we turn to the biological sciences, in them the phenomenon of life appears to us as new. If we stick to the observed facts, keeping ourselves free from all hypotheses, we observe as the general characteristics of the phenomena of life the continuous stream of energy which courses through a relatively constant structure. Change of substance is only a part, although a very important part, of this stream. Especially in plants we can observe at first hand the great importance of energy in its most incorporeal form, the sun's rays. Along with this, self-preservation and development and reproduction, the begetting of offspring of like nature, are characteristic. All of these properties must be present in order that an organism may come into existence; they must also be present if the reflecting man is to be able by repeated experience to form a concept of any definite organism, whether of a lion or of a mushroom. Other organisms are met with which do not fulfill these conditions; on account of their rarity, however, they do not lead to a species concept, but are excluded from scientific consideration (except for special purposes) as deformities or monsters.

While organisms usually work with kinds of energy which we know well from the inorganic world, organs are found in the higher forms which without doubt cause or assist transfers of energy, but we cannot yet say definitely what particular kind of energy is active in them. These organs are called nerves, and their function is regularly that, after certain forms of energy have acted upon one end of them, they should act at the other end and release the energies stored up there which then act in their special manner. That energetical transformations also take place in the nerve during the process of nervous transmission can be looked upon as demonstrated. We shall thus be justified in speaking of a nerve energy, while leaving it undecided whether there is here an energy of a particular kind, or perhaps chemical energy, or finally a combination of several energies.

While these processes can be shown objectively by the stimulation of the nerve and its corresponding releasing reaction in the end apparatus (for instance, a muscle), we find in ourselves, connected with certain nervous processes, a phenomenon of a new sort which we call self-consciousness. From the agreement of our reactions with those of other people we conclude with scientific probability that they also have self-consciousness; and we are justified in making the same conclusion with regard to some higher animals. How far down something similar to this is present cannot be determined by the means at hand, since the analogy of organization and of behavior diminishes very quickly; but the line is probably not very long, in view of the great leap from man to animal. Moreover, there are many reasons for the view that the gray cortical substance in the brain, with its characteristic pyramidal cell, is the anatomical substratum of this kind of nervous activity.

The study of the processes of self-consciousness constitutes the chief task of psychology. To this science belong those fields which are generally allotted to philosophy, especially logic and epistemology, while æsthetics, and still more ethics, are to be reckoned with the social sciences.

The latter have to do with living beings in so far as they can be united in groups with common functions. Here in place of the individual mind appears a collective mind, which owing to the adjustment of the differences of the members of society shows simpler conditions than that. From this comes especially the task of the historical sciences. The happenings in the world accessible to us are conditioned partly by physical, partly by psychological factors, and both show a temporal mutability in one direction. Thus arises on the one hand a history of heaven and earth, on the other hand a history of organisms up to man.

All history has primarily the task of fixing past events through the effects which have remained from them. Where such are not accessible, only analogy is left, a very doubtful means for gaining a conception of those events. But it must be kept in mind that an event which has left no evident traces has no sort of interest for us, for our interest is directly proportional to the amount of change which that event has caused in what we have before us. The task of historical science is just as little exhausted, however, with the fixing of former events as, for instance, the task of physics with the establishment of a single fact, as the temperature of a given place at a given time. Rather the individual facts must serve to bring out the general characteristics of the collective mind, and the much-discussed historical laws are laws of collective psychology. Just as physical and chemical laws are deduced in order with their help to predict the course of future physical events (to be called forth either experimentally or technically), so should the historical laws contribute to the formation and control of social and political development. We see that the great statesmen of all time have eagerly studied history for this purpose, and from that we derive the assurance that there are historical laws in spite of the objections of numerous scholars.

After this brief survey, if we look back over the road we have come, we observe the following general facts. In every case the development of a science consists in the formation of concepts by certain abstractions from experience, and setting of these concepts in relation with each other so that a systematical control of certain sides of our experience is made possible. These relations, according to their generality and reliability, are called rules or laws. A law is the more important the more it definitely expresses concerning the greatest possible number of things, and the more accurately, therefore, it enables us to predict the future. Every law rests upon an incomplete induction, and is therefore subject to modification by experience. From this there results a double process in the development of science.

First, the actual conditions are investigated to find out whether, besides those already known, new rules or laws, that is, constant relations between individual peculiarities, cannot be discovered between them. This is the inductive process, and the induction is always an incomplete one on account of the limitlessness of all possible experience.

Immediately the relationship found inductively is applied to cases which have not yet been investigated. Especially such cases are investigated as result from a combination of several inductive laws. If these are perfectly certain, and the combination is also properly made, the result has claim to unconditional validity. This is the limit which all sciences are striving to reach. It has almost been reached in the simpler sciences: in mathematics and in certain parts of mechanics. This is called the deductive process.

In the actual working of every science the two methods of investigation are continually changing. The best means of finding new successful inductions is in the making of a deduction on a very insufficient basis, perhaps, and subsequently testing it in experience. Sometimes the elements of his deductions do not come into the investigator's consciousness; in such cases we speak of scientific instinct. On the other hand we have much evidence from great mathematicians that they were accustomed to find their general laws by the method of induction, by trying and considering single cases; and that the deductive derivation from other known laws is an independent operation which sometimes does not succeed until much later. Indeed there is to-day a number of mathematical propositions which have not yet reached the second stage and therefore have at present a purely inductive empirical character. The proportion of such laws in science increases very quickly with the rise in the scale (page 339).

Another peculiarity which may be mentioned here is that in the scale all previous sciences have the character of applied sciences (page 341) with reference to those which follow, since they are everywhere necessary in the technique of the latter, yet do not serve to increase their own field but are merely auxiliaries to the latter.

If we ask finally what influence upon the shaping of the future such investigations as those which have been sketched in outline above can have, the following can be said. Up till now it has been considered a completely uncontrollable event whether and where a great and influential man of science has developed. It is obvious that such a man is among the most costly treasures which a people (and, indeed, humanity) can possess. The conscious and regular breeding of such rarities has not been considered possible. While this is still the case for the very exceptional genius, we see in the countries of the older civilization, especially in Germany at present, a system of education in vogue in the universities by which a regular harvest of young scientific men is gained who not only have a mastery of knowledge handed down, but also of the technique of discovery. Thereby the growth of science is made certain and regular, and its pursuit is raised to a higher plane. These results were formerly attained chiefly by empirically and oftentimes by accidental processes. It is a task of scientific theory to make this activity also regular and systematic, so that success is no more dependent solely upon a special capacity for the founding of a "school" but can also be attained by less original minds. By the mastery of methods the way to considerably higher performances than he could otherwise attain will be open for the exceptionally gifted.

THE CONTENT AND VALIDITY OF THE CAUSAL LAW

BY BENNO ERDMANN

(*Translated from the German by Professor Walter T. Marvin, Western Reserve University*)

[**Benno Erdmann,** Professor of Philosophy, University of Bonn, since 1898. b. October 5, 1851, Glogau in Schlesien, Germany. Ph.D.; Privy Councilor. Academical Lecturer, Berlin, 1876– ; Special Professor, Kiel, 1878–79; Regular Professor, *ibid.* 1879–84; *ibid.* Breslau, 1884–90; *ibid.* Halle, 1890–98. Member various scientific and learned societies. **Author of** *The Axioms of Geometry; Kant's Criticism; Logic; Psychological Researches on Reading* (together with Prof. Ramon Dodge); *The Psychology of the Child and the School; Historical Researches on Kant's Prolegomena,* and many other works and papers in Philosophy.]

We have learned to regard the real, which we endeavor to apprehend scientifically in universally valid judgments, as a whole that is connected continuously in time and in space and by causation, and that is accordingly continuously self-evolving. This continuity of connection has the following result, namely, every attempt to classify the sum total of the sciences on the basis of the difference of their objects leads merely to representative types, that is, to species which glide into one another. We find no gaps by means of which we can separate sharply physics and chemistry, botany and zoölogy, political and economic history and the histories of art and religion, or, again, history, philology, and the study of the prehistoric.

As are the objects, so also are the methods of science. They are separable one from another only through a division into representative types; for the variety of these methods is dependent upon the variety of the objects of our knowledge, and is, at the same time, determined by the difference between the manifold forms of our thought, itself a part of the real, with its elements also gliding into one another.[1]

The threads which join the general methodology of scientific thought with neighboring fields of knowledge run in two main directions. In the one direction they make up a closely packed cable, whereas in the other their course diverges into all the dimensions of scientific thought. That is to say, first, methodology has its roots in logic, in the narrower sense, namely, in the science of the elementary forms of our thought which enter into the make-up of all scientific methods. Secondly, methodology has its source in the methods themselves which actually, and therefore technically, develop in the various fields of our knowledge out of the problems peculiar to those fields.

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[1] Cf. the author's "Theorie der Typeneinteilungen," *Philosophische Monatshefte,* vol. xxx, Berlin, 1894.

It is the office of scientific thought to interpret validly the objects that are presented to us in outer and inner perception, and that can be derived from both these sources. We accomplish this interpretation entirely through judgments and combinations of judgments of manifold sorts. The concepts, which the older logic regarded as the true elementary forms of our thinking, are only certain selected types of judgment, such stereotyped judgments as those which make up definitions and classifications, and which appear independent and fundamental because their subject-matter, that is, their intension or extension, is connected through the act of naming with certain words. Scientific methods, then, are the ways and means by which our thought can accomplish and set forth, in accordance with its ideal, this universally valid interpretation.

There belongs, accordingly, to methodology a list of problems which we can divide, to be sure only *in abstracto*, into three separate groups. First, methodology has to analyze the methods which have been technically developed in the different fields of knowledge into the elementary forms of our thinking from which they have been built up. Next to this work of *analyzing*, there comes a second task which may be called a *normative* one; for it follows that we must set forth and deduce systematically from their sources the nature of these manifold elements, their resulting connection, and their validity. To these two offices must be added a third that we may call *a potiori* a *synthetic* one; for finally we must reconstruct out of the elements of our thinking, as revealed by analysis, the methods belonging to the different fields of knowledge and also determine their different scope and validity.

The beginning of another conception of the office of methodology can be found in those thoughts which have become significant, especially in Leibnitz's fragments and drafts of a *calculus ratiocinator* or a *spécieuse générale*. The foregoing discussion has set aside all hope that these beginnings and their recent development may give, of the possibility of constructing the manifold possible methods *a priori*, that is, before or independent of experience. However, it remains entirely undecided, as it should in this our preliminary account of the office of general methodology, whether or not all methods of our scientific thought will prove to be ultimately but branches of one and the same universal method, a thought contained in the undertakings just referred to. Although modern empiricism, affiliated as it is with natural science, tends to answer this question in the affirmative even more definitely and dogmatically than any type of the older rationalism, still the question is one that can be decided only in the course of methodological research.

The conception of a methodology of scientific thought can be said to be almost as old as scientific thought itself; for it is already contained essentially, though undifferentiated, in the Socratic challenge of knowledge. None the less, the history of methodology, as the history of every other science, went through the course of which Kant has given a classical description. "No one attempts to construct a science unless he can base it on some idea; but in the elaboration of it the schema, nay, even the definition which he gives in the beginning of his science, corresponds very seldom to his idea, which, like a germ, lies hidden in the reason, and all the parts of which are still enveloped and hardly distinguishable even under microscopical observation." [1]

We are indebted to the Greek, and especially to the Platonic-Aristotelian philosophy for important contributions to the understanding of the deductive method of mathematical thought. It was precisely this trend of philosophic endeavor which, though furnishing for the most part the foundation of methodological doctrine well on into the seventeenth century, offered no means of differentiating the methods that are authoritative for our knowledge of facts. What Socrates was perhaps the first to call "induction," is essentially different, as regards its source and aim, from the inductive methods that direct our research in natural and mental science. For it is into these two fields that we have to divide the totality of the sciences of facts, the material sciences, let us call them, in opposition to the formal or mathematical sciences, — that is, if we are to do justice to the difference between sense and self-perception, or "outer" and "inner" perception.

Two closely connected forces especially led astray the methodological opinions regarding the material sciences till the end of the eighteenth century, and in part until the beginning of the nineteenth century. We refer, in the first place, to that direction of thought which gives us the right to characterize the Platonic-Aristotelian philosophy as a "concept philosophy;" namely, the circumstance that Aristotelian logic caused the "concept" to be set before the "judgment." In short, we refer to that tendency in thought which directs the attention not to the permanent in the world's occurrences, the uniform connections of events, but rather to the seemingly permanent in the things, their essential attributes or essences. Thus the concept philosophy, as a result of its tendency to hypostasize, finds in the abstract general concepts of things, the ideas, the eternal absolute reality that constitutes the foundation of things and is contained in them beside the accidental and changing properties.[2]

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[1] Kant, *Kr. d. r. V.*, 2d ed., p. 862.

[2] According to Plato, it is true, the ideas are separated from the sensible things; they must be thought in a conceptual place, for the space of sense-perception is to

Here we have at once the second force which inspired the ancient methodology. These ideas, like the fundamentally real, constitute that which ultimately alone acts in all the coming into existence and the going out of existence of the manifold things. In the Aristotelian theory of causation, this thought is made a principle; and we formulate only what is contained in it, when we say that, according to it, the efficient and at the same time final causes can be deduced through mere analysis from the essential content of the effects; that, in fact, the possible effects of every cause can be deduced from the content of its definition. The conceptual determination of the causal relation, and with it in principle the sum total of the methods in the material sciences, becomes a logical, analytical, and deductive one. These sciences remain entirely independent of the particular content of experience as this broadens, and so do also the methods under discussion.

As a consequence, every essential difference between mathematical thought and the science of causes is done away with in favor of a rationalistic construction of the methods of material science. Accordingly, throughout the seventeenth century, the ideal of all scientific method becomes, not the inductive method that founded the new epoch of the science of to-day, but the deductive mathematical method applied to natural scientific research. The flourish of trumpets with which Francis Bacon hailed the onslaught of the inductive methods in the natural science of the time, helped in no way; for he failed to remodel the traditional, Aristotelian-Scholastic conception of cause, and, accordingly, failed to understand both the problem of induction and the meaning of the inductive methods of the day.[1] Descartes, Hobbes, Spinoza, and related thinkers develop their *mathesis universalis* after the pattern of geometrical thinking. Leibnitz tries to adapt his *spécieuse générale* to the thought of mathematical analysis. The old methodological conviction gains its clear-cut expression in Spinoza's doctrine: "*Aliquid efficitur ab aliqua re*" means "*aliquid sequitur ex ejus definitione.*"

The logically straight path is seldom the one taken in the course of the history of thought. The new formulation and solution of problems influence us first through their evident significance and consequences, not through the traditional presuppositions upon which they are founded. Thus, in the middle of the seventeenth century, when insight into the precise difference between mental and physical events gave rise to pressing need for its definite formulation, no question arose concerning the dogmatic presupposition

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be understood as non-being, matter. The things revealed to sense, however, occupy a middle position between being and non-being, so that they partake of the ideas. In this sense, the statement made above holds also of the older view of the concept philosophy.

[1] Cf. the articles on Francis Bacon by Chr. Sigwart in the *Preussische Jahrbücher*, XII, 1863, and XIII, 1864.

of a purely logical (*analytisch*) relationship between cause and effect; but, on the contrary, this presupposition was then for the first time brought clearly before consciousness. It was necessary to take the roundabout way through occasionalism and the preëstablished harmony, including the latter's retreat to the omnipotence of God, before it was possible to raise the question of the validity of the presupposition that the connection between cause and effect is analytic and rational.

Among the leading thinkers of the period this problem was recognized as the cardinal problem of contemporaneous philosophy. It is further evidence how thoroughly established this problem must have been among the more deeply conceived problems of the time in the middle of the eighteenth century, that Hume and Kant were forced to face it, led on, seemingly independently of each other, and surely from quite different presuppositions and along entirely different ways. The historical evolution of that which from the beginning has seemed to philosophy the solving of her true problem has come to pass in a way not essentially different from that of the historical evolution in all other departments of human knowledge. Thus, in the last third of the seventeenth century, Newton and Leibnitz succeeded in setting forth the elements of the infinitesimal calculus; and, in the fifth decade of the nineteenth century, Robert Mayer, Helmholtz, and perhaps Joule, formulated the law of the conservation of energy. In one essential respect Hume and Kant are agreed in the solution of the new, and hence contemporaneously misunderstood, problem. Both realized that the connection between the various causes and effects is not a rational analytic, but an empirical synthetic one. However, the difference in their presuppositions as well as method caused this common result to make its appearance in very different light and surroundings. In Hume's empiricism the connection between cause and effect appears as the mere empirical result of association; whereas in Kant's rationalism this general relation between cause and effect becomes the fundamental condition of all possible experience, and is, as a consequence, independent of all experience. It rests, as a means of connecting our ideas, upon an inborn uniformity of our thought.

Thus the way was opened for a fundamental separation of the inductive material scientific from the deductive mathematical method. For Hume mathematics becomes the science of the relations of ideas, as opposed to the sciences of facts. For Kant philosophical knowledge is the knowledge of the reason arising from concepts, whereas the mathematical is that arising from the construction of concepts. The former, therefore, studies the particular only in the universal; the latter, the universal in the particular, nay, rather in the individual.

Both solutions of the new problem which in the eighteenth century supplant the old and seemingly self-evident presupposition, appear accordingly embedded in the opposition between the rationalistic and empiristic interpretation of the origin and validity of our knowledge, the same opposition that from antiquity runs through the historical development of philosophy in ever new digressions.

Even to-day the question regarding the meaning and the validity of the causal connection stands between these contrary directions of epistemological research; and the ways leading to its answer separate more sharply than ever before. It is therefore more pressing in our day than it was in earlier times to find a basis upon which we may build further epistemologically and therefore methodologically. The purpose of the present paper is to seek such a basis for the different methods employed in the sciences of facts.

As has already been said, the contents of our consciousness, which are given us immediately in outer and inner perception, constitute the raw material of the sciences of facts. From these various facts of perception we derive the judgments through which we predict, guide, and shape our future perception in the course of possible experience. These judgments exist in the form of reproductive ideational processes, which, if logically explicit, become *inductive inferences* in the broader sense. These inferences may be said to be of two sorts, though fundamentally only two sides of one and the same process of thought; they are in part analogical inferences and in part *inductive inferences in the narrower sense.* The former infers from the particular in a present perception, *which in previous perceptions was uniformly connected with other particular contents of perception*, to a particular that resembles *those other contents of perception.* In short, they are inferences from a particular to a particular. After the manner of such inferences we logically formulate, for example, the reproductive processes, whose conclusions run: "This man whom I see before me, is attentive, feels pain, will die;" "this meteor will prove to have a chemical composition similar to known meteors, and also to have corresponding changes on its surface as the result of its rapid passage through our atmosphere." The inductive inferences in the narrower sense argue, on the contrary, from the perceptions of a series of uniform phenomena to a universal, which includes the given and likewise all possible cases, in which a member of the particular content of the earlier perceptions is presupposed as given. In short, they are conclusions from a particular to a universal that is more extensive than the sum of the given particulars. For example: "All men have minds, will die;" "all meteoric stones will prove to have this chemical composition and those changes of surface."

There is no controversy regarding the inner similarity of both these types of inference or regarding their outward structure; or, again, regarding their outward difference from the deductive inferences, which proceed not from a particular to a particular or general, but from a general to a particular.

There is, however, difference of opinion regarding their inner structure and their inner relation to the deductive inferences. Both questions depend upon the decision regarding the meaning and validity of the causal relation. The contending parties are recruited essentially from the positions of traditional empiricism and rationalism and from their modern offshoots.

We maintain first of all:

1. The *presupposition* of all inductive inferences, from now on to be taken in their more general sense, is, that the contents of perception are given to us *uniformly* in repeated perceptions, that is, in uniform components and uniform relations.

2. The *condition* of the validity of the inductive inferences lies in the thoughts that *the same causes will be present* in the unobserved realities as in the observed ones, and that *these same causes will bring forth the same effects*.

3. The *conclusions* of all inductive inferences have, logically speaking, purely *problematic* validity, that is, their contradictory opposite remains equally thinkable. They are, accurately expressed, merely *hypotheses*, whose validity needs verification through future experience.

The first-mentioned *presupposition* of inductive inference must not be misunderstood. The paradox that nothing really repeats itself, that each stage in nature's process comes but once, is just as much and just as little justified as the assertion, everything has already existed. It does not deny the fact that we can discriminate in the contents of our perceptions the uniformities of their components and relations, in short, that similar elements are present in these ever new complexes. This fact makes it possible that our manifold perceptions combine to make up one continuous experience. Even our paradox presupposes that the different contents of our perceptions are comparable with one another, and reveal accordingly some sort of common nature. All this is not only a matter of course for empiricism, which founds the whole constitution of our knowledge upon habits, but must also be granted by every rationalistic interpretation of the structure of knowledge. Every one that is well informed knows that what we ordinarily refer to as facts already includes a theory regarding them. Kant judges in this matter precisely as Hume did before him and Stuart Mill after him. "If cinnabar were sometimes red and sometimes black, sometimes light and sometimes heavy, if a man could be changed now into this, now into another animal shape, if on the longest day the fields were sometimes covered with fruit, sometimes with ice and snow, the faculty of my empirical imagination would never be in a position, when representing red color, to think of heavy cinnabar."[1]

The assumption that in recurring perceptions similar elements of content, as well as of relation, are given, is a necessary condition of the possibility of experience itself, and accordingly of all those processes of thought which lead us, under the guidance of previous perceptions, from the contents of one given perception to the contents of possible perceptions.

A tradition from Hume down has accustomed us to associate the relation of cause and effect not so much with the uniformity of coexistence as with the uniformity of sequence. Let us for the present keep to this tradition. Its first corollary is that the relation of cause and effect is to be sought in the uninterrupted flow and connection of events and changes. The cause becomes the uniformly preceding event, the constant *antecedens*, the effect the uniformly following, the constant *consequens*, in the course of the changes that are presented to consciousness as a result of foregoing changes in our sensorium.

According to this tradition that we have taken as our point of departure, the uniformity of the sequence of events is a necessary presupposition of the relation between cause and effect. This uniformity is given us as an element of our experience; for we actually find uniform successions in the course of the changing contents of perception. Further, as all our perceptions are in the first instance sense-perceptions, we may call them the sensory presupposition of the possibility of the causal relation.

In this presupposition, however, there is much more involved than the name just chosen would indicate. The uniformity of sequence lies, as we saw, not in the contents of perception as such, which are immediately given to us. It arises rather through the fact that, in the course of repeated perceptions, we apprehend through abstraction the uniformities of their temporal relation. Moreover, there lie in the repeated perceptions not only uniformities of sequence, but also uniformities of the qualitative content of the successive events themselves, and these uniformities also must be apprehended through abstraction. Thus these uniform contents of perception make up series of the following form:

$$\begin{array}{ccc} a_1 & \rightarrow & b_1 \\ a_2 & \rightarrow & b_2 \\ \text{"} & & \text{"} \\ \text{"} & & \text{"} \\ \text{"} & & \text{"} \\ a_n & \rightarrow & b_n \end{array}$$

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[1] Kant, *Kr. d. r. V.*, 1st ed., pp. 100 f.

The presupposition of the possibility of the causal relations includes, therefore, more than mere perceptive elements. It involves the relation of different, if you will, of peculiar contents of perception, by virtue of which we recognize $a_2 \rightarrow b_2 \ . \ . \ . \ a_n \rightarrow b_n$ as events that resemble one another and the event $a_1 \rightarrow b_1$ qualitatively as well as in their sequence. There are accordingly involved in our presupposition *reproductive* elements which indicate the action of memory. In order that I may in the act of perceiving $a_3 \rightarrow b_3$ apprehend the uniformity of this present content with that of $a_2 \rightarrow b_2$ and $a_1 \rightarrow b_1$, these earlier perceptions must in some way, perhaps through memory,[1] be revived with the present perception.

In this reproduction there is still a further element, which can be separated, to be sure only *in abstracto*, from the one just pointed out. The present revived content, even if it is given in memory as an independent mental state, is essentially different from the original perception. It differs in all the modifications in which the memory of lightning and thunder could differ from the perception of their successive occurrence, or, again, the memory of a pain and the resulting disturbance of attention could differ from the corresponding original experience. However, as memory, the revived experience presents itself as a picture of that which has been previously perceived. Especially is this the case in memory properly so called, where the peculiar space and time relations individualize the revived experience. If we give to this identifying element in the associative process a logical expression, we shall have to say that there is involved in revival, and especially in memory, an awareness that the present ideas recall the same content that was previously given us in perception. To be sure, the revival of the content of previous perceptions does not have to produce ideas, let alone memories. Rapid, transitory, or habitual revivals, stimulated by associative processes, can remain unconscious, that is, they need not appear as ideas or states of consciousness. Stimulation takes place, but consciousness does not arise, provided we mean by the term "consciousness" the genus of our thoughts, feelings, and volitions. None the less it must not be forgotten that this awareness of the essential identity of the present revived content with that of the previous perception can be brought about in every such case of reproduction. How all this takes place is not our present problem.

We can apply to this second element in the reproductive process, which we have found to be essential to the causal relation, a Kantian term, "Recognition." This term, however, is to be taken only in the sense called for by the foregoing statements; for the rationalistic presuppositions and consequences which mark Kant's "Synthesis of Recognition" are far removed from the present line of thought.

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[1] It is not our present concern to ascertain how this actually happens. The psychological presuppositions of the present paper are contained in the theory of reproduction that I have worked out in connection with the psychology of speech in the articles on "Die psychologischen Grundlagen der Beziehungen zwischen Sprechen und Denken," *Archiv für systematische Philosophie*, II, III, und VII; cf. note 1, page 151.

We may, then, sum up our results as follows: In the presupposition of a uniform sequence of events, which we have accepted from tradition as the necessary condition of the possibility of the causal relation, there lies the thought that the contents of perception given us through repeated sense stimulation are related to one another through a reproductive recognition.

The assumption of such reproductive recognition is not justified merely in the cases so far considered. It is already necessary in the course of the individual perceptions *a* and *b*, and hence in the apprehension of an occurrence. It makes the sequence itself in which *a* and *b* are joined possible; for in order to apprehend *b* as following upon *a*, in case the perception of *a* has not persisted in its original form, *a* must be as far revived and recognized upon *b*'s entrance into the field of perception as it has itself passed out of that field. Otherwise, instead of *b* following upon *a* and being related to *a*, there would be only the relationless change from *a* to *b*. This holds generally and not merely in the cases where the perception of *a* has disappeared before that of *b* begins, for example, in the case of lightning and thunder, or where it has in part disappeared, for example, in the throwing of a stone.

We have represented *a* as an event or change, in order that uniform sequences of events may alone come into consideration as the presupposition of the causal relation. But every event has its course in time, and is accordingly divisible into many, ultimately into infinitely many, shorter events. Now if *b* comes only an infinitely short interval later than *a*, and by hypothesis it must come later than *a*, then a corresponding part of *a* must have disappeared by the time *b* appears. But the infinitesimal part of a perception is just as much out of all consideration as would be an infinitely long perception; all which only goes to show that we have to substitute intervals of finite length in place of this purely conceptual analysis of a continuous time interval. This leaves the foregoing discussion as it stands. If *b* follows *a* after a perceptible finite interval, then the flow or development of *a* by the time of *b*'s appearance must have covered a course corresponding to that interval; and all this is true even though the earlier stages of *a* remain unchanged throughout the interval preceding *b*'s appearance. The present instant of flow is distinct from the one that has passed, even though it takes place in precisely the same way. The former, not the latter, gives the basis of relation which is here required, and therefore the former must be reproduced and recognized. This thought also is included in the foregoing summary of what critical analysis shows to be involved in the presupposition of a uniform sequence.

In all this we have already abandoned the field of mere perception which gave us the point of departure for our analysis of uniform sequence. We may call the changing course of perception only in the narrower meaning the sensory presupposition of the causal relation. In order that these changing contents of perception may be known as like one another, as following one another, and as following one another uniformly, they must be related to one another through a recognitive reproduction.

Our critical analysis of uniform sequence is, however, not yet complete. To relate to one another the contents of two ideas always requires a process at once of identifying and of differentiating, which makes these contents members of the relation, and which accordingly presupposes that our attention has been directed to each of the two members as well as to the relation itself — in the present case, to the sequence. Here we come to another essential point. We should apply the name "thought" to every ideational process in which attention is directed to the elements of the mental content and which leads us to identify with one another, or to differentiate from one another, the members of this content.[1] The act of relating, which knows two events as similar, as following one another, indeed, as following one another uniformly, is therefore so far from being a sensation that it must be claimed to be an act of thinking. The uniformity of sequence of *a* and *b* is therefore an act of relating on the part of our thought, so far as this becomes possible solely through the fact that we at one and the same time identify with one another and differentiate from one another *a* as cause and *b* as effect. We say "at one and the same time," because the terms identifying and differentiating are correlatives which denote two different and opposing sides of one and the same ideational process viewed logically. Accordingly, there is here no need of emphasizing that the act of relating, which enables us to think *a* as cause and *b* as effect, is an act of thought also, because it presupposes on our part an act of naming which raises it to being a component of our formulated and discursive thought. We therefore *think a* as cause and *b* as effect in that we apprehend the former as uniform *antecedens* and the latter as uniform *consequens*.

Have we not the right, after the foregoing analysis, to interpret the uniform sequence of events solely as the *necessary* presupposition of the causal relation? Is it not at the same time the *adequate* presupposition? Yes, is it not the causal relation itself? As we know, empiricism since Hume has answered the last question in the affirmative, and rationalism since Kant has answered it in the negative.

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[1] Cf. the author's "Umrisse zur Psychologie des Denkens," in *Philosophische Abhandlungen Chr. Sigwart . . . gewidmet*, Tübingen, 1900.

We, too, have seemingly followed in our discussion the course of empiricism. At least, I find nothing in that discussion which a consistent empiricist might not be willing to concede; that is, if he is ready to set aside the psychological investigation of the actual processes which we here presuppose and make room for a critical analysis of the content of the relation of cause and effect.[1] However, the decision of the question, whether or not empiricism can determine exhaustively the content that we think in the causal relation, depends upon other considerations than those which we have until now been called upon to undertake. We have so far only made clear what every critical analysis of the causal relation has to concede to empiricism. In reality the empiristic hypothesis is inadequate. To be sure,

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[1] The difference between the two points of view can be made clearer by an illustration. The case that we shall analyze is the dread of coming into contact with fire. The psychological analysis of this case has to make clear the mental content of the dread and its causes. Such dread becomes possible only when we are aware of the burning that results from contact with fire. We could have learned to be aware of this either immediately through our own experience, or mediately through the communication of others' experience. In both cases it is a matter of one or repeated experiences. In all cases the effects of earlier experiences equal association and recall, which, in turn, result in recognition. The recognition explaining the case under discussion arises thus. The present stimuli of visual perception arouse the retained impressions of previous visual perceptions of fire and give rise to the present perception (apperception) by fusing with them. By a process of interweaving, associations are joined to this perception. The apperceptively revived elements which lie at the basis of the content of the perception are interwoven by association with memory elements that retain the additional contents of previous perceptions of fire, viz., the burning, or, again, are interwoven with the memory elements of the communications regarding such burning. By means of this interweaving, the stimulation of the apperceptive element transmits itself to the remaining elements of the association complex. The character of the association is different under different conditions. If it be founded only upon one experience, then there can arise a memory or a recall, in the wider sense, of the foregoing content of the perception and feeling at the time of the burning, or, again, there can arise a revival wherein the stimulated elements of retention remain unconscious. Again, the words of the mother tongue that denote the previous mental content, and which likewise belong to the association complex (the apperceiving mass, in the wider sense), can be excited in one of these three forms and in addition as abstract verbal ideas. Each one of these forms of verbal discharge can lead to the innervations of the muscles involved in speech, which bring about some sort of oral expression of judgment. Each of these verbal reproductions can be connected with each of the foregoing sensory (*sachlichen*) revivals. Secondly, if the association be founded upon repeated perceptions on the part of the person himself, then all the afore-mentioned possibilities of reproduction become more complicated, and, in addition, the mental revivals contain, more or less, only the common elements of the previous perceptions, *i. e.*, reappear in the form of abstract ideas or their corresponding unconscious modifications. In the third case the association is founded upon a communication of others' experience. For the sake of simplicity, let this case be confined to the following instance. The communication consisted in the assertion: "All fire will burn upon contact." Moreover, this judgment was expressed upon occasion of imminent danger of burning. There can then arise, as is perhaps evident, all the possibilities mentioned in the second case, only that here there will be a stronger tendency toward verbal reproduction and the sensory reproduction will be less fixed.

In the first two cases there was connected with the perception of the burning an intense feeling of pain. In the third the idea of such pain added itself to the visual perception of the moment. The associated elements of the earlier mental contents belong likewise to the apperceiving mass excited at the moment, in fact to that part of it excited by means of association processes, or, as we can again say, depending upon the point from which we take our view, the associative or apperceptive completion of the content of present perception. If these pain elements are revived as memories, *i. e.*, as elements in consciousness, they give rise to a new disagreeable feeling, which is referred to the possible coming sensation of burning. If the mental modifications corresponding to these pain elements remain unconscious, as is often possible, there arises none the less the same result as regards our feeling, only with less intensity. This feeling tone we call the dread.

As a result of the sum total of the revivals actual and possible, there is finally produced, according to the particular circumstances, either a motor reaction or an inhibitant of such reaction. Both innervations can take place involuntarily or voluntarily.

The critical analysis of the fact that we dread contact with fire, even has another purpose and accordingly proceeds on other lines. It must make clear under what presuppositions the foresight that lies at the basis of such dread is valid for future experience. It must then formulate the actual process of revival that constitutes the foundation of this feeling as a series of judgments, from which the meaning and interconnection of the several judgments will become clear. Thus the critical analysis must give a logical presentation of the apperceptive and associative processes of revival.

For this purpose the three cases of the psychological analysis reduce themselves to two: viz., first, to the case in which an immediate experience forms the basis, and secondly, to that in which a variety of similar mediately or immediately communicated experiences form such basis.

In the first of these logically differentiated cases, the transformation into the speech of formulated thought leads to the following inference from analogy:

Fire A burned.
Fire B is similar to fire A.

Fire B will burn.

In the second case there arises a syllogism of some such form as:

All fire causes burning upon contact.
This present phenomenon is fire.

This present phenomenon will cause burning upon contact.

Both premises of this syllogism are inductive inferences, whose implicit meaning becomes clear when we formulate as follows:

All heretofore investigated instances of fire have burned, therefore all fire burns.
The present phenomenon manifests some properties of fire, will consequently have all the properties thereof.

The present phenomenon will, in case of contact, cause burning.

The first syllogism goes from the particular to the particular. The second proves itself to be (contrary to the analysis of Stuart Mill) an inference that leads from the general to the particular. For the conclusion is the particular of the second parts of the major and minor premises; and these second parts of the premises are inferred from their first parts in the two possible ways of inductive inference. The latter do not contain the case referred to in the conclusion, but set forth the conditions of carrying a result of previous experience over to a new case with inductive probability, in other words, the conditions of making past experience a means of foreseeing future experience. It would be superfluous to give here the symbols of the two forms of inductive inference.

We remain within the bounds of logical analysis, if we state under what conditions conclusions follow necessarily from their premises, viz., the conclusions of arguments from analogy and of syllogisms in the narrower sense, as well as those of the foregoing inductive arguments. For the inference from analogy and the two forms of inductive inference, these conditions are the presuppositions already set forth in the text of the present paper, that in the as yet unobserved portion of reality the like causes will be found and they will give rise to like effects. For the syllogism they are the thought that the predicate of a predicate is the (mediate) predicate of the subject. Only the further analysis of these presuppositions, which is undertaken in the text, leads to critical considerations in the narrower sense.

the proof of this inadequacy is not to be taken from the obvious argument which Reid raised against the empiricism of Hume, and which compelled Stuart Mill in his criticism of that attack [1] to abandon his empiristic position at this point. No doubt the conclusion to which we also have come for the time being, goes much too far, the conclusion that the cause is nothing but the uniform *antecedens* and the effect merely the uniform *consequens*. Were it true, as we have hitherto assumed, that every uniformly preceding event is to be regarded as cause and every uniformly following event as effect, then day must be looked upon as cause of night and night as cause of day.

Empiricism can, however, meet this objection without giving up its position; in fact, it can employ the objection as an argument in its favor; for this objection affects only the manifestly imperfect formulation of the doctrine, not the essential arguments.

It should have been pointed out again and again in the foregoing exposition that only in the first indiscriminating view of things may we regard the events given us in perception as the basis of our concepts of cause and effect. All these events are intricately mixed, those that are given in self perception as well as those given in sense perception. The events of both groups flow along continuously. Consequently, as regards time, they permit a division into parts, which division proceeds, not indeed for our perception, but for our scientific thought, in short, conceptually, into infinity. The events of sense perception permit also conceptually of infinite division in their spatial relations.

It is sufficient for our present purpose, if we turn our attention to the question of divisibility in time. This fact of divisibility shows that the events of our perception, which alone we have until now brought under consideration, must be regarded as systems of events. We are therefore called upon to apportion the causal relations among the members of these systems. Only for the indiscriminating view of our practical *Weltanschauung* is the perceived event a the cause of the perceived event b. The more exact analysis of our theoretical apprehension of the world compels us to dissect the events a and b into the parts a_α, a_β, a_γ, — b_α, b_β, b_γ, and, where occasion calls for it, to continue the same process in turn for these and further components. We have accordingly to relate those parts to one another as causes and effects which, from the present standpoint of analysis, follow one another uniformly and *immediately*, viz., follow one another so that from this standpoint no other intervening event must be presupposed. In this way we come to have a *well-ordered experience*. The dispositions to such experience which reveal themselves within the field of practical thought taught man long before the beginning of scientific methods not to connect causally day and night with one another, but the rising and setting of the sun with day and night. The theoretical analysis, indeed, goes farther. It teaches that in what is here summed up as rising of the sun and yonder as day, there lie again intricate elements requiring special attention, in our own day extending perhaps to the lines of thought contained in the electro-dynamic theory of light and of electrons. Still the ways of thought remain the same on all the levels of penetrating analysis. We have throughout to relate to one another as cause and effect those events which, in a well-ordered experience, must be regarded as following one another immediately. The cause is then the *immediate* uniform *antecedens*, the effect the *immediate* uniform *consequens*. Otherwise stated, the perceived events that we are accustomed, from the standpoint of the practical *Weltanschauung*, to regard as causes and effects, *e. g.*, lightning and thunder, from the theoretical apprehension of the world prove to be infinitely involved collections of events, whose elements must be related to one another as causes and effects in as far as they can be regarded as following one another immediately. No exception is formed by expressions of our rough way of viewing and describing which lead us without hesitation to regard as cause one out of the very many causes of an event, and this, too, not necessarily the immediate uniformly preceding event. All this lies rather in the nature of such a hasty view.

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[1] *A System of Logic, Ratiocinative and Inductive*, bk. III, ch. V, § 6.

The present limitation of uniform sequence to cases of immediate sequence sets aside, then, the objection from which we started, in that it adopts as its own the essential point in question.

Moreover, the way that leads us to this necessary limitation goes farther: it leads to a strengthening of the empiristic position. It brings us to a point where we see that the most advanced analysis of intricate systems of events immediately given to us in perception as real nowhere reveals more than the simple fact of uniform sequence. Again where we come to regard the intervals between the events that follow one another immediately as very short, there the uniformity of the time relation makes, it would seem, the events for us merely causes and effects; and as often as we have occasion to proceed to the smaller time differences of a higher order, the same process repeats itself; for we dissect the events that make up our point of departure into ever more complex systems of component events, and the coarser relations of uniform sequence into ever finer immediate ones. Nowhere, seemingly, do we get beyond the field of events in uniform sequence, which finally have their foundation in the facts of perception from which they are drawn. Thus there follows from this conceptual refinement of the point of departure only the truth that nothing connects the events as causes and effects except the immediate uniformity of sequence.

None the less, we have to think the empiristic doctrine to the bottom, if we desire to determine whether or not the hypothesis which it offers is really sufficient to enable us to deduce the causal relation. For this purpose let us remind ourselves that the question at issue is, whether or not this relation is merely a temporal connection of events that are given to us in perception or that can be derived from the data of perception.

Besides, let us grant that this relation is as thoroughly valid for the content of our experience as empiricism has always, and rationalism nearly always, maintained. We presuppose, therefore, as granted, that every event is to be regarded as cause, and hence, in the opposite time relation, as effect, mental events that are given to us in self perception no less than the physical whose source is our sense perception. In other words, we assume that the totality of events in our possible experience presents a closed system of causal series, that is, that every member within each of the contemporary series is connected with the subsequent ones, as well as with the subsequent members of all the other series, backward and forward as cause and effect; and therefore, finally, that every member of every series stands in causal relationship with every member of every other series. We do not then, for the present purpose, burden ourselves with the hypothesis which was touched upon above, that this connection is to be thought of as a continuous one, namely, that other members can be inserted *ad infinitum* between any two members of the series.

We maintain at the same time that there is no justification for separating from one another the concepts, causality and interaction. This separation is only to be justified through the metaphysical hypothesis that reality consists in a multitude of independently existing substances inherently subject to change, and that their mutual interconnection is conditioned by a common dependence upon a first infinite cause.[1] Every connection between cause and effect is mutual, if we assume with Newton that to every action there is an equal opposing reaction.

In that we bring the totality of knowable reality, as far as it is analyzable into events, under the causal relation, we may regard the statement that every event requires us to seek among uniformly preceding events for the sufficient causes of its own reality, namely, *the general causal law*, as the principle of all material sciences. For all individual instances of conformity to law which we can discover in the course of experience are from this point of view only special cases of the general universal conformity to law which we have just formulated.

[1] This doctrine began in the theological evolution of the Christian concept of God. It was first fundamentally formulated by Leibnitz. It is retained in Kant's doctrine of the *harmonia generaliter stabilita* and the latter's consequences for the critical doctrine of the *mundus intelligibilis*. Hence it permeates the metaphysical doctrines of the systems of the nineteenth century in various ways.

For the empiristic interpretation, the (general) causal law is only the highest genus of the individual cases of empirically synthetic relations of uniform sequence. Starting from these presuppositions, it cannot be other than a generalization from experience, that is, a carrying over of observed relations of uniform, or, as we may now also say, constant sequence to those which have not been or cannot be objects of observation, as well as to those which we expect to appear in the future. Psychologically regarded, it is merely the most general expression of an expectation, conditioned through associative reproduction, of uniform sequence. It is, therefore, — to bring Hume's doctrine to a conclusion that the father of modern empiricism himself did not draw, — a species of temporal contiguity.

The general validity which we ascribe to the causal law is accordingly a merely empirical one. It can never attain apodeictic or even assertorical validity, but purely that type of problematic validity which we may call "real" in contradistinction to the other type of problematic validity attained in judgments of objective as well as of subjective and hypothetical possibility.[1] No possible progress of experience can win for the empiristically interpreted causal law any other than this real problematic validity; for experience can never become complete *a parte post*, nor has it ever been complete *a parte ante*. The causal law is valid assertorically only in so far as it sums up, purely in the way of an inventory, the preceding experiences. We call such assumptions, drawn from well-ordered experience and of inductive origin, "hypotheses," whether they rest upon generalizing inductive inferences in the narrower sense, or upon specializing inferences from analogy. They, and at the same time the empiristically interpreted causal law, are not hypotheses in the sense in which Newton rightly rejected all formation of hypotheses,[2] but are such as are necessarily part of all methods in the sciences of facts in so far as the paths of research lead out beyond the content given immediately in perception to objects of only possible experience.

The assertion of Stuart Mill, in opposition to this conclusion, that the cause must be thought of as the "invariable antecedent" and, correspondingly, the effect as the "invariable consequent,"[3] does all honor to the genius of the thinker; but it agrees by no means with the empiristic presuppositions which serve as the basis for his conclusions. For, starting from these presuppositions, the "invariable sequence" can only mean one that is uniform and constant according to past experience, and that we henceforth carry over to not yet observed events as far as these prove in conformity with it, and in this way verify the anticipation contained in our general assertion. The same holds of the assertion through which Mill endeavors to meet the above-mentioned objection of Reid, namely, that the unchanging sequence must at the same time be demonstrably an "unconditional" one. The language in which experience speaks to us knows the term "the unconditioned" as little as the term "the unchangeable," even though this have, as Mill explains, the meaning that the effect "will be, whatever supposition we may make in regard to all other things," or that the sequence will "be subject to no other than negative conditions." For in these determinations there does not lie exclusively, according to Mill, a probable prediction of the future. "It is *necessary* to our using the word cause, that we should believe not only that the antecedent always *has* been followed by the consequent, but that as long as the present constitution of things endures, it always *will* be so." Likewise, Mill, the man of research, not the empiristic logician, asserts that there belongs to the causal law, besides this generality referring to all possible events of uniform sequence, also an "undoubted assurance;" although he could have here referred to a casual remark of Hume.[1] Such an undoubted assurance, "that for every event . . . there is a law to be found, if we only know where to find it," evidently does not know of a knowledge referred exclusively to experience.

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[1] Cf. the author's *Logik*, bd. I, § 61.

[2] "*Rationem* vero harum gravitatis proprietatum ex phaenomenis nondum potui deducere, et hypotheses non fingo. *Quicquid enim ex phaenomenis non deducitur, hypothesis vocanda est;* et hypotheses seu metaphysicae, seu physicae, seu qualitatum occultarum, seu mechanicae, in philosophia experimentali locum non habent. In hac philosophia propositiones deducuntur ex phaenomenis, et redduntur generales per *inductionem.*" Newton, at the end of his chief work.

[3] *Logic*, bk. III, ch. V, § 2.

Hence, if the causal law is, as empiricism to be consistent must maintain, only a general hypothesis which is necessarily subject to verification as experience progresses, then it is not impossible that in the course of experience events will appear that are not preceded or followed uniformly by others, and that accordingly cannot be regarded as causes or effects. According to this interpretation of the causal law, such exceptional events, whether in individual or in repeated cases of perception, must be just as possible as those which in the course of preceding experience have proved themselves to be members of series of constant sequence. On the basis of previous experience, we should only have the right to say that such exceptional cases are less probable; and we might from the same ground expect that, if they could be surely determined, they would only have to be regarded as exceptions to the rule and not, possibly, as signs of a misunderstood universal non-uniformity of occurrence. No one wants to maintain an empirical necessity, that is, a statement that so comprehends a present experience or an hypothesis developed on the basis of present experience that its contradictory is rationally impossible. An event preceded by no other immediately and uniformly as cause would, according to traditional usage, arise out of nothing. An event that was followed immediately and constantly by no other would accordingly be an event that remained without effect, and, did it pass away, it must disappear into nothing. The old thought, well known in its scholastic formulation, *ex nihilo nihil fit, in nihilum nihil potest reverti,* is only another expression for the causal law as we have interpreted it above. The contradictories to each of the clauses of the thought just formulated, that something can arise out of nothing and pass into nothing, remain therefore, as a consequence of empiricism, an improbable thought, to be sure, but none the less a thought to which a real possibility must be ascribed.

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[1] *Logic*, bk. III, ch. V, § 6, and end of § 2. Hume says in a note to section VI of his *Enquiry concerning Human Understanding*: "We ought to divide arguments into *demonstrations, proofs, and probabilities*. By proofs meaning such arguments from experience as leave no room for doubt or opposition." The note stands in evident contrast to the well-known remarks at the beginning of section IV, pt. I.

It was in all probability this that Stuart Mill wished to convey in the much-debated passage: "I am convinced that any one accustomed to abstraction and analysis, who will fairly exert his faculties for the purpose, will, when his imagination has once learnt to entertain the notion, find no difficulty in conceiving that in some one, for instance, of the many firmaments into which sidereal astronomy now divides the universe, events may succeed one another at random without any fixed law; nor can anything in our experience, or in our mental nature, constitute a sufficient, or indeed any, reason for believing that this is nowhere the case." For Mill immediately calls our attention to the following: "Were we to suppose (what it is perfectly possible to imagine) that the present order of the universe were brought to an end, and that a chaos succeeded in which there was no fixed succession of events, and the past gave no assurance of the future; if a human being were miraculously kept alive to witness this change, he surely would soon cease to believe in any uniformity. the uniformity itself no longer existing."[1]

We can throw light from another side upon the thought that lies in this outcome of the empiristic interpretation of the causal law. If we still desire to give the name "effect" to an event that is preceded uniformly by no other, and that we therefore have to regard as arising out of nothing, then we must say that it is the effect of itself, that is, its cause lies in its own reality, in short, that it is *causa sui.* Therefore the assumption that a *causa sui* has just as much real possibility as have the causes of our experience which are followed uniformly by another event, is a necessary consequence of the empiristic view of causation. This much only remains sure, there is nothing contained in our previous experience that in any way assures us of the validity of this possible theory.

The empiristic doctrine of causation requires, however, still further conclusions. Our scientific, no less than our practical thought has always been accustomed to regard the relation between cause and effect not as a matter of mere sequence, not therefore as a mere formal temporal one. Rather it has always, in both forms of our thought, stood for a *real* relation, that is, for a relation of *dynamic dependence* of effect upon cause. Accordingly, the effect *arises out* of the cause, is *engendered through* it, or *brought forth by* it.

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[1] *Logic,* bk. III, ch. xxi, § 1.

The historical development of this dynamic conception of cause is well known. The old anthropopathic interpretation, which interpolates anthropomorphic and yet superhuman intervention between the events that follow one another uniformly, has maintained itself on into the modern metaphysical hypotheses. It remains standing wherever God is assumed as the first cause for the interaction between parts of reality. It is made obscure, but not eliminated, when, in other conceptions of the world, impersonal nature, fate, necessity, the absolute identity, or an abstraction related to these, appears in the place of God. On the other hand, it comes out clearly wherever these two tendencies of thought unite themselves in an anthropopathic pantheism. That is, it rests only upon a difference in strength between the governing religious and scientific interests, whether or not the All-One which unfolds itself in the interconnection and content of reality is thought of more as the immanent God, or more as substance. Finally, we do not change our position, if the absolute, self-active being (in all these theories a first cause is presupposed as *causa sui*) is degraded to a non-intellectual will.

However, the dynamic interpretation of cause has not remained confined to the field of these general speculations, just because it commanded that field so early. There is a second branch, likewise early evolved from the stem of the anthropopathic interpretation, the doctrine that the causal relations of dependence are effected through "forces." These forces adhere to, or dwell in, the ultimate physical elements which are thought of as masses. Again, as spiritual forces they belong to the "soul," which in turn is thought of as a substance. In the modern contrast between attractive and repulsive forces, there lies a remnant of the Empedoklean opposition between Love and Hate. In the various old and new hylozoistic tendencies, the concepts of force and its correlate, mass, are eclectically united. In consistent materialism as well as spiritualism, and in the abstract dynamism of energetics, the one member is robbed of its independence or even rejected in favor of the other.[1]

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[1] Alongside of these dynamic theories, there are to be found mechanical ones that arose just as early and from the same source, viz., the practical *Weltanschauung*. It is not part of our purpose to discuss them. Their first scientific expression is to be found in the doctrine of effluences and pores in Empedokles and in Atomism.

It is evident in what light all these dynamic conceptions appear, when looked at from the standpoint of consistent extreme empiricism. These "forces," to consider here only this one of the dynamic hypotheses, help to explain nothing. The physical forces, or those which give rise to movement, are evidently not given to us as contents of sense perception, and at the most they can be deduced as non-sensuous foundations, not as contents of possible sense perception. The often and variously expressed belief that self perception reveals to us here what our senses leave hidden has proved itself to be in all its forms a delusion. The forces whose existence we assume have then an intuitable content only in so far as they get it through the uniformities present in repeated perceptions, which uniformities are to be "explained" through them. But right here their assumption proves itself to be not only superfluous but even misleading; for it makes us believe that we have offered an explanation, whereas in reality we have simply duplicated the given by means of a fiction, quite after the fashion of the Platonic doctrine of ideas. This endeavor to give the formal temporal relations between events, which we interpret as causes and effects, a dynamic real substructure, shows itself thus to be worthless in its contributions to our thought. The same holds true of every other dynamic hypothesis. The critique called forth by these contributions establishes therefore only the validity of the empiristic interpretation.

If, however, we have once come so far, we may not hold ourselves back from the final step. Empiricism has long ago taken this step, and the most consistent among its modern German representatives has aroused anew the impulses that make it necessary. Indeed, if we start from the empiristic presuppositions, we must recognize that there lies not only in the assumption of forces, but even in the habit of speaking of causes and effects, "a clear trace of fetishism." We are not then surprised when the statement is made: The natural science of the future, and accordingly science in general, will, it is to be hoped, set aside these concepts also on account of their formal obscurity. For, so it is explained, repetitions of like cases in which *a* is always connected with *b*, namely, in which like results are found under like circumstances, in short, the essence of the connection of cause and effect, exists only in the abstraction that is necessary to enable us to repicture the facts. In nature itself there are no causes and effects. *Die Natur ist nur einmal da.*

It is, again, Stuart Mill, the man of research, not the empiricist, that opposes this conclusion, and indeed opposes it in the form that Auguste Comte had given it in connection with thoughts that can be read into Hume's doctrine. Comte's "objection to the *word* cause is a mere matter of nomenclature, in which, as a matter of nomenclature, I consider him to be entirely wrong. . . . By rejecting this form of expression, M. Comte leaves himself without any term for marking a distinction which, however incorrectly expressed, is not only real, but is one of the fundamental distinctions in science."[1]

For my own part, the right seems to be on the side of Comte and his recent followers in showing the old nomenclature to be worn out, if viewed from the standpoint of empiricism. If the relation between cause and effect consists alone in the uniformity of sequence which is hypothetically warranted by experience, then it can be only misleading to employ words for the members of this purely formal relation that necessarily have a strong tang of real dynamic dependence. In fact, they give the connection in question a peculiarity that, according to consistent empiricism, it does not possess. The question at issue in the empiristically interpreted causal relation is a formal functional one, which is not essentially different, as Ernst Mach incidentally acknowledges, from the interdependence of the sides and angles of a triangle.

Here two extremes meet. Spinoza, the most consistent of the dogmatic rationalists, finds himself compelled in his formulation of the analytic interpretation of the causal relation handed down to him to transform it into a mathematical one. Mach, the most consistent of recent German empiricists, finds himself compelled to recognize that the empirically synthetic relation between cause and effect includes no other form of dependence than that which is present in the functional mathematical relations. (In Germany empiricism steeped in natural science has supplanted the naïve materialism saturated with natural science.) That the mathematical relations must likewise be subjected to a purely empirical interpretation, which even Hume denied them, is a matter of course.

However, this agreement of two opposing views is no proof that empiricism is on the right road. The empiristic conclusions to which we have given our attention do not succeed in defining adequately the specific nature of the causal relation; on the contrary, they compel us to deny such a relation. Thus they cast aside the concept that we have endeavored to define, that is, the judgment in which we have to comprehend whatever is peculiar to the causal connection. But one does not untie a knot by denying that it exists.

It follows from this self-destruction of the empiristic causal hypothesis that an additional element of thought must be contained in the relation of cause and effect besides the elements of reproductive recognition and those of identification and discrimination, all of which are involved in the abstract comprehension of uniform sequence. The characteristics of the causal connection revealed by our previous analysis constitute the necessary and perhaps adequate conditions for combining the several factual perceptions into the abstract registering idea of uniform sequence. We may, therefore, expect to find that the element sought for lies in the tendency to extend the demand for causal connections over the entire field of possible experience; and perhaps we may at the same time arrive at the condition which led Hume and Mill to recognize the complete universality of the causal law in spite of the exclusively empirical content that they had ascribed to it. In this further analysis also we have to draw from the nature of our thought itself the means of guiding our investigation.

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[1] *Logic*, bk. III, ch. V, § 6.

In the first place, all thought has a formal necessity which reveals itself in the general causal law no less than in every individual thought process, that is, in every valid judgment. The meaning of this formal necessity of thought is easily determined. If we presuppose, for example, that I recognize a surface which lies before me as green, then the perception judgment, "This surface is green," that is, the apprehension of the present perceptive content in the fundamental form of discursive thought, repeats with predicative necessity that which is presented to me in the content of perception. The necessity of thought contained in this perception judgment, as *mutatis mutandis* in every affirmative judgment meeting the logical conditions, is recognizable through the fact that the contradictory judgment, "This surface is not green," is impossible for our thought under the presupposition of the given content of perception and of our nomenclature. It contradicts itself. I can express the contradictory proposition, for instance, in order to deceive; but I cannot really pass the judgment that is contained in it. It lies in the very nature of our thought that the predicate of an assertive judgment can contain only whatever belongs as an element of some sort (characteristic, attribute, state, relation) to the subject content in the wider sense. The same formal necessity of thought, to give a further instance, is present in the thought process of mediate syllogistic predication. The conclusion follows necessarily from the premises, for example, the judgment, "All bodies are divisible," from the propositions, "All bodies are extended," and, "Whatever is extended is divisible."

These elementary remarks are not superfluous; for they make clear that the casually expressed assertion of modern natural scientific empiricism, declaring in effect that there is no such thing as necessity of thought, goes altogether too far. Such necessity can have an admissible meaning only in so far as it denotes that in predicting or recounting *the content* of possible experience every hypothesis is possible for thought. Of course it is, but that is not the subject under discussion.

The recognition of the formal necessity of thought that must be presupposed helps us to define our present question; for it needs no proof that this formal necessity of thought, being valid for every affirmative judgment, is valid also for each particular induction, and again for the general causal law. If in the course of our perceptions we meet uniform sequences, then the judgment, "These sequences are uniform," comprehends the common content of many judgments with formal necessity of thought. Empiricism, too, does not seriously doubt that the hypothesis of a general functional, even though only temporal, relation between cause and effect is deduced as an expectation of possible experience with necessity from our real experience. It questions only the doctrine that the relation between the events regarded as cause and effect has any other than a purely empirical import. The reality of an event that is preceded and followed uniformly by no other remains for this view, as we have seen, a possibility of thought.

In opposition to empiricism, we now formulate the thesis to be established: Wherever two events *a* and *b* are known to follow one another uniformly and immediately, there we must require with formal necessity that some element in the preceding *a* be thought of as fundamental, which will determine sufficiently *b*'s appearance or make that appearance necessary. The necessity of the relation between the events regarded as cause and effect is, therefore, the question at issue.

We must keep in mind from the very start that less is asserted in this formulation than we are apt to read into it. It states merely that something in *a* must be thought of as fundamental, which makes *b* necessary. On the other hand, it says nothing as to what this fundamental something is, or how it is constituted. It leaves entirely undecided whether or not this something that our thought must necessarily postulate is a possible content of perception or can become such, accordingly whether or not it can become an object of our knowledge, or whether or not it lies beyond the bounds of all our possible experience and hence all our possible knowledge. It contains nothing whatsoever that tells us how the determination of *b* takes place through *a*. The word "fundamental" is intended to express all this absence of determination.

Thus we hope to show a necessity of thought peculiar to the relation between cause and effect. This is the same as saying that our proof will establish the logical impossibility of the contradictory assertion; for the logical impossibility of the contradictory assertion is the only criterion of logical necessity. Thus the proof that we seek can be given only indirectly. In the course of this proof, we can disregard the immediacy of the constant sequence and confine our attention to the uniformity of the sequence, not only for the sake of brevity, but also because, as we have seen, we have the right to speak of near and remote causes. We may then proceed as follows.

If there is not something fundamental in a constant antecedent event *a*, which determines necessarily the constant subsequent appearance of one and the same *b*, — that is, if there is nothing fundamental which makes this appearance necessary, — then we must assume that also *c* or *d* . . . , in short, any event you will, we dare not say "follows upon," but appears after *a* in irregular alternation with *b*. This assumption, however, is impossible for our thought, because it is in contradiction with our experience, on the basis of which our causal thought has been developed. Therefore the assumption of a something that is fundamental in *a*, and that determines sufficiently and necessarily the appearance of *b*, is a necessity for our thought.

The assertion of this logical impossibility (*Denkunmöglichkeit*) will at once appear thoroughly paradoxical. The reader, merely recalling the results of the empiristic interpretation given above, will immediately say: "The assumption that a *b* does not follow constantly upon an *a*, but that sometimes *b*, sometimes *c*, sometimes *d* . . . irregularly appears, is in contradiction only with all our previous experience, but it is not on this account a *logical* impossibility. It is merely improbable." The reader will appeal especially to the discussion of Stuart Mill, already quoted, in which Mill pictures *in concreto* such an improbable logical impossibility, and therefore at the same time establishes it in fact. Again, the reader may bring forward the words in which Helmholtz introduces intellectual beings of only two dimensions. "By the much misused expression, 'to be able to imagine to one's self,' or, 'to think how something happens,' I understand (and I do not see how anybody can understand anything else thereby without robbing the expression of all meaning) that one can picture to one's self the series of sense impressions which one would have if such a thing actually took place in an individual case."[1]

Nevertheless, pertinent as are these and similar objections, they are not able to stand the test. We ask: "Is in fact a world, or even a portion of our world, possible for thought that displays through an absolutely irregular alternation of events a chaos in the full sense; or is the attempt to picture such a chaos only a mere play of words to which not even our imagination, not to mention our thought, can give a possible meaning?"

Perhaps we shall reach a conclusion by the easiest way, if we subject Mill's description to a test. If we reduce it to the several propositions it contains, we get the following: (1) Every one is able to picture to himself in his imagination a reality in which events follow one another without rule, that is, so that after an event *a* now *b* appears, now *c*, etc., in complete irregularity. (2) The idea of such a chaos accordingly contradicts neither the nature of our mind nor our experience. (3) Neither the former nor the latter gives us sufficient reason to believe that such an irregular alternation does not actually exist somewhere in the observable world. (4) If such a chaos should be presented to us as fact, that is, if we were in a position to outlive such an alternation, then the belief in the uniformity of time relations would soon cease.

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[1] *Vorträge und Reden*, bd. II, "Über den Ursprung und die Bedeutung der geometrischen Axiome."

Every one would subscribe to the last of these four theses, immediately upon such a chaos being admitted to be a possibility of thought; that is, he would unless he shared the rationalistic conviction that our thought constitutes an activity absolutely independent of all experience. We must simply accept this conclusion on the ground of the previous discussion and of a point still to be brought forward.

If we grant this conclusion, however, then it follows, on the ground of our previous demonstration of the reproductive and recognitive, as well as thought elements involved in the uniform sequence, that the irregularity in the appearance of the events, assumed in such a chaos, can bring about an absolutely relationless alternation of impressions for the subject that we should presuppose to be doing the perceiving. If we still wish to call it perception, it would remain only a perception in which no component of its content could be related to the others, a perception, therefore, in which not even the synthesis of the several perception contents could be apprehended as such. That is, every combination of the different perception contents, by which they become components of one and the same perception, presupposes, as we have seen, those reproductive and recognitive acts in revival which are possible only where uniformities of succession (and of coexistence) exist. Again, every act of attention involved in identifying and discriminating, which likewise we have seen to be possible only if we presuppose uniformities in the given contents of perception, must necessarily disappear when we presuppose the chaotic content; and yet they remain essential to the very idea of such a chaos. A relationless chaos is after all nothing else than a system of relations thought of without relations! That the same contradiction obtains also in the mere mental picturing of a manifold of chaotic impressions needs no discussion; for the productive imagination as well as the reproductive is no less dependent than is our perceptive knowledge upon the reproductive recognition and upon the processes of identifying and discriminating.

Thus the mental image of a chaos could be formed only through an extended process of ideation, which itself presupposes as active in it all that must be denied through the very nature of the image. A relationless knowledge, a relationless abstraction, a relationless reproduction or recognition, a relationless identification or discrimination, in short, a relationless thought, are, as phrases, one and all mere contradictions. We cannot picture "through our relating thought,'' to use Helmholtz's expression, nor even in our imagination, the sense impressions that we should have if our thought were relationless, that is, were nullified in its very components and presuppositions. In the case of Helmholtz's two dimensional beings, the question at issue was not regarding the setting aside of the conditions of our thought and the substituting conditions contradictory to them, but regarding the setting aside of a part of the content of our sense intuition, meanwhile retaining the conditions and forms peculiar to our thought. In this case, therefore, we have a permissible fiction, whereas in Mill's chaos we have an unthinkable thought.

Again, the sense impressions that must be presupposed in an inherently relationless chaos have no possible relation to the world of our perception, whose components are universally related to each other through the uniformities of their coexistences and sequences. Accordingly, the remark with which Helmholtz concludes the passage above quoted holds, *mutatis mutandis*, here also. "If there is no sense impression known that stands in relation to an event which has never been observed (by us), as would be the case for us were there a motion toward a fourth dimension, and for those two dimensional beings were there a motion toward our third dimension; then it follows that such an 'idea' is impossible, as much so as that a man completely blind from childhood should be able to 'imagine' the colors, if we could give him too a conceptual description of them."

Hence the first of the theses in which we summed up Stuart Mill's assumptions must be rejected. With it go also the second and third. In this case we need not answer the question: In how far do these theses correspond to Mill's own statements regarding the absolute surety and universality of the causal law?

We have now found what we sought, in order to establish as a valid assertion the seeming paradox in the proof of the necessity that we ascribe to the relation between cause and effect. We have proved that the assumption of a completely irregular and therefore relationless alternation of impressions contradicts not only our experience, but even the conditions of our thought; for these presuppose the uniformities of the impressions, and consequently our ability to relate them, all which was eliminated from our hypothetical chaos. Hence we have also established that a necessary relation is implied in the thought of a constant sequence of events, which makes the uniformly following b really dependent upon the uniformly preceding a.

From still another side, we can make clear the necessity asserted in the relation of cause and effect. We found that the connection between each definite cause and its effect is an empirically synthetic one and has as its warrant merely experience. We saw further that the necessity inherent in the causal connection contains merely the demand that there shall be something fundamental in the constantly preceding *a* which makes the appearance of *b* necessary; not, however, that it informs us what this efficacy really is, and hence also not that it informs us how this efficacy brings about its effect. Finally, we had to urge that every induction, the most general no less than the most particular, depends upon the presupposition that the same causes will be given in the reality not yet observed as in that already observed. This expectation is warranted by no necessity of thought, not even by that involved in the relation of cause and effect; for this relation begins for future experience only when the presupposition that the same causes will be found in it is assumed as fulfilled.[1] This expectation is then dependent solely upon previous experience, whose servants we are, whose lords we can never be. Therefore, every induction is an hypothesis requiring the verification of a broader experience, since, in its work of widening and completing our knowledge, it leads us beyond the given experience to a possible one. In this respect we can call all inductive thought empirical, that is, thought that begins with experience, is directed to experience, and in its results is referred to experience. The office of this progressing empirical thought is accordingly to form hypotheses from which the data of perception can be regressively deduced, and by means of which they can be exhibited as cases of known relations of our well-ordered experience, and thus can be explained.

The way of forming hypotheses can be divided logically into different sections which can readily be made clear by an example. The police magistrate finds a human corpse under circumstances that eliminate the possibility of accident, natural death, or suicide; in short, that indicate an act of violence on the part of another man. The general hypothesis that he has here to do with a crime against life forms the guide of his investigation. The result of the circumstantial evidence, which we presuppose as necessary, furnishes then a special hypothesis as following from the general hypothesis.

It is clear that this division holds for all cases of forming hypotheses. A general hypothesis serves every special hypothesis as a heuristic principle. In the former we comprehend the causal explanation indicated immediately by the facts revealed to our perception in the special case. It contains, as we might also express it, the genus to the specific limitations of the more exact investigation. But each of these general hypotheses is a modification of the most general form of building hypotheses, which we have already come to know as the condition of the validity of all inductive inferences, that is, as the condition for the necessity of their deduction, and, consequently, as the condition for the thought that like causes will be given in the reality not yet observed as in that already observed. We have further noticed that in this most general form of building hypotheses there lie two distinct and different valid assumptions: beside the empirical statement that like causes will be given, which gives the inductive conclusion the hypothetical form, there stands the judgment that like causes bring forth like effects, a corollary of the causal law. The real dependence of the effect upon the cause, presupposed by this second proposition and the underlying causal law, is not, as was the other assumption, an hypothesis, but a necessary requirement or *postulate* of our thought. Its necessity arises out of our thought, because our experience reveals uniformity in the sequence of events. From this point of view, therefore, the causal law appears as a postulate of our thought, grounded upon the uniformity in the sequence of events. It underlies every special case of constructing hypotheses as well as the expectation that like causes will be given in the reality not yet observed.

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[1] The only empiricism which can maintain that the same causes would, in conformity with the causal law, be given in the unobserved reality, is one which puts all events that can be regarded as causes in the immediately given content of perception as its members. Such a view is not to be found in Mill; and it stands so completely in the way of all further analysis required of us by every perception of events that no attention has been paid in the text to this extreme of extremes.

Mill's logic of induction contains the same fault as that already present in Hume's psychological theory of cause. Hume makes merely the causal law itself responsible for our inductive inferences, and accordingly (as Mill likewise wrongly assumes) for our inferences in general. But we recognize how rightly Mill came to assert, in contradiction to his empiristic presuppositions, that the causal law offers "an undoubted assurance of an invariable, universal, and unconditional," that is, necessary, sequence of events, from which no seeming irregularity of occurrence and no gap in our experience can lead us astray, as long as experience offers uniformities of sequence.

Rationalism is thus in the right, when it regards the necessary connection as an essential characteristic of the relation between cause and effect, that is, recognizes in it a relation of real dependence. At this point Kant and Schopenhauer have had a profounder insight than Hume and Stuart Mill. Especially am I glad to be in agreement with Lotze on a point which he reached by a different route and from essentially different presuppositions. Lotze distinguishes in pure logic between postulates, hypotheses, and fictions. He does not refer the term "postulate" exclusively to the causal law which governs our entire empirical thought in its formation of hypotheses, but gives the term a wider meaning. "Postulates" are only corollaries from the inductive fundamental form of all hypothesis construction, and correspond essentially to what we have called general or heuristic hypotheses. His determination of the validity of these postulates, however, implies the position to be assigned to the causal law and therefore not to those heuristic hypotheses. "The postulate is not an assumption that we can make or refrain from making, or, again, in whose place we can substitute another. It is rather an (absolutely) necessary assumption without which the content of the view at issue would contradict the laws of our thought." [1]

Still the decision that we have reached is not on this account in favor of rationalism, as this is represented for instance by Kant and his successors down to our own time, and professed by Lotze in the passage quoted, when he speaks of an absolute necessity for thought. We found that the causal law requires a necessary connection between events given us in constant sequence. It is not, however, on that account a law of our thought or of a "pure understanding" which would be absolutely independent of all experience. When we take into consideration the evolution of the organic world of which we are members, then we must say that our intellect, that is, our ideation and with it our sense perception, has evolved in us in accordance with the influences to which we have been subjected. The common elements in the different contents of perception which have arisen out of other psychical elements, seemingly first in the brute world, are not only an occasion, but also an efficient cause, for the evolution of our processes of reproduction, in which our memory and imagination as well as our knowledge and thought, psychologically considered, come to pass. The causal law, which the critical analysis of the material-scientific methods shows to be a fundamental condition of empirical thought, in its requirement that the events stand as causes and effects in necessary connection, or real dependence, comprehends these uniform contents of perception only in the way peculiar to our thought.

Doubtless our thought gives a connection to experience through this its requirement which experience of itself could not offer. The necessary connection of effect with cause, or the real dependence of the former upon the latter, is not a component of possible perception. This requirement of our thought does not, however, become thereby independent of the perceptive elements in the presuppositions involved in the uniformity of sequence. The *a priori* in the sense of "innate ideas," denoting either these themselves or an absolutely *a priori* conformity to law that underlies them, for instance, our "spontaneity," presupposes in principle that our "soul" is an independently existing substance in the traditional metaphysical sense down to the time of Locke. Kant's rationalistic successors, for the most part, lost sight of the fact that Kant had retained these old metaphysical assumptions in his interpretation of the transcendental conditions of empirical interaction and in his cosmological doctrine of freedom. The common root of the sensibility and of the understanding as the higher faculty of knowledge remains for Kant the substantial force of the soul, which expresses itself (just as in Leibnitz) as *vis passiva* and *vis activa*. The modern doctrine of evolution has entirely removed the foundation from this rationalism which had been undermined ever since Locke's criticism of the traditional concept of substance.

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[1] *Logic*, 1874, buch II, kap. viii.

To refer again briefly to a second point in which the foregoing results differ from the Kantian rationalism as well as from empiricism since Hume: The postulate of a necessary connection between cause and effect, as we have seen, in no way implies the consequence that the several inductions lose the character of hypotheses. This does not follow merely from the fact that all inductions besides the causal law include the hypothetical thought that the same causes will be given in the reality not yet observed as appear in that already observed. The hypothetical character of all inductive inferences is rather revealed through the circumstance that in the causal postulate absolutely nothing is contained regarding *what* the efficacy in the causes is, and *how* this efficacy arises.

Only such consequences of the foregoing interpretation of the causal law and of its position as one of the bases of all scientific construction of hypotheses may be pointed out, in conclusion, as will help to make easier the understanding of the interpretation itself.

The requirement of a necessary connection, or dependence, is added by our thought to the reproductive and recognitive presuppositions that are contained in the uniformity of the sequence of events. If this necessary connection be taken objectively, then it reveals as its correlate the requirement of a real dependence of effect upon cause. We come not only upon often and variously used rationalistic thoughts, but also upon old and unchangeable components of all empirical scientific thought, when we give the name "force" to the efficacy that underlies causes. The old postulate of a dynamic intermediary between the events that follow one another constantly retains for us, therefore, its proper meaning. We admit without hesitation that the word "force" suggests fetishism more than do the words "cause" and "effect;" but we do not see how this can to any degree be used as a counter-argument. All words that were coined in the olden time to express thoughts of the practical *Weltanschauung* have an archaic tang. Likewise all of our science and the greater part of our nomenclature have arisen out of the sphere of thought contained in the practical *Weltanschauung*, which centred early in fetishism and related thoughts. If, then, we try to free our scientific terminology from such words, we must seek refuge in the Utopia of a *lingua universalis*, in short, we must endeavor to speak a language which would make science a secret of the few. Or will any one seriously maintain that a thought which belongs to an ancient sphere of mental life must be false for the very reason that it is ancient?

In any case, it is fitting that we define more closely the sense in which we are to regard forces as the dynamic intermediaries of uniform occurrence. Force cannot be given as a content of perception either through our senses or through our consciousness of self; in the case of the former, not in our kinesthetic sensations, in the case of the latter, not in our consciousness of volition. Volition would not include a consciousness of force, even though we were justified in regarding it as a simple primitive psychosis, and were not compelled rather to regard it as an intricate collection of feelings and sensations as far as these elementary forms of consciousness are connected in thought with the phenomena of reaction. Again, forces cannot be taken as objects that are derived as *possible* perceptions or after the analogy of possible perceptions. The postulate of our thought through which these forces are derived from the facts of the uniform sequence of events, reveals them as limiting notions (*Grenzbegriffe*), as specializations of the necessary connection between cause and effect, or of the real dependence of the former upon the latter; for the manner of their causal intermediation is in no way given, rather they can be thought of only as underlying our perceptions. They are then in fact *qualitates occultae;* but they are such only because the concept of quality is taken from the contents of our sense and self perception, which of course do not contain the necessary connection required by our thought. Whoever, therefore, requires from the introduction of forces new contents of perception, for instance, new and fuller mechanical pictures, expects the impossible.

The contempt with which the assumption of forces meets, on the part of those who make this demand, is accordingly easily understood, and still more easily is it understood, if one takes into consideration what confusion of concepts has arisen through the use of the term "force" and what obstacles the assumption of forces has put in the way of the material sciences. It must be frankly admitted that this concept delayed for centuries both in the natural and moral sciences the necessary analysis of the complicated phenomena forming our data. Under the influence of the "concept philosophy" it caused, over and over again, the setting aside of the problems of this analytical empirical thought as soon as their solution had been begun. This misuse cannot but make suspicious from the very start every new form of maintaining that forces underlie causation.

However, misuse proves as little here against a proper use as it does in other cases. Moreover, the scruples that we found arising from the standpoint of empiricism against the assumption of forces are not to the point. In assuming a dynamic intermediary between cause and effect, we are not doubling the problems whose solution is incumbent upon the sciences of facts, and still less is it true that our assumption must lead to a logical circle. That is, a comparison with the ideas of the old concept philosophy, which even in the Aristotelian doctrine contain such a duplication, is not to the point. Those ideas are hypostasized abstractions which are taken from the uniformly coexisting characteristics of objects. Forces, on the other hand, are the imperceivable relations of dependence which we must presuppose between events that follow one another uniformly, if the uniformity of this sequence is to become for us either thinkable or conceivable. The problems of material scientific research are not doubled by this presupposition of a real dynamic dependence, because it introduces an element not contained in the data of perception which give these problems their point of departure. This presupposition does not renew the thought of an analytic rational connection between cause and effect which the concept philosophy involves; on the contrary, it remains true to the principle made practical by Hume and Kant, that the real connection between causes and their effects is determinable only through experience, that is, empirically and synthetically through the actual indication of the events of uniform sequence. How these forces are constituted and work, we cannot know, since our knowledge is confined to the material of perception from which as a basis presentation has developed into thought. The insight that we have won from the limiting notion of force helps us rather to avoid the misuse which has been made of the concept of force. A fatal circle first arises, when we use the unknowable forces and not the knowable events for the purpose of explanation, that is, when we cut off short the empirical analysis which leads *ad infinitum*. To explain does not mean to deduce the known from the unknown, but the particular from the general. It was therefore no arbitrary judgment, but an impulse conditioned by the very nature of our experience and of our thought, that made man early regard the causal connection as a dynamic one, even though his conception was of course indistinct and mixed with confusing additions.

The concept of force remains indispensable also for natural scientific thought. It is involved with the causal law in every attempt to form an hypothesis, and accordingly it is already present in every description of facts which goes by means of memory or abstraction beyond the immediately given content of present perception. In introducing it we have in mind, moreover, that the foundations of every possible interpretation of nature possess a dynamic character, just because all empirical thought, in this field as well, is subordinate to the causal law. This must be admitted by any one who assumes as indispensable aids of natural science the mechanical figures through which we reduce the events of sense perception to the motion of mass particles, that is, through which we associate these events with the elements of our visual and tactual perception. All formulations of the concept of mass, even when they are made so formal as in the definition given by Heinrich Hertz, indicate dynamic interpretations. Whether the impelling forces are to be thought of in particular as forces acting at a distance or as forces acting through collision depends upon the answer to the question whether we have to assume the dynamic mass particles as filling space discontinuously or continuously. The dynamic basis of our interpretation of nature will be seen at once by any one who is of the opinion that we can make the connection of events intelligible without the aid of mechanical figures, for instance, in terms of energy.

Thus it results that we interpret the events following one another immediately and uniformly as causes and effects, by presupposing as fundamental to them forces that are the necessary means of their uniformity of connection. What we call "laws" are the judgments in which we formulate these causal connections.

A second and a third consequence need only be mentioned here. The hypothesis that interprets the mutual connection of psychical and physical vital phenomena as a causal one is as old as it is natural. It is natural, because even simple observations assure us that the mental content of perception *follows* uniformly the instigating physical stimulus and the muscular movement the instigating mental content which we apprehend as will. We know, however, that the physical events which, in raising the biological problem, we have to set beside the psychical, do not take place in the periphery of our nervous system and in our muscles, but in the central nervous system. But we must assume, in accordance with all the psychophysiological data which at the present time are at our disposal, that these events in our central nervous system do not follow the corresponding psychical events, but that both series have their course simultaneously. We have here, therefore, instead of the real relation of dependence involved in constant sequence, a real dependence of the simultaneity or correlative series of events. This would not, of course, as should be at once remarked, tell as such against a causal connection between the two separate causal series. But the contested parallelistic interpretation of this dependence is made far more probable through other grounds. These are in part corollaries of the law of the conservation of energy, rightly interpreted, and in part epistemological considerations. Still it is not advisable to burden methodological study, for instance, the theory of induction, with these remote problems; and on that account it is better for our present investigation to subordinate the psychological interdependences to the causal ones in the narrower sense.

The final consequence, too, that forces itself upon our attention is close at hand in the preceding discussion. The tradition prevailing since Hume, together with its inherent opposition to the interpretation of causal connection given by the concept philosophy, permitted us to make the uniform sequences of events the basis of our discussion. In so doing, however, our attention had to be called repeatedly to one reservation. In fact, only a moment ago, in alluding to the psychological interdependences, we had to emphasize the uniform *sequence*. Elsewhere the arguments depended upon the *uniformity* that characterizes this sequence; and rightly, for the reduction of the causal relation to the fundamental relation of the sequence of events is merely a convenient one and not the only possible one. As soon as we regard the causal connection, along with the opposed and equal reaction, as an interconnection, then cause and effect become, as a matter of principle, simultaneous. The separation of interaction from causation is not justifiable.

In other ways also we can so transform every causal relation that cause and effect must be regarded as simultaneous. Every stage, for instance, of the warming of a stone by the heat of the sun, or of the treaty conferences of two states, presents an effect that is simultaneous with the totality of the acting causes. The analysis of a cause that was at first grasped as a whole into the multiplicity of its constituent causes and the comprehension of the constituent causes into a whole, which then presents itself as the effect, is a necessary condition of such a type of investigation. This conception, which is present already in Hobbes, but especially in Herbart's "method of relations," deserves preference always where the purpose in view is not the shortest possible argumentation but the most exact analysis.

If we turn our attention to this way of viewing the problem, — not, however, in the form of Herbart's speculative method, — we shall find that the results which we have gained will in no respect be altered. We do, however, get a view beyond. From it we can find the way to subordinate not only the uniform sequence of events, but also the persistent characteristics and states with their mutual relations, under the extended causal law. In so doing, we do not fall back again into the intellectual world of the concept philosophy. We come only to regard the *persisting coexistences* — in the physical field, the bodies, in the psychical, the subjects of consciousness — as systems or modes of activity. The thoughts to which such a doctrine leads are accordingly not new or unheard of. The substances have always been regarded as sources of modes of activity. We have here merely new modifications of thoughts that have been variously developed, not only from the side of empiricism, but also from that of rationalism. They carry with them methodologically the implication that it is possible to grasp the totality of reality, as far as it reveals uniformities, as a causally connected whole, as a cosmos. They give the research of the special sciences the conceptual bases for the wider prospects that the sciences of facts have through hard labor won for themselves. The subject of consciousness is unitary as far as the processes of memory extend, but it is not simple. On the contrary, it is most intricately put together out of psychical complexes, themselves intricate and out of their relations; all of which impress upon us, psychologically and, in their mechanical correlates, physiologically, an ever-recurring need for further empirical analysis. Among the mechanical images of physical reality that form the foundation of our interpretation of nature, there can finally be but one that meets all the requirements of a general hypothesis of the continuity of kinetic connections. With this must be universally coördinated the persistent properties or sensible modes of action belonging to bodies. The mechanical constitution of the compound bodies, no matter at what stage of combination and formation, must be derivable from the mechanical constitution of the elements of this combination. Thus our causal thought compels us to trace back the persistent coexistences of the so-called elements to combinations whose analysis, as yet hardly begun, leads us on likewise to indefinitely manifold problems. Epistemologically we come finally to a universal phenomenological dynamism as the fundamental basis of all theoretical interpretation of the world, at least fundamental for our scientific thought, and we are here concerned with no other.

SECTION E — ETHICS

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(*Hall 6, September 23, 10 a. m.*)

Chairman: Professor George H. Palmer, Harvard University.
Speakers: Professor William R. Sorley, University of Cambridge.
Professor Paul Hensel, University of Erlangen.
Secretary: Professor F. C. Sharp, University of Wisconsin.

THE RELATIONS OF ETHICS

BY WILLIAM RITCHIE SORLEY

[**William Ritchie Sorley**, Knightbridge Professor of Moral Philosophy in the University of Cambridge; Fellow of the British Academy. **b.** Selkirk, Scotland, 1855. M.A. Edinburgh; Litt.D. Cambridge; Hon. LL.D. Edinburgh. Post-Graduate, Shaw Fellow, Edinburgh University, 1878; Fellow, Trinity College, Cambridge, 1883; Lecturer, Local Lectures Syndicate and for the Moral Science Board, Cambridge, 1882–86; Deputy for the Professor of Philosophy, University College, London, 1886–87; Professor of Philosophy, University College, Cardiff, 1888–94; Regius Professor Moral Philosophy, Aberdeen, 1894–1900. **Author of** *Ethics of Naturalism*, 1885 (new ed. 1904); *Mining Royalties*, 1889; *Recent Tendencies in Ethics*, 1904; Edition of *Adamson Development of Modern Philosophy*, 1903.]

There are many departments of inquiry whose scope is so well defined by the consensus of experts that one may proceed, almost without preliminary, to mark off the boundaries of one science from other departments, to investigate the relations in which it stands to them, and to exhibit the place which each occupies in the whole scheme of human knowledge. In other departments opinion differs not only regarding special problems and results, but concerning the whole nature of the science and its relation to connected subjects. The study of ethics still belongs to this latter group. In it there is no consensus of experts. Competent scholars hold diametrically opposed views as to its scope. They differ not merely in the answers they give to ethical questions, but in their views as to what the fundamental question of ethics is. And this opposition of opinion as to its nature is connected with a difference of view regarding the relation of ethics to the sciences. By many investigators it is set in line with the sciences of biology, psychology, and sociology; and its problems are formulated and discussed by the application of the same historical method as those sciences employ. On the other hand, it is maintained that ethics implies and requires a concept so different from the concepts used by the historical and natural sciences as to give its problem an altogether distinct character and to indicate for it a far more significant position in the whole scheme of human thought.

The question of the relation of ethics to the sciences implies a view of the nature of ethics itself and, in particular, of the fundamental concept used in ethical judgments. If the nature of this concept and its relation to the concepts employed in other branches of inquiry can be determined, the relations of ethics will become clear of themselves. The problem of this paper will receive its most adequate solution — so far as the time at my disposal permits — by an independent inquiry into the nature of the ethical concept in relation to the concepts used in other sciences.

The immediate judgments of experience fall into two broadly contrasted classes, which may be described in brief as judgments of fact and judgments of worth. The former are the foundations on which the whole edifice of science (as the term is commonly used) is built. Science has no other object than to understand the relations of facts as exhibited in historical sequence, in causal interconnection, or in the logical interdependence which may be discovered amongst their various aspects. In its beginnings it may have arisen as an aid to the attainment of practical purposes: it is still everywhere yoked to the chariot of man's desires and aims. But it has for long vindicated an independent position for itself. It may be turned to what uses you will; but its essential spirit stands aloof from these uses. It has one interest only, — to know what happens and how. Otherwise it is indifferent to all purposes alike. It studies with equal mind the slow growth of a plant or the swift destruction wrought by the torpedo, the reign of a Caligula or of a Victoria; it takes no side, but observes and describes all "just as if the question were of lines, planes, and solids." Mathematical method does not limit its range, but it typifies its attitude of indifference to every interest save one, — that of knowing the what and how of things.

We can conceive an intelligence of this nature, a pure intelligence, or mere intelligence, to whose understanding all the relations of things are evident, with the prophetic power of the Laplacian Demon and the gift of tongues to make its knowledge clear, and yet unable to distinguish between good and evil or to see beauty or ugliness in nature. We can conceive such an intelligence; but it is an unreality, a mere abstraction from the scientific aspect of human intelligence. Pure intelligence of this sort does not exist in man, and we have no grounds for asserting its existence anywhere. In the experience which forms the basis of mental life, judgments of reality are everywhere combined with and colored by judgments of worth. And the latter are as insistent as the former, and make up as large a part of our experience. If we go back to the original judgments of experience, we find that they are not only of the form "it is here or there," "it is of this nature or that," "it has such and such effects;" just as a large part of our experience is of another order which may be expressed in judgments of the form "it is good or evil," "it is fair or foul."

Nor does the way in which scientific judgments are elaborated give any rationale of the distinction between good and evil. If we ask of science "What is good?" it can give no relevant answer to the question. Strictly speaking, it does not understand the meaning of the question at all. The ball has gone out of bounds; and science cannot touch it until it has been thrown back into the field. It can say what is, and what will happen, and it can describe the methods or laws by which things come to pass; that is all; it has only one law for the just and the unjust.

But science is very resourceful, and is able to deal with judgments of worth from its own point of view. For these judgments also are facts of individual experience: they are formed by human minds under certain conditions, betray certain relations to the judgments of fact with which they are associated, and are connected with an environment of social institutions and physical conditions of life: they have a history therefore. And in these respects they become part of the material for science: and a description of them can be given by psychological and historical methods.

The general nature and results of the application of these methods to ethics are too well known to need further comment, too well established to require defense. But these results may be exaggerated and have been exaggerated. When all has been said and done that the historical method can say and do, the question "What is good?" is found to remain exactly where it was. We may have learned much as to the way in which certain kinds of conduct in certain circumstances promote certain ends, and as to the gradual changes which men's ideas about good and evil, virtue and vice, have passed through; but we have not touched the fundamental question which ethics has to face — the question of the nature of worth or goodness or duty.

And yet it is this question only which gives significance to the problems on which historical evolution has been able to throw light. Moral ideas and moral institutions have all along been effective factors in human development, as well as the subject of development themselves. And the secret of their power has lain in this that men have believed in those ideas as expressing a moral imperative or a moral end, and that they have looked upon moral institutions as embodiments of something which has worth for man or a moral claim upon his devotion. These ideas and institutions would have had no power apart from this belief in their validity.

But was this belief true? Were the ideas or institutions valid? This question the man of science, as sociologist or historian, does not answer and has no means of answering. He can show their adaptation or want of adaptation to certain ends, but he can say nothing about the validity of these ends themselves. It is implied in their efficiency that these ends were conceived as having moral value or moral authority. But to what ends does this moral value or authority truly belong? and what is its significance? — these are questions which the positive sciences (such as psychology and sociology) cannot touch and which must be answered by other methods than those which they employ.

The moral concept is expressed in various ways and by a variety of terms, — right, duty, merit, virtue, goodness, worth. And these different terms indicate different aspects opened up by a single new point of view. Thus "right" seems to imply correspondence with a standard or rule, which standard or rule is some moral law or ideal of goodness; and "merit" indicates performance of the right, perhaps in victory over some conflicting desire; and "virtue" means a trait of character in which performance of this sort has become habitual. The term "worth" has conveniences which have led to its having considerable vogue in ethical treatises since the time of Herbart; it lends itself easily to psychological manipulation; but it does not seem to refer to a concept fundamentally distinct from goodness. But between "goodness" and "duty" there seems to be this difference at any rate, that the latter term refers definitely to something to be done by a voluntary agent, whereas, in calling something "good," we may have no thought of action at all, but only see and name a quality.

There lies here therefore a difference which is not a mere difference of expression.

On the one hand it may be held that good is a quality which belongs to certain things and has no special and immediate reference to volition: that we say this or that is good as we say that something else is heavy or green or positively electrified. No relation to human life at all may be implied in the one form of judgment any more than in the other. That relation will only follow by way of application to circumstances. Just as a piece of lead may serve as a letter-weight because it is heavy, so certain actions may come to be our duty because they lead to the realization of something which is objectively good in quality.

According to the other view goodness has reference in its primary meaning to free self-conscious agency. The good is that which ought to be brought into existence: goodness is a quality of things, but only in a derivative regard because these things are produced by a good will. It is objective, too, inasmuch as it unites the individual will with a law or ideal which has a claim upon the will; but it does not in its primary meaning indicate something out of relation to the will: if there were no will there would be no law; apart from conscious agency good and evil would disappear.

The question thus raised is one of real and fundamental importance. "Ethics" by its very name may seem to have primary reference to conduct; and that is the view which most moralists have, in one way or another, adopted. But the other view which gives to the concept "good" an independence of all relation to volition is not always definitely excluded, even by these moralists; by others it has been definitely maintained: it seems implied in Plato's idealism, at one stage of its development; and quite recently a doctrine of the principles of ethics has been worked out which is based on its explicit recognition.[1]

If we would attempt to decide between these two conflicting views of the ethical concept, we must, in the first place, imitate the procedure of science and examine the facts on which the concept is based. To get to the meaning of such scientific concepts as "mass," "energy," or the like, we begin by a consideration of the facts which the concepts are introduced to describe. These facts are in the last resort the objects of sense perception. No examination of these sense percepts will, as we have seen, yield the content of the ethical concept; good and evil are not given in sense perception — they are themselves an estimate of, or way of regarding, the immediate material of experience. Moral experience is thus in a manner reflex, as so many of the English moralists have called it. Its attitude to things is not merely receptive; and the concepts to which it gives rise have not mere understanding in view. Objects are perceived as they occur; and experience of them is the groundwork of science. There is also, at the same time, an attitude of approbation or disapprobation; this attitude is the special characteristic of moral experience; and from moral experience the ethical concept is formed.

This reflex experience, or reflex attitude to experience, is exhibited in different ways. There is, to begin with, the appreciation of beauty in its various kinds and degrees and the corresponding depreciation of ugliness or deformity. These give rise to the concepts and judgments of æsthetics. They are closely related to moral approbation and disapprobation, so closely that there has always been a tendency amongst a school of moralists to strain the facts by identifying them. A certain looseness in our use of terms favors this tendency. For we do often use good of a work of art or even scene in nature when we mean beautiful. But if we reflect on and compare our mental attitudes in commending, say, a sunset and self-sacrifice, it seems to me that there can be no doubt that the two attitudes are different. Both objects may be admired; but both are not, in the same sense, approved. It is hard to express this difference otherwise than by saying that the moral attitude is present in the one and absent in the other. But the difference is brought out by the fact that our æsthetical and moral attitudes towards the same experience may diverge from one another. We may admire the beauty of that which we condemn as immoral. De Quincey saw a fine art in certain cases of murder; the finish and perfection of wickedness may often stir a certain artistic admiration, especially if we lull the moral sense to sleep. And, on the other hand, moral approval is often tempered by a certain æsthetic depreciation of those noble characters who do good awkwardly, without the ease and grace of a gentleman. John Knox and Mary Queen of Scots (if I may assume for the moment an historical judgment which may need qualification) will each have his or her admirers according as the moral or æsthetic attitude preponderates — the harsh tones of the one appealing to the law of truth and goodness, the other an embodiment of the beauty and gaiety of life, "without a moral sense, however feeble."

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[1] *Principia Ethica*, by G. E. Moore (1903).

Nor is æsthetic appreciation the only other reflex attitude which has a place in our experience side by side with the moral. Judgments about matters of fact and relations of ideas are discriminated as true or false; an ideal of truth is formed; and conditions of its realization are laid down. Here again we have a concept and class of judgments analogous to our æsthetical and ethical concepts and judgments, but not the same as them, and not likely to be confused with them.

Beside these may be put a whole class of judgments of worth which may be described as judgments of utility. We estimate and approve or disapprove various facts of experience according to their tendency to promote or interfere with certain ends or objects of desire. That moral judgments are to be identified with a special class of these judgments of utility is a thesis too well known to require discussion here, and too important to admit of discussion in a few words. But it may be pointed out that it is only in a very special and restricted sense of the term "utility" that judgments of utility have ever been identified with moral judgments. The "jimmy" is useful to the burglar, as his instruments are useful to the surgeon; and they are in both cases appreciated by the same kind of reflective judgment. Judgments of utility are all of them, properly speaking, judgments about means to ends; and the ends may and do differ; while it is only by a forced interpretation that all these ends are sometimes and somehow made to resolve themselves into pleasure.

It is enough, however, for my present purpose to recognize the *prima facie* distinction of moral judgments or judgments of goodness from other judgments of worth, such as those of utility, of beauty, and of truth (in the sense in which these last also are judgments of worth). Had the question of the origin and history of the moral judgment been before us, a great deal more might have been necessary. For our present purpose what has been already said may be sufficient: it was required in order to enable us to approach the consideration of the question already raised concerning the application and meaning of the moral concept.

The question is, Does our moral experience support the assignment of the predicate "good" or "bad" to things regarded as quite independent of volition or consciousness? At first sight it may seem easy to answer the question in the affirmative. We do talk of sunshine and gentle rain and fertile land as good, and of tornadoes and disease and death as bad. But I think that when we do so, in nine cases out of ten, our "good" or "bad" is not a moral good or bad; they are predicates of utility or sometimes æsthetic predicates, not moral predicates; and we recognize this in recognizing their relativity: the fertile land is called good because its fertility makes it useful to man's primary needs; but the barren and rocky mountain may be better in the eyes of the tourist, though the farmer would call it bad land. There is an appreciation, a judgment of worth in the most general sense, in such experiences; but they are in most cases without the special feature of moral approbation or disapprobation.

There remains, however, the tenth case in which the moral predicate does seem to be applied to the unconscious. One may instance J. S. Mill's passionate impeachment of the course of nature, in which "habitual injustice" and "nearly all the things which men are hanged or imprisoned for doing to one another" are spoken of as "nature's every-day performances;"[1] and a similar indictment was brought by Professor Huxley, twenty years after the publication of Mill's essay, against the cosmic process for its encouragement of selfishness and ferocity.[2] These are only examples. Literature is full of similar reflections on the indiscriminate slaughter wrought by the earthquake or the hurricane, and on the sight of the wicked flourishing or of the righteous begging his bread; and these reflections find an echo in the experience of most men.

But the nature of this experience calls for remark.

In the first place, if we look more closely at the arguments of Mill or Huxley, we see that both are cases of criticism of a philosophical theory. Mill was refuting a view which he held (and rightly held) to have influence still on popular thought, though it might have ceased to be a living ethical theory — the doctrine that the standard of right and wrong was to be found in nature; it was in keeping with his purpose, therefore, to speak of the operations of nature as

[1] J. S. Mill, *Three Essays on Religion*, pp. 35, 38.

[2] T. H. Huxley, *Evolution and Ethics* (Romanes Lecture).

if they were properly the subject of moral praise or blame. In the same way, when Huxley wrote, the old doctrine which Mill regarded as philosophically extinct and only surviving as a popular error had been revived by the impetus which the theory of evolution had given to every branch of study; and Huxley was criticising the evolutionist ethics of Spencer and others who looked for moral guidance to the course of evolution. He, therefore, was led to speak of the cosmic process as a possible subject of moral predicates, not necessarily because he thought that application appropriate, but in order to demonstrate the hollowness of the ethics of evolution by showing that if the moral predicate could be applied at all, then the appropriate adjective would be not "good" but "bad."

Perhaps there is more than this in Huxley; and Mill's expressions often betray a direct and genuine moral condemnation of the methods of nature as methods of wickedness; and, still more clearly, this immediate moral disapproval may be found in expressions of common experience as yet uncolored by philosophy. But if we examine these we find that, while there is no reference to philosophical theories about nature, the things approved or condemned are yet looked upon as implying consciousness. In the lower stages of development this implication is simply animistic; at a later period it becomes theological. But throughout experience moral judgments upon nature are not passed upon mere nature. Its forces are regarded as expressing a purpose or mind; and it is this that is condemned or approved. The primitive man and the child do not merely condemn the misdoings of inanimate objects; they wreak their vengeance upon them or punish them: and this is a consequence of their animistic interpretation of natural forces. Gradually, in the mental growth of the child, this animistic interpretation of things gives place to an understanding of the natural laws of their working; and at the same time and by the same degrees, the child ceases to inflict punishment upon the chair that has fallen on him or to condemn its misdemeanor. Here the moral judgment is displaced by the causal judgment; and the reason of its displacement is the disappearance of mind or purpose from amongst the phenomena. When the child comes to understand that the chair falls by "laws of nature" which are not the expression of will, like the acts done by himself or his companions, he ceases to disapprove or to resent, though he does not cease to feel pain or to improve the circumstances by setting the chair firmly on the floor. The recognition of natural causation as all that there is in the case leaves no room for the moral attitude. So true is this that the same result is sometimes thought to be a consequence of the scientific understanding even of what is called moral causation, "tout comprendre c'est tout pardonner" — as if knowledge of motive and circumstances were sufficient to dispense with praise or blame.

Moral judgments of a more mature kind on the constitution and course of nature form the material for optimistic and pessimistic views of the world — at least, when these views rise above the assertion of a preponderance of pleasure or of pain in life. But, so far as I can see, in such moral judgments nature is never looked upon as consisting of dead mechanical sequences. It is because it is looked upon as the expression of a living will or as in some way — perhaps very vaguely conceived — animated by purpose or consciousness, that we regard it as morally good or evil. Apart from some such theological conception, it does not seem to me that the nature of things calls out the attitude of moral approval or disapproval. Things are estimated as useful for this or that end, they are seen and appreciated as beautiful or the reverse, without any reference to them as due to an inspiring or originating mind; and in one or other of these references the terms "good" or "bad" may be used. But when we use the term good in its specifically moral signification, we do not apply it to the inanimate, except in a derivate way, on account of the relation in which these inanimate things stand to the moral ends and character of conscious beings.

So far, therefore, as the evidence of moral experience goes, it does not support the view that the "good" is a quality which belongs to things out of relation to self-conscious activity. And, in so far, the peculiarity of the moral experience would seem to be better brought out by the conception "ought" than by the conception "good."

But here a difficulty arises at once. For how can we say that anything ought to be done or to be except on the assumption that it is antecedently good? Is not such antecedent and independent goodness necessary in order to justify the assertion that any one ought to produce it?

The question undoubtedly points to a difficulty; and if that difficulty can be solved it may help to bring out the true significance of the moral concept. The judgment which assigns the duty of an individual — according to which I or any one ought to adopt a certain course of action — involves a special application of the moral concept. It binds the individual to a certain objective rule or end. The individual's desires as mere facts of experience may point in an altogether different direction; the purpose or volition contemplated and approved by the moral judgment has in view the union of individual striving with an end which is objective and, as objective, universal. This union involves an adaptation of two things which may fall asunder, and which in every case of evil volition do fall asunder. And the adaptation may be regarded from either side: on the side of the individual, application to his individuality is implied; the duty of one man is not just the same as the duty of any other; he has his own special place and calling. But he is connected with a larger purpose which in his consciousness becomes both an ideal and a law, while its application is not limited to his individuality or his circumstances.

All this is implied in the moral judgment. It is not limited to one individual consciousness or volition. But it does not follow that the predicate "good," in the ethical meaning of the term, is or can be applied out of relation to consciousness altogether. At the earliest stages of moral development we find it applied unhesitatingly wherever conscious activity is supposed to be present — to anything that is regarded as the embodiment of spirit; and it is applied to the universe as a whole when the universe is thought of as the product of mind. "Good" is not even limited to an actual existent; it neither implies nor denies actual existence. "Such and such, if it existed, would be a good" is as legitimate though not so primitive an expression of the moral judgment as "this existent is good." But it does imply a relation to existence. It does not even seem possible to distinguish except verbally between "good" and "ought to be." And this "ought" seems to imply a reference to a purpose through which the idea is to be realized.

This conception "ought to be" is not the same as the concept "ought to be done by me." The latter is an application of the more general concept to a special individual in special circumstances; and this is the common meaning of the concept duty. The former is the more general concept of "goodness." It may be called objective, because it does not refer to any individual state of mind; it is universal because independent of the judgments and desires of the individual; and when the goodness is not due to its tendency towards some further end, it may also be called absolute.

The point of the whole argument can thus be made clear if we bear in mind the familiar distinction between "good in itself" and "good for me now." That the latter has always a relation to consciousness is obvious: it is something to be done or experienced by me. But there must be some ground why anything is to be or ought to be done or experienced by me at any time. Present individual activity must rest upon or be connected with some wider or objective basis. What is good for me points to and depends upon something which is not merely relatively good, but good in itself or absolutely. Yet it does not follow that this good in itself is necessarily absolute in the sense of having significance apart altogether from consciousness. Its absoluteness consists in independence of individual consciousness or feeling, not in independence of consciousness altogether. It is objective rather than absolute in the literal sense of the term. The good in itself, like the relative good, is one aspect which can only belong to a consciousness — to purpose. The moral judgment on things — either on the universe as a whole, or on anything in the universe which is not regarded as due to the will of man — is only justified if we regard these things as in some way expressing consciousness; either as directly due to it, or as aiding it, or as in conflict with it. From any other point of view, to speak of things as good or evil (unless in some non-ethical sense of these terms) seems out of place, and is unsupported by the mode of application which belongs to the immediate judgments of the moral consciousness. If the moral concept has significance beyond the range of the feelings and desires of men, it is because the objects to which it applies are the expression of mind.

This is not put forward as a vindication of a spiritual idealism. It is only a small contribution towards the meaning of "good." A comprehensive idealism may not be the only view of reality with which the conclusions reached so far will harmonize. But it is the view with which they harmonize most simply. The conception of a purpose to which all the events of the world are related is a form in which the essential feature of idealism may be expressed; the view of this purpose as good makes the idealism at the same time a moral interpretation of reality, and allows of our classing each distinguishable event as good or evil according as it tends to the furtherance or hindrance of that purpose.

This doctrine of the significance and application of the ethical concept would enable us to reach a definite view of the nature of ethics and of the way in which it is related to the sciences and to metaphysics. The ethical concept is based upon the primary facts of the moral consciousness, just as scientific concepts have as their basis the facts of direct experience. The primary facts of the moral consciousness are themselves of the nature of judgment — they are approbations or disapprobations. But all facts of experience involve judgments, though these judgments may be only of the form "it is here" or "it is of this or that nature." Again, the primary ethical facts or judgments cannot be assumed to be of unquestionable validity: we may approve what is not worthy of approval, or disapprove what ought to have been approved. Our moral judgments claim validity; and their claim is of the nature of an assertion, not that one simply feels in such and such a way, but that something ought or ought not to be. They imply an objective standard. But the objective standard, when more clearly understood, may modify or even reverse them. Our primary ethical judgments — all our ethical judgments, indeed — stand in need of revision and criticism; and they receive this revision and criticism in the course of the elaboration of the ethical concept and of its application to the worlds of fact and possibility. In the same way it may be contended that the direct judgments of experience upon which science is based need criticism and correction; though their variation may be less in amount than the variation of moral judgments. The color-blind man identifies red with green, and his judgment on this point has to be reversed; the hypersensitive subject often confuses images with percepts; exact observation needs a highly trained capacity. The correction and criticism which is needed come from objective standards; and these are the result of the comparison of many experiences and the work of many minds.

It is no otherwise in the case of ethics. Criticism brings to light inconsistencies in the primary judgments of approbation and disapprobation as well as in the later developments of the moral judgment. And these inconsistencies must be dealt with in a way similar to that in which we deal with inconsistencies in the judgments of perception and of science. The objective standard is not itself given once for all; it has to be formed by accumulation and comparison of moral experiences. Like the experiences on which science is based, these have to be made as far as possible harmonious, and analysis has to be employed to bring out the element of identity which often lurks behind apparent contradiction. They have also to be made as comprehensive as possible, so that they may be capable of application to all relevant facts, and that the scattered details of the moral consciousness may be welded into an harmonious system. In these general respects the criticism of ethical concepts proceeds upon the same lines as the criticism of scientific concepts. The difference lies in the concepts themselves, for ethics involves a point of view to which science must always remain a stranger.

BIBLIOGRAPHICAL NOTE

The relations of ethics are discussed in almost every ethical treatise; special reference may be made to the writers who have worked out the theory of worth or value, especially von Ehrenfels, *System der Wert-theorie* (1897); Meinong, *Psychologisch-Ethische Untersuchungen zur Werth-theorie* (1894), and an article in *Archiv für Syst. Phil.* 1895; Krueger, *Begriff der Absolut Wertvollen* (1898); also to articles by Standinger and by Natorp in *Archiv für Syst. Phil.* (1896); by Wentscher, *Archiv für Syst. Phil.* 1899; by Westermarck, *Mind*, 1900; and by Belot, *Revue de Métaphysique et de Morale*, 1905. — W. R. S.

PROBLEMS OF ETHICS

BY PAUL HENSEL

(*Translated from the German by Professor J. H. Woods, Harvard University*)

[**Paul Hensel**, Professor of Systematic Philosophy, University of Erlangen, since 1902. **b.** May 17, 1860, Great Barten, East Prussia. Ph.D. Freiburg, Baden, 1885. Privat-docent, Strassburg, 1888–95; Special Professor, Strassburg, 1895–98. **Author of** *The Ethical Basis and Ethical Transactions; Carlyle; The Principal Problems of Ethics.*]

SINCE the appearance of the three chief works of Kant a certain rhythm in the treatment of philosophical problems, first of all in Germany, but also, in less degree, in other civilized countries, is unmistakable. After an intense occupation with theoretical problems a flood of ethical discussion usually follows; and this then is usually resolved into a renewed revision of æsthetical problems. If I am not deceived, we are now at the period of transition from the second to the third epoch; so much the more favorable is the time to review the present condition of ethical problems. In the first place, then, it seems rather remarkable that recent ethical discussion, so intensely carried on, has resulted in a definite victory for neither one school nor the other. One thing alone, however, may with some accuracy be said, that the school of utilitarianism of the older interpretation by Bentham, which earlier prevailed almost alone in England with a fairly strong representation in France and Germany, seems to be withdrawn from the field. Not as if there were no men to-day who in other times would have sworn by Bentham's flag, rather we are here facing a fact that a theory which formerly appeared in independence, now may be deemed a special case of a more inclusive theory, which with the help of its wider horizon can remove a whole series of difficulties, which apparently raised insolvable problems for the special theory. Utilitarianism, since it had started with the examination of the individual, could not, even in the master-hand of Bentham, transfer itself without remainder into the greatest happiness of the greatest number; the interest paid on the sacrifice offered to fellow men, again and again seemed dubitable and probable; again and again the best calculation seemed to consist in egoism pure and simple. The impossibility of an exact calculation of consequences in pleasure and in pain was likewise repeatedly emphasized by opponents; the suggestion that we do not count the shrewd calculator so good as the man who acts impulsively was also not lacking: all these were difficulties, which, on the ground of the older utilitarianism, could be evaded but not quite entirely put out of the world.

It is then easily understood that the further combinations into which evolution was able to advance ethical questions have resulted in the cessation of utilitarianism as an independent system. Around the huge system of thought of Herbert Spencer one of the great camps of ethical workers is collected. It is not correct to count Herbert Spencer as systematizer of Darwin's thoughts; his main thoughts were finished, before a line of Darwin had appeared. But it is correct that the wonderful inductions of Darwin were precisely that which Spencer's system needed in order to begin its triumphal march through the civilized world. Here the case is the reverse of that of Copernicus and Giordano Bruno: the systematizer precedes the man of special research. It is superfluous on American soil to give a description of Spencer's thoughts; they have become parts of the general consciousness. So it may suffice to emphasize a few characteristic features, to which my remarks shall be attached, since, otherwise, in view of the richness of the system, there might easily be other sides of it in the mind of my hearers than those to which I have here to attach importance.

The characteristic feature of the system of Spencer is its unity and compactness. Just as every picture has a definite point from which it should be seen, so also the system of Spencer is a view of the world from a quite definite point of view, — that of evolution. Systems of evolution had already occurred in philosophy, — I mention the vast performance of Hegel only, — but that which gives Spencer's system its characteristic significance is that here evolution is conceived not as logical, but as biological; while in the case of Hegel nature is the vestibule of the realm of purpose, and therein alone has its significance, Spencer takes nature as his point of departure, and the realm of human activity represents itself to him merely as the finest conformation of natural events. Here the whole evolution from the nebula in world-space to the most delicate relations between man and man are comprehended in one grand conception. The same amount of force which then existed in world-space exists still to-day, only in infinitely more differentiated form. The new which is produced is nothing else than the transformed old, but transformed in an essential relation, in the direction towards constantly increasing complexity of relations in which single things and centres of force stand to each other.

If it be asked what this principle is which is the ground for this differentiation, a glance at the behavior of organisms informs us. In them we can most clearly recognize effects which result, with the necessity of laws of nature, from increasing differentiation. The undifferentiated individual is powerless in the presence of every change of his environment. Banished to its accidental place, the plant must wait for what happens to it. Only within a narrow limit can it maintain its existence. Better equipped we find the animal, especially when it has gathered into social groups, either for protection against carnivora or for the breeding of progeny in common. The young steer has an infinitely better prospect to maintain itself, to grow up, than the single egg in the spawn of the sturgeon.

So it is, before all else, the fact of social combination which attracts to itself the attention of the revolutionary ethicist. His ethics is social ethics. The analysis of the historical development of mankind forms the standard, in which the social combinations have resulted, and in which greater and world-inclusive formations have replaced those earlier, smaller, and smallest, usually engaged in war with each other. It is a long way from the time when *hospes* was equivalent to *hostis* to international expositions, and the single stages of this way reflect themselves in the moral behavior of the individuals. The old question, which in so many ways agitated the English ethics of the seventeenth and eighteenth centuries, the question, whether man should be regarded as an originally egoistic being, or whether equally original, benevolent instincts must be ascribed to him, is transferred by evolutionists of to-day beyond the realm of man to that of his animal ancestors and, in this case, in favor of the originality of egoism. But long before man appeared as an independent species the effects of the life of the horde must have shown themselves in him, since those communities only in which the single members were bound to each other by sympathy had any prospect of survival. It is therefore possible to speak of animal ethics. The interesting attempts which Darwin had made in this field were taken by Spencer, as a whole, into his system. It must, however, be conceded that we must observe the full development of this process, first of all, in man, and the tendency then consists in a constant decrease of egoistic, as compared with altruistic, actions. How it was possible that the individual was ever willing to renounce the amounts of pleasure, which he could obtain, in favor of others, Spencer skillfully tried to explain by the introduction of the egoistic-altruistic feelings. These give the impulse to actions which are useful to the community, but which give to the doer honor and distinction, and thus, from egoistic motives, make actions which promote the welfare of the community commendable. But those actions which damage the community are visited with punishment of all kinds. The theory of sanctions in Bentham and Mill here passes over into the more extensive system of evolution. For modern theory of evolution, by the broader biological foundation of its system, succeeds in explaining why even, in the case of those who cannot overlook the consequences of such actions as are injurious to their own person, these consequences are still ignored. The fact of the conscience, for the consistent Benthamite a negligible quantity, forms the keystone of Spencer's ethics, and affords the chance of making the theory of heredity applicable in a new field of ethical speculation.

It is, as a matter of fact, impossible for the single individual to calculate, by Bentham's receipt, all the consequences of pleasure or of pain which result from the actions for his own welfare. The individual need not, however, undertake this calculation at all. He does not begin at the beginning of making his experiences in this world; he enjoys the heaped-up treasure of experiences which, before him, long-forgotten generations of ancestors had made; and the sum of these experiences he calls his conscience. This voice of the conscience restrains the individual from anti-social actions, which, in accordance with experience, must lead to an injury to his own person; in accordance, of course, with the experience not of single ancestors but of the whole line. Here, again, a selective process in the struggle for existence is being completed. Men with no conscience at all or with an only imperfectly developed conscience have to contend with disadvantages similar to those in whom the corporal adjustment to the modern conditions of civilization have proved defective; they are exterminated by seclusion in prison or by execution, as the others by diseases which their bodies cannot resist. The criminal of to-day might perhaps have been, in primitive times, a respected member of his horde, perhaps, even a great chief. To-day he can be regarded only as an atavistic survivor, who fits into our conditions as little as a living ichthyosaurus into this lecture-hall. Again, it is to be hoped, it is even definitely to be predicted, that many who to-day are quite irreproachable in moral respects, in later times will no longer succeed in satisfying the requirements in the form of their grandson or great-grandson. For the progress is a biological necessity; and he who cannot attach himself to its ways is submerged.

It is small disparagement for this vast construction of the connection between the moral life of the individual and the total evolution of the associations of men, of organisms, of the whole, that, now especially in English ethics, a bitter strife has broken forth, which we may regard as the one-sided elaboration of the individualistic parts of Spencer's ethics on the one side, of the social on the other side. While the orthodox disciples of Spencer insist that such progress only can be kept in aim which must assure to the individual, to the fit the most unrestricted possible amount of free movement, while the whole rigor of the process of selection must fall upon the unadjusted and the unfit, the socialist tendencies of our time tend to advocate a reversal of this harsh result and to advocate both the united struggle of human society, by suppressing over-energetic individuals, and the preservation of the economically weak. Though it would be interesting to trace this division to its final grounds, I must limit myself to note the fact that the socialist movement seems here also to be in advance, — at least, so far as European movements of thought are concerned; and that they are in the condition to compensate for their departure from the teachings of the master by an appeal to the main thoughts of his system, concerns me just here. Doubtless socialistic thought is on the whole in advance when compared with liberal and individualistic thought. And, under these circumstances, the inference for every disciple of Darwin's theory of evolution is simple; that here again is a case of survival of the fittest; that socialistic ideals represent a higher form of adjustment; that just by the fact of their victory the necessity and justification of this victory is placed beyond doubt. It helped little that the venerable thinker himself in the last years of his rich and active life descended into the arena of the contest and warned his beloved England against the dangers of this socialistic tendency. It was inconsistent that he tried to brand these thoughts as a retrograde movement, as a step backward, since his own system with its powerful optimism affords no possibility for victorious retrograde movements. Even imperfection and evil has for Spencer only the significance of an imperfect progress; and the thought that imperfection could even win the victory over the perfect, that must be warned against it, could only be nonsense in connection with his system. For him, as for Hegel, the final formula, obtained it is true by a very different way, is the thesis: The actual is rational.

But just this reference to Hegel's system makes clear to us the opposition which Herbert Spencer's system found in Germany, first of all, but also in wide circles in England and in America. If it could be objected against Hegel that the activity of the individual, in contrast to the might of the developing process of the logical idea, is reduced to insignificance, this consideration returns with doubled force in contrast to the concept of the thought of development, which is found in the modern theory of evolution of Spencer. For here it is not teleological necessities which prevail, but causal. To have proved evolution by the laws of nature is precisely his system's title to fame. The question must then be raised whether an obligation to any definite practical action can be deduced from the proof of the necessity of any event. If the development is necessary, it will be completed whether I coöperate with it or not. If it needs my coöperation, it need not be regarded as a law of nature. It is exactly the same difficulty which beset the Stoics, when they tried to harmonize the determinism of world events with the demands which their ethics put upon the moral resolves of the individual. It is absurd to will any necessary event of the laws of nature; I can suspend my action so that I count upon the occurrence of such an incident, but I cannot make this incident the object of my will. I can decide that

I will observe an eclipse of the moon, but I cannot will the occurrence of this eclipse of the moon, or not will it.

If we reduce the difficulty to the simplest formula, it would be as follows: the theory of evolution did not distinguish between two completely different kinds of attitudes on the part of human mental activity; between the knowledge of the necessity of what exists and its judgment by standards of value. But it is precisely with the latter that ethics has to do. It is, like logic and æsthetics, a science of values; the interest in the question how something has come to be, is quite different from the interest in determining its value. Everything has come to be, the valueless as well as the valued, with the same necessity; that is a self-evident presupposition of all explanatory science. The bungling drawing of a school-boy and the Sistine Madonna, the hallucinations of a lunatic and the thought of a Herbert Spencer, a demonic crime and a deed of the purest ethical fulfillment of duty, are, in the same sense, necessary; but with the knowledge of this necessity we have not come a single step nearer to the task of their valuation.

The difference between these two kinds of attitudes has perhaps never been more clearly sketched than in Fichte's book *On the Calling of Man.* If we assume that I have a fully adequate scientific knowledge of the course of nature, I might discern that this grain of sand which the storm has set in motion could not drift a hair's breadth farther, unless the whole previous course of nature had been quite different; what then would be gained for my own moral action? The answer must be: Nothing. More than that, if this point of view were the only possible for man, then this action would have no longer, as a moral action, any significance, and could have none; since as a part of the world event alike in value to all other parts it would remain like in value, and it would be meaningless to select and emphasize out of this continuum of facts and environments, alike in value, single elements as especially valuable and significant. The man who could not resign himself to this knowledge, who could not be satisfied to continue, in cool content, at the point of view of the silent contemplation of causes, must fall into conflicts similar to those which Carlyle so vividly described in *Sartor Resartus.* We must then, in order to an understanding for this new problem, provisionally disregard, above all else, whatever the theory of evolution has accomplished by way of scientific explanation, and reserve for a later investigation the ethical valuation of this sequence of development. The question which is now to occupy us is directed, first of all, to the subject of our moral valuation. What do we call good or bad?

This is the main question of all normative ethics in general, and its answer by Kant will always remain a brilliant feat in this field. He proved, in the first place, that this predicate can be properly applied to no action whatever, that we can speak of a good action in figurative language only, when we believe that we can make from this action an inference with regard to something else, — the disposition of the actor; and that the same action which we do not hesitate to describe as good, on the supposition of the correctness of this inference, loses directly this character as soon as doubt of the correctness of the inference arises. This disposition, which we distinguish in this way, which forms the substrate of our moral valuation, we call the good will, and the Magna Charta of the Kantian ethics consists in the celebrated thesis: Nothing can possibly be good except a good will. This reasoning appears to be as self-evident as its result is important.

The whole ethical process is removed within the soul. While the theory of evolution and, still more, utilitarianism could still hope to obtain, with the character of the work, at the same time an expression with regard to the ethical value of the action; while, in this combination of ideas, the ethical goodness of the disposition could be judged by the usefulness or value to civilization of the performance done, so that both these systems would have essentially the character of an ethics of results, we have in Kant and his successors, most decidedly, an ethics of dispositions. It has rightly been pointed out that this ethics could grow only upon Protestant soil, that here the same contradiction prevails which Luther once summed up in the words: "Good works do not make the good man, but the good man creates good works." All the excellences, but all the weaknesses also, of Protestantism, cling to Kant's ethics.

First, let us follow the further stages of Kant's thought. How must a good will be constituted, so that we may count it as ethically good? All our acts happen in order to fulfill a purpose. The character of the action depends upon the character of the purpose, which the actor proposes for himself, which he affirms with his will, which he makes his own. But if the purpose be no longer willed, then all the actions cease, which hitherto had had to be accomplished for its fulfillment. All those purposes, which under the circumstances cannot be willed, cannot therefore produce that lasting constitution of the will which we understand under the term the good will. But among the different motivations of the will, there are some which for the observer become separated. They have not a character such that they could, under any circumstances, cease to motivate the will; they are necessary and universal determinations of the will. The imperative which they contain and with which they demand action has not the hypothetical form: "If thou wilt obtain this or that, you must;" but the absolute: "Thou shalt." It is a categorical imperative, to which the will is here subordinated, which determines my actions; and such a categorical imperative we term duty. Only the dutiful will is good. It is clear that this determination shows an exact analogy to the other norms of judgment in the logical and the æsthetical field. The principle of contradiction states nothing at all with regard to the single thoughts, it only asserts that our thinking can then alone make a claim upon a logical valuation while it fills the condition which the principle of contradiction states. Likewise, the impulses of our wills can be morally valued only when they refer to an absolute "Thou shalt;" if this is not the case, they are excluded from the range of valuation, just as the play of our fancy, which does not recognize the principle of contradiction, is excluded from the realm of the norm of scientific thinking.

Here again the normal action of ethics is represented as a selective process. While the evolutionist ethicist can estimate every single content of human consciousness with reference to the point whether it is preservative of the species or not, and thus give it ethical value, the realm of the Kantian ethics is much more confined. Only those impulses of the will occur with conscious subordination under the command of duty, or in conscious opposition to it fall within the realm of moral valuation. All others — and their name is legion — must be termed unmoral. Not as if they become thereby actually valueless; they may stand as high as you please in the intellectual, æsthetic, or religious scale of values. But to bring them under just the moral norms of judgment would be an attempt at an unappliable object. This is the point, perhaps, where the Kantian ethics gives the hardest shock to the healthy human understanding. It will always seem a paradox that we have a moral act when a man with strong desires for theft, after a severe inner struggle, does not put a silver spoon into his pocket, while the man who omits all this quite as a matter of course may have no claim upon moral desert. And yet each one of us would feel it as an insult, if he should be praised for such omission. The solution of this difficulty lies in the distinction of the value of the single resolve and that of the whole moral personality. The man who is still led into temptation by silver spoons stands morally upon the same plane upon which the scholar stands who struggles with extreme mental effort to calculate a simple example in multiplication. In the case of the more advanced person our moral approval is not aroused because he no longer needs, in this simple case, to appeal to the law of duty, but because we believe that we may conclude that his moral personality is attacking other more difficult problems with full force, and that he is here in himself feeling the full weight of the contest. If we were deceived in this, if it prove true that he, content with what had been attained, had withdrawn to the position of the ethical capitalist, our ethical interest in him would likewise cease, just as our intellectual interest ceases in the scholar for whom there are no more problems in his science. From this point of view the result is necessary that the category of duties, to speak with Hegel, is absolutely infinite; and in this perhaps lies the considerable difference between modern and ancient ethics. For ancient ethics the ideal of the wise man was a distinctly finitely determined amount. However difficult it might be to fulfill the conditions for it, it could still be fulfilled in a human life; and a further advance beyond this fulfilled ideal would have been to the Greeks an absurdity: it is the "nothing too much" transferred to the ethical point of view. It is otherwise in modern ethics, and with this is connected the change in that the concept of the infinite has become a concept of value. It is as Carlyle says: "Fulfill the next duty which presents itself to thee, and when thou hast fulfilled it, wait for ten, twenty, a hundred to be fulfilled." But we recognize the degree of ethical development which a man has attained by noting that it is no longer duty to him.

If the limits of the moral valuation have been much restricted by the introduction of the concept of unmoral actions, it has been extended in the other direction by the insight that now every action which happens in fulfillment of a command of duty is to be valued as the result of a moral disposition. We come thus to the problem which, since the time of the ancient sophists, has not ceased to occupy minds, and which may most simply be termed the anthropological problem. What in the world is there that is not by individuals and by people deemed to be moral! With what strange contents the formal "Thou shalt" of morality is filled! In face of these contradictions, is there any sense at all in speaking of ethical commands? All skeptical attacks upon ethics find in such considerations their strongest support; and here again the answer is easy when we reflect upon the analogy with science, art, and religion. Aristotle and Democritus, Hegel and Hobbes, have taught very differently, and yet all have been busy with science. Raphael and Menzel are surely to be valued as artists; Mahomet and Buddha were both religious geniuses of the first magnitude. Why should it be different in the field of ethics? What other men have held to be moral, how they have acted, this can be valuable to me, in order for me to become clear with regard to my own moral determination, just as the artist sees the works of other masters, just as the scientific man must know the theorems of others. But all this cannot be the standard for the formation of my own life. I am, once for all, placed in this world, to be active there; I am responsible to myself for what I wish to accomplish with this life. And so it can, it is true, be an encouragement to me that other men have felt in themselves the same motive to moral activity; I can give them my hand as striving for the same with me through the separating centuries and across the estranging seas. But their way of solving the great problems of life cannot be the standard for me save in the sense that I receive them into my will, recognize them as valid for my own life.

So, then, the whole weight of the distinction, the whole moral process, is transferred to the individual. He is the point of departure and the goal of the struggle for a content in life. Is this now egoism? This much-discussed question also suffers, as I believe, by a defect in the statement of the problem. If it is intended that that action is meant by egoism, the motive for which is one's own welfare or happiness, by altruism, however, the action which aims at the happiness of others, it is quite clear that these two contrasts have as little meaning for the ethics of disposition as the complementary contrast of beautiful and ugly. Moral action is completely indifferent with regard to these contrasts. Moral actions can be characterized as altruistic as well as egoistic, and the same is the case for unmoral or bad actions. By knowing that distinct advantages have resulted to the doer from an action, or that "the greatest happiness of the greatest number" has resulted from it, I have not gained one step for the moral valuation of this action. I should surely act immorally if I omitted an action acknowledged as moral by me because it would involve pain for others and thus would have an anti-altruistic character. Whence this confusion of the altruistic with the moral arose is easy to see. Long before the child could himself act morally, it must be accustomed to feel that its beloved self cannot be the sole standard for its action; and to the end that it keep peace and content with its brothers and playmates, it is properly accustomed to regard in its action the welfare of the human beings about it. That is a preparatory step to moral action; but, strictly speaking, it can be counted as moral by those only who are determined not to recognize the limits between psychological motivation and normative determination.

It would be an interesting task to trace the relations into which the autonomous moral individual enters with the great moral institutions which dominate the community and have combined in usage, society, and state, and which Hegel described in a happy expression as "objective morality." Here it is no longer the regard for the weal or woe of fellow men which strives to gain influence over my action; here the ethical will of past generations of my own ancestors accosts and asks me whether I can bring my action into harmony with that which they willed and for which they strove. It is a slight disadvantage to the ethically directed man that, in order to protect these moral institutions from injury, an arsenal of punishments, of social influences, of boycotts, and of whatever finer or coarser means of compulsion there may be, are set up. This arsenal is necessary to sustain the social structure which alone affords the chance for moral action; and he who calculates with pleasure and pain, who tries to arrange his life as happily as possible, will be restrained by shrewd calculation from injuring the prevailing moral institutions. The moral man has nothing to do with such considerations. When he affirms the objective morality, he does so because he recognizes his moral will as identical with that of previous generations which have made these forms. But the time can come when he discovers that a moral life within these forms is no longer possible for him, when with deep regret he sees the bond of continuity break which knit him in affection with the past, when he must resolve to enter new untrodden paths, just as Copernicus was forced to resolve to substitute a new knowledge for those which had satisfied centuries. Such a man will endure calmly and patiently the consequences which result from such a course; he will not expect to be justified, through the purity of his intentions, in the eyes of his fellows, if he undertakes to lay hands on the institutions which the moral consciousness of his contemporaries recognizes as valid. But he will also know that these same institutions owe what sacredness they possess to the blood of previous martyrs, that these shadows of a past can only then speak to a living generation when they have tasted the sacred blood of sacrifice.

So then we see two great movements in our time struggling about the ethical questions. The one has on its side the whole apparatus of scientific conceptions, the presupposition of necessary events without exceptions, the knowledge that the single individual is an infinitely small element in a necessary sequence of development. It can explain everything, deduce everything from its conditions. At one point only its power breaks down: it cannot make the individual comprehend why he should raise a finger to keep in motion this machine which goes of itself.

And, opposed to this, is the other movement, which rests upon the one fact that the point of view of its opponent, the scientific, is also a relation of reality to values, and that man alone introduces these values into reality, measures and tests it by these values. According to this movement, every new human life has the question put to it, what it can accomplish with these values, whether it is capable of making something out of reality, out of itself, which has in itself a value such as to raise it above the flux of appearances as the bearer of these values. Everything previous as well as everything subsequent vanishes before these thoughts that it is now day, that the night is soon coming when no man can work, that at the day's end the day's work must be done. But what each recognizes as his day's work, he must himself find within himself. This decision is his destiny.

I cannot better close than with the words of the man whose life had little joy, but who grappled with these questions in the solitude of Craigenputtock, in the supreme solitude of the human wilderness of London, with a seriousness which still to-day proves to be soul-wooing and soul-winning: "Centuries have passed that thou mightest be born, and centuries are waiting in dumb expectation of what thou wilt accomplish with this life, now that it has begun." And what this life can offer Carlyle, by combining the thoughts of Fichte and of Goethe, has united in the call:

"Work and despair not."

SECTION F — ÆSTHETICS

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(*Hall 4, September 23, 3 p. m.*)

CHAIRMAN: PROFESSOR JAMES H. TUFTS, University of Chicago.
SPEAKERS: DR. HENRY RUTGERS MARSHALL, New York City.
PROFESSOR MAX DESSOIR, University of Berlin.
SECRETARY: PROFESSOR MAX MEYER, University of Missouri.

THE RELATION OF ÆSTHETICS TO PSYCHOLOGY AND PHILOSOPHY

BY HENRY RUTGERS MARSHALL

[**Henry Rutgers Marshall**, Practicing Architect, President of the New York Chapter, American Institute of Architects, Member of Art Commission, City of New York. **b.** July 22, 1852, New York City. B.A. Columbia University, 1873; M.A. *ibid.* 1875; L.H.D. Rutgers College, 1903. Member American Psychological Association, Society of American Naturalists, Fellow American Institute of Architects, Honorary Member National Society of Mural Painters, Member American Philosophical Association. **Author of** *Pain, Pleasure, and Æsthetics; Æsthetic Principles; Instinct and Reason.*]

IF conventional divisions of time are to serve as means by which we may mark the movement of thought as it develops, we may well say that the nineteenth century saw a real awakening in relation to æsthetics among those who concern themselves with accurate thinking, — a coming to consciousness, as it were, of the importance to the philosophy of life of the existence of beauty in the world, and of the sense of beauty in man.

And with this awakening came a marked breadth of inquiry; an attempt to throw the light given by psychological analysis upon the broad field of æsthetics, and an effort to grasp the relations within the realm in which beauty holds sway to philosophy as a whole.

That the questions thus presented to us have been answered, I imagine few, if any, would claim; rather may we say that the nineteenth century set the problems which it concerns the æsthetician of the twentieth century to solve; and this without underestimating the value of the work of the masters in æsthetics who lived and wrote in the century so lately closed, some of whom are fortunately with us still.

Of these present problems M. Dessoir will treat in his address to follow mine; in the regretted absence of Professor Lipps the privilege has been granted to me to consider with you briefly the relations of æsthetics to psychology, and to philosophy, which must in the end determine the nature of the problems to be studied by the æsthetician, and the import of the solutions of these problems which they present for our consideration.

I. *The Relation of Æsthetics to Psychology*

We live in what may well be called the era of psychological development, an era marked by the recognition of the truth that no philosophical view of life can be adequate which does not take full account of the experience of the individual human spirit which interprets this life. And so quite naturally for ourselves, and in all probability quite in accord with the habit of thought of the immediate future, we begin our study by the consideration of the relation of æsthetics to psychology.

In turning for light to psychology, the æsthetician finds himself of course asking what is the nature of the states of mind related to his inquiry; and here at once he finds himself confronted with a distinction which must be made if a correct æsthetic doctrine is to become established. He notes that there is a sharp difference between (1) the mental attitude of an artist who produces works of beauty; and (2) the mental attitude of a man at the moment when he appreciates beauty in his experience.[1] The failure to note this distinction has in my view led to much confusion of thought among the æstheticians of the past, and to the defense of dogmas which otherwise would not have been maintained.

That this distinction is an important one becomes clear in the fact that the sense of beauty is aroused in us by objects in nature which bear no relation to what men call fine art. The mental state of the appreciator of beauty has therefore a breadth which does not belong to the mental state which accompanies, or leads to, the production of works of beauty by the artist.

And yet it should not surprise us that this distinction has so often been overlooked; for the theorists first follow the trend of thought of the uncritical man, and this uncritical man does not naturally make the distinction referred to.

For, on the one hand, even the least talented of men has some little tendency to give part of his strength to artistic creation in one form or another; the creative artist is guided by what is a truly racial instinct, which under favorable conditions will appear in any man who is not defective: each of us thus in the appreciation of beauty throws himself to some degree into the attitude of the creative artist.

And, on the other hand, the artist, when not in creative mood, falls back into the ranks of men who keenly appreciate beauty but who are not productive artists; he thus alternately creates and appreciates, and with difficulty separates his diverse moods.

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[1] Cf. my *Æsthetic Principles*, chap. I, "The Observer's Standpoint," and chap. III, "The Artist's Standpoint."

We may well consider these two distinguishable mental attitudes separately.

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In asking what is the nature of the experience which we call the sense of beauty, we are stating what may well be held to be the most important problem in æsthetics that is presented to the psychologist.

Man is practical before he deals with theory, and his first theoretical questionings are aroused by practical demands in connection with his failures to reach the goal toward which he strives. The development of modern æsthetic theory has in the main quite naïvely followed this course, and we may properly consider first the psychological inquiries which seem to have the most direct bearing upon practical questions.

The artist asks why his efforts so often fail, and he is led to inquire what are the qualities in his work which he so often misses, but now and again gains with the resulting attainment of beauty.

It is thus that we naturally find the æsthetician appealing to the psychologist, asking him what special types of impression yield beauty, what special characteristics of our mental states involve the fullest æsthetic experience.

The psychologist is naturally first led to consider certain striking relations found within the beautiful object which impresses us, and to inquire into the nature of the psychic functioning which is involved with the impressions thus given. He thus comes to consider the relations of the lineal parts of pleasing plane-surface figures; and the study of these relations has given to us such investigations as the notable ones of Fechner in respect to the "Golden Section," which have been supplemented by the more rigid tests of Dr. Witmer and Doctors Haines and Davies in our own day. In similar manner the basis of the beauty found in symmetry and in order, and the problems related to rhythm, have been closely studied, especially in late years by Lipps; and the fundamental principles of tonal relation, and of melodic succession, by Helmholtz, Stumpf, and later writers.

But all these studies of the striking characteristics found in the object are for the psychologist necessarily involved with the study of the distinctly subjective accompaniments in the sense of beauty aroused by the objective forms thus brought to our attention, and he is led to dwell upon the active part the mind takes in connection with æsthetic appreciation. We see this tendency in Berenson's emphasis, and perhaps on the whole over-emphasis, of the importance of the interpretation of works of art, in the group of what I would call the arts of sight, in terms of the tactile sensibilities. But we see it much more markedly in the important studies of Lipps, who shows us how far our appreciation of beauty in nature, and in artistic products, is due to the sympathetic introjection of ourselves as it were into the object, — to what he calls *Einfühlung*.

But, broad as he shows the applicability of this principle to be, it is clear that we have not in it the solution of the fundamental æsthetic problem with which the psychologist must deal when appealed to by the æsthetician. For no one would claim that all of this sympathetic introjection — this *Einfühlung* — is æsthetic: the æsthetic *Einfühlung* is of a special type. Nor to my mind does it seem clearly shown that there are no sources of beauty which do not involve this introjection, as would be the case if we had reached in this principle the solution of the fundamental æsthetico-psychologic problem. For instance, the sense of beauty experienced when I look at some one bright star in the deep blue of the heaven seems to me to be inexplicable in terms of such introjection.

All this work, however, brings help to the practical artist and to the critic. They do not acknowledge it fully to-day, but year by year, more and more will the influence of the results of these studies be felt as they gain the attention of thinking men.

Nevertheless, we cannot but face the fact that the practical benefit to be gained from them is of a negative sort. There is no royal road to the attainment of beauty; but the psychologist is able to point out, by the methods here considered, the inner nature of certain sources of beauty; thus teaching the artist how he may avoid ugliness, and even indicating to him the main direction in which he may best travel toward the attainment of his goal.

But, after all, the relations thus discovered in the beautiful object, and the related special analyses of mental functioning which are involved with our appreciation of beauty, tell us of but relatively isolated bits of the broad realm of beauty. The objects which arouse within us the sense of beauty are most diverse, and equally diverse are the modes of mental functioning connected with the appreciation of their beauty.[1]

And this has led to the formulation of such principles as that of the "unity of manifoldness" of which Fechner makes so much, and that of the *monarchischen Unterordnung* which Lipps has more lately enunciated.

It is indeed of great interest to inquire why it is that the processes which lead to the recognition of these principles are so clearly defined in many cases where the sense of beauty is aroused. But very evidently these general principles, important though they be in themselves, are not ones upon which we can afford to rest: for clearly they apply in very many cases where beauty does not claim sway.

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[1] Nothing has shown this more clearly than the investigations of Haines and Davies in reference to the Golden Section of which we have spoken above. See *Psychological Review*, XI, 415.

Our whole mental life exemplifies the unification of the manifold, and monarchic subordination, whether the processes be æsthetic or not. It does not suffice us to show, what is thus shown, that the æsthetic states conform with conditions of our mental life that have a broad significance, although it is of great importance to demonstrate the fact: for our mental functioning in the appreciation of beauty appears thus as in truth an important type, but for all that but a special and peculiar type of the functioning which we thus bring into prominence.

The problem then remains, what is the special nature of this functioning which yields to us the sense of beauty?

And here in my view we have the problem which is of prime importance to æsthetics to-day, and which psychology alone can answer; namely, what is the characteristic that differentiates the sense of beauty from all other of our mental states? Until this question is answered, all else must seem of secondary importance from the standpoint of theoretical psychology, however important other forms of inquiry may be from a practical point of view.

When the psychologist turns his attention to this problem, he at once perceives that he is unable to limit his inquiry to the experience of the technically trained artist, or even to that of the man of culture who gives close attention to æsthetic appreciation.

Beauty is experienced by all men. But beauty is very clearly of varied types, and the sense of beauty is evidently called out by impressions of most varied nature; but the fields of what is considered beautiful by different people so far overlap that we can rest assured that we all refer to an experience of the same characteristic mental state when we proclaim the existence of beauty; for when we by general agreement use a special term as descriptive of an objective impression, we do so because this impression excites in us certain more or less specific mental states; and when different people use the same term in reference to objects of diverse nature, we are wont to assume, and are in general correct in assuming, that these objects affect these different people in approximately the same way.

It seems probable, therefore, that if the child, who has learned how to apply words from his elders, speaks of having a beautiful time at his birthday party; and if the grown man speaks of a beautiful day; and if the pathologist speaks of his preparation of morbid tissue as beautiful; and if the artist or connoisseur speaks of the beauty of a picture, a statue, a work of architecture, a poem, a symphony; then the word beauty must be used to describe a certain special mental state which is aroused in different people by very diverse objective impressions.

This view is strengthened when we consider that the application of the term by individuals changes as they develop naturally or by processes of education; and that the standards of beauty alter in like manner in a race from generation to generation as it advances in its development.

We must then look for the essence of beauty in some quality of our mental states which is called up by different objective impressions in different people, and under diverse conditions by different objects at different times in the same individual.

Search for such a quality has led not a few psychologists to look to pleasure as the quality of our mental states which is most likely to meet our demand. It is true that the consideration of pleasure as of the essence of the sense of beauty has not often been seriously carried out; apparently because so many of what we speak of as our most vivid pleasures appear as non-æsthetic; and because pleasure is recognized to be markedly evanescent, while beauty is thought of as at least relatively permanent.

It is true, also, that there is a hesitancy in using the word pleasure in this connection; many writers preferring the less definite word "feeling" in English, and "gefühl" in German. But by a large number of psychologists the words pleasure and feeling are used as synonyms; and those who, with me, agree that what we loosely call feeling is broader than mere pleasure, must note that it is the pleasurable aspect alone of what is called "feeling" that is essentially related to our experience of the sense of beauty.

All of us agree, in any event, that the sense of beauty is highly pleasant; and, in fact, most of our æstheticians have come to assume tacitly in their writings that the field of æsthetics must be treated as a field of pleasure-getting; and this whether or not they attempt to indicate the relation of pleasure-getting to the sense of beauty.

The suggestion that pleasure of a certain type is of the essence of beauty seems the more likely to prove to be satisfactory when we consider that pleasure is universally acknowledged to be the contradictory opposite of pain; and that we have in ugliness, which is always unpleasant, a contradictory opposite of beauty.[1]

Clearly then it behooves the psychologist to give to the æsthetician an account of the nature of pleasure which shall be compatible with the pleasurable nature of the sense of beauty; and which shall either explain the nature of this sense of beauty in terms of pleasure, or explain the nature of pleasure in a manner which shall throw light upon the nature of this sense of beauty to which pleasure is so indissolubly attached.

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[1] It is of course agreed that beauty and ugliness may be held together in a complex impression: but in such cases the beauty and the ugliness are inherent in diverse elements of the complex.

The æsthetician thus demands urgently of the psychologist an analysis of the nature of pleasure; and an analysis of this so-called "feeling," which shall show the relation between the two experiences.

Concerning the latter problem I hope some day to have something to say.

Those of you who happen to be familiar with my published works will realize that my efforts in this field in the past have been given largely to the study of the former problem. My own view may be succinctly stated thus.

While all æsthetic experiences are pleasant, very evidently much that we *call* pleasant is not æsthetic. We must look then for some special differentiation of æsthetic pleasure, and this I find in its relative permanency.

This view is led up to by a preliminary study of the psychological nature of pleasure.

Pleasure I find to be one phase of a general quality — Pleasure-Pain — which, under proper conditions, may inhere in any emphasis within the field of attention, or, to use more common language, may belong to any element of attention.

Now pleasure, as we have said, is notably evanescent, but this does not preclude the existence of pleasurable states of attention which are relatively permanent. This permanency may be given by the shifting of attention from one pleasurable element to another; by the summation of very moderate pleasures, etc., etc.

Any pleasant psychic element may become an element of an æsthetic complex: and any psychic complex which displays a relative permanency of pleasure is in that fact æsthetic. Our æsthetic states are those in which many pleasant elements are combined to produce a relative permanency of pleasure.

Our "non-æsthetic pleasures," so called, are those states which have been experienced in the past as vividly pleasant, and to which the name pleasure has become indissolubly attached: but they are states which do not produce a relatively permanent pleasure in revival; and correctly speaking, are not pleasures at the moment when they are described as such, and at the same time as non-æsthetic.

I am glad to feel that this view of mine is not discrepant from that of Dr. Santayana, as given in quite different terms in his book entitled *The Sense of Beauty*. For what is relatively permanent has the quality which I call realness; and that in experience which has realness we tend to objectify. Hence it is quite natural to find Dr. Santayana defining beauty as objectified pleasure.

You will not blame me I believe for thinking that my own definition cuts down closer to the root of the matter than Dr. Santayana's.

But if this theory of mine is found wanting, the æsthetician will not cease to call upon the psychologist for some other which shall meet the demands of introspection; and which shall accord with our experiences of the sense of beauty, which in all their wealth of impression the æsthetician offers to the psychologist as data for the laborious study asked of him.

Before leaving this subject I may perhaps be allowed to call attention to the fact that the theoretical view, which places the essence of the sense of beauty in pleasure-getting, if it prove to be true, is not without such practical applications as are so properly demanded in our time. For if this view is correct, it teaches to the critic a lesson of sympathetic tolerance; for he learns from it that the sources from which the sense of beauty are derived differ very markedly in people of diverse types: and it warns him also against the danger of an artificial limitation of his own æsthetic sense, which will surely result unless he carefully avoids the narrowing of his interests.

It teaches further that there is no validity in the distinction between fine art and æsthetics on the one hand, and beauty on the other, on the ground, commonly accepted by the highly trained artist and connoisseur, that a work of art may deal with what is not beautiful.

For it appears that while the sense of beauty is the same for each of us, the objects which call it out are in some measure different for each.

Now it happens naturally that the objects which arouse the sense of beauty in a large proportion of men of culture get the word beauty firmly attached to them in common speech.

But under the view here maintained, it must be that the highly trained artist or critic will pass beyond these commoner men, and find his sense of beauty aroused by objects and objective relations quite different from those which arouse the sense of beauty in the commoner man; so that often he may deal with the beauty of elements in connection with which beauty is unknown to the commoner man, and even with elements which arouse a sense of ugliness in the commoner man; while on the other hand the objects which the commoner man signalizes as most beautiful, and which are currently so called, may not arouse in the trained artist or critic the sense of beauty which is now aroused in him by effects of broader nature, and of less common experience.

The critic and the skilled artist thus often find their æsthetic sense aroused no longer by the objects to which the word beauty has by common consent come to be attached; although with the commoner man he still uses the word beauty as descriptive of the object which arouses the æsthetic thrill in the mass of normally educated men. He may even find his æsthetic sense aroused by what the common man calls ugly; although it is for himself really beautiful. And he comes thus quite improperly to think of the highest art as in a measure independent of what he calls "mere beauty." What he has a right to say, however, is merely this, that the highest art deals with sources of beauty which are not appreciated by even the generally well-cultivated man.

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I have dwelt, perhaps, too long on the psychological problems presented when the psychologist attempts to describe to the æsthetician the nature of the experience of one who appreciates beauty; and have left perhaps too little time for the consideration of the problems presented when he is asked to consider the nature of the experience of the artist who creates.

The man who finds strongly developed within him the creative tendency, is wont, when he turns to theory, to lay emphasis upon *expression* as of the essence of beauty.

It is, of course, to be granted that the process of *Einfühlung*, — of introjection, — above referred to, leads us to find a source of beauty in the vague imagination of ourselves as doing what others have done; and we may take great æsthetic delight in reading, through his work, the mind of the man who has created the object of beauty for us. But evidently, when we lay stress upon this introjection, we are dealing with the appreciation of beauty, and not with the force which leads to its production.

Just as clearly is it impossible to hold that expression is of the essence of the *making* of beauty. For expressiveness is involved in all of man's creative activity, much of which has no relation whatever to the æsthetic. The expression of the character of the genius of the inventor of a cotton loom, or of the successful leader of an army in a bloody battle, excites our interest and wonder; but the expression of his character as read in the result accomplished does not constitute it a work of beauty.

I speak of this point at this length because in my opinion views of the nature of that here objected to could not have been upheld by such men as Bosanquet and Véron had they kept clear the distinction referred to above between the experience of one who appreciates beauty, and the experience of the creative artist; and especially because the teaching of the doctrine thus combated is wont to lead the artist whose cry is "Art for Art's sake" to excessive self-satisfaction, and to lack of restraint which leads to failure.[1]

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[1] In order to avoid misunderstanding, I may say here that notwithstanding these remarks I am in full sympathy with the artist who thus expresses himself, as will presently appear clear.

The strong hold which this theory has in many minds has its value, however, in the emphasis of the fact that æsthetic creation is due to impulses which are born of innate instincts expressing themselves in the production of works of beauty. And if this be so, we see how true it must be that each of us must have in him some measure of this instinct; and that the appearance of its appropriate impulses should not mislead us, and induce us to devote our lives to the worship of the Muses, unless we become convinced that no other work can adequately express the best that is in us.

But the true artist is not troubled by such questionings. He finds himself carried away by what is a true passion; by what is instinctive and not ratiocinative.

The fact that the artist is thus impelled by what may well be called the "art instinct" is one he could only have learned from the psychologist, or when in introspective mood he became a psychologist himself; and it carries with it corollaries of great value, which the psychologist alone can elucidate.

It teaches the artist, for instance, that his success must be determined by the measure of this instinct that is developed within him; that he must allow himself to be led by this instinct; that his best work will be his "spontaneous" work. This, of course, is very far from saying that he cannot gain by training; but it does mean that he must learn to treat this training as his tool; that he must not trust overmuch to his ratiocinative work, the result of which must be assimilated by, and become part of, his impulsive nature, if he is to be a master.

An artist is one in whom is highly developed the instinct which leads him to create objects that arouse the sense of beauty. The expression of this instinct marks his appropriate functioning. He may incidentally do many useful things, and produce results apart from his special aptitude; but as an artist his work is solely and completely bound up in the production of works of beauty.

We naturally ask here what may be the function in life of the expressions of such an instinct as we have been studying, and this leads us to consider a point of more than psychological interest, and turns our thought to our second division.

II. *The Relation of Æsthetics to Philosophy*

For while the science of psychology must guide, it can never dominate the thought of the philosopher who strives to gain a broad view of the world of experience; and, as will appear below, the æsthetician calls upon the philosopher for aid which the psychologist as such cannot give.

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In approaching this subject we may take at the start what we may call the broadly philosophical view, and may consider the question raised immediately above, where we ask what may be the function in life of the art instinct, and what the significance of the æsthetic production to which its expression leads.

We, in our day, are still strongly influenced by the awakening of interest in the problems of organic development with which Darwin's name is identified, and thus naturally look upon this problem from a genetic point of view; from which, to my mind, artistic expression appears, as I have elsewhere argued at length, as one of nature's means to enforce social consolidation. But it is possible that we are led, by the present-day interest above spoken of, to over-emphasize the importance of the processes of the unfolding of our capacities, and it is not improbable that those who follow us, less blinded by the brilliancy of the achievement of the evolutionists, may be able to look deeper than we can into the essence of the teleological problem thus raised.

That art is worthy for art's sake is the conviction of a large body of artists, who labor in their chosen work often with a truly martyr-like self-abnegation; and as an artist I find myself heartily in sympathy with this attitude. But æsthetics looks to philosophy for some account of this artistic τέλος, which shall harmonize the artist's effort with that of mankind in general, from whom the artist all too often feels himself cut off by an impassable gulf.

The study of æsthetics by the philosopher from the genetic standpoint has, however, already brought to our attention some facts which are both significant and helpful.

It has shown us how slow and hesitant have been the steps in the development of æsthetic accomplishment and appreciation in the past, and how dependent these steps have been upon economic conditions. This on the one hand arouses in us a demand for a fuller study of the relations of the artistic to the other activities of men; and on the other hand is a source of encouragement to critic and artist alike, each of whom in every age is apt to over-emphasize the artistic failures of his time, and to minimize the importance of its artistic accomplishment.

This genetic study has a further value in the guidance of our critical judgment, in that it shows us that the artistic tendencies of our time are but steps in what is a continuous process of development. It shows us arts which have differentiated in the past, and teaches us to look for further artistic differentiations of the arts in the future; thus leading us to critical conclusions of no little importance. This consideration seems to me to be of sufficient interest to warrant our dwelling upon it a little at length.

The arts of greatest importance in our time may well be divided into the arts of hearing (that is, literature, poetry, music), and the arts of sight (that is, architecture, sculpture, painting, and the graphic arts).

These diverse groups of arts were differentiated long before any age of which we have a shadow of record. But many animals display what seem to be rudimentary art instincts, in which rhythmical movement (which is to be classed as an art of sight) and tonal accompaniment are invariably combined — as they are also in the dance and song of the savage; and this fact would seem to indicate that in the earliest times of man's rise from savagery the differentiation between the arts of sight and the arts of hearing was at least very incomplete.

But leaving such surmises, we may consider the arts of sight and the arts of hearing in themselves. We see them still in a measure bound together; for many an artist, for instance, devotes his life to the making of paintings which "tell a story," and many a poet to the production of "word-pictures."

In general, however, it may be said that the arts of hearing and the arts of sight express themselves in totally different languages, so to speak, and they have thus differentiated because each can give a special form of æsthetic delight.

Turning to the consideration of each great group, we note that the arts of sight have become clearly differentiated on lines which enable us to group them broadly as the graphic arts, painting, sculpture, and architecture. Each of these latter has become important in itself, and has separated itself from the others, just so far as it has shown that it can arouse the sense of beauty in a manner which its kindred arts of sight cannot approach. It is true that all the arts of sight hold together more closely than do the arts of sight, as such, with the arts of hearing, as such. But it is equally clear that the bond between the several arts of sight was closer in earlier times than it is to-day, in the fact that modeled painting, and colored sculpture, were common media of artistic expression among the ancients, the latter being still conventional even so late as in the times of the greatest development of art among the Greeks.

But the modern has learned that in painting and graphics the artist can gain a special source of beauty of color and line which he is able to gain with less distinctness when he models the surface upon which he works: and the experience of the ages has gradually taught the sculptor once for all that he in his own special medium is able to gain a special source of beauty of pure form which no other arts can reach, and that this special type of beauty cannot be brought into as great emphasis when he colors his modeled forms.

In my view we may well state, as a valid critical principle, that, other things being equal, in any art the artist does best who presents in his chosen medium a source of beauty which cannot be as well presented by any other art. That this principle is appreciated and widely accepted (although implicitly rather than explicitly) is indicated by the unrationalized objection of the cultivated critic in our day to colored sculpture or to modeled painting, and in a more special direction to the use of body-color in *aquarelle* work. The objection in all cases is apparently to the fact that the artist fails to bring into prominence that type of beauty which his medium can present as no other medium can.

Personally I have no objection to raise to a recombination of the arts of sight, provided a fuller sense of beauty can thereby be reached. But it is clear that this recombination becomes more and more difficult as the ages of development pass; and I believe the principle of critical judgment above enunciated is valid, based as it is upon the inner sense of cultivated men.

Better than attempts to recombine the already differentiated arts of sight are attempts to use them in conjunction, so that our shiftings of attention from one type of beauty to another may carry with them more permanent and fuller æsthetic effects; and such attempts we see common to-day in the conjunction of architecture and of sculpture and of painting, in our private and public galleries, in which are collected together works of the arts of sight.

Now if we turn to the consideration of the arts of hearing, we find a correspondence which leads to certain suggestions of no little importance to the critical analyst in our day.

The arts of hearing have become differentiated on lines which enable us to group them broadly as rhetoric, poetry and literature, and music. Each has become important in itself, and has gradually separated itself from the others; — and this just so far as it has shown that it can arouse in men, in a special and peculiar manner, the sense of beauty.

It is true, as with the arts of sight, that the special arts of hearing still hold well together.

But in relatively very modern times music, having discovered a written language of its own, has differentiated very distinctly from the other arts of hearing. Men have discovered that *pure music* can arouse in a special manner the sense of beauty, and can bring to us a form of æsthetic delight which no other art can as well give.

Poetry has long been written which is not to be sung; and it has gained much in freedom of development in that fact.

Music in our modern times is composed by the greatest masters for its own intrinsic worth, and not as of old as a mere accompaniment of the spoken word of the poet; the existence of the works of Bach, to mention no others, tells of the value of this differentiation.

And here I think we may apply with justice the principle of criticism above presented. The poet and the musician each do their best work, other things being equal, when they emphasize the forms of beauty which their several arts alone can give. We have here in my view a rational ground for the repulsion many of us feel for the so-called "programme music" of our day.

Music and literature of the highest types nowadays present sources of beauty of very diverse character, and any effort to make one subsidiary to the other is likely to lessen the æsthetic worth of each, and of the combination.

Here again I may say that I have no objection to raise to a recombination of the arts of hearing, provided a fuller sense of beauty can thereby be reached. But this recombination becomes year by year more difficult as the several arts become more clearly differentiated, and must in my view soon reach its limit.

The opera of to-day attempts such a recombination, but does so either to the detriment of the musical or of the literary constituent; that is clear when we consider the musical ineptitude of such operas as deal with a finely developed drama, and the literary crudeness of the plot-interest in Wagner's very best works. Such a consideration makes very clear to us how much each of the great divisions of the arts of hearing has gained by their differentiation, and by their independent development.

Here as with the arts of sight we may, in my view, hope for better æsthetic results from the development of each of the differentiated arts in conjunction; rather from the persistent attempt to recombine them, with the almost certain result that the æsthetic value of each will be reduced.

b

But æsthetics demands more of philosophy than an account of the genesis of art, with all the valuable lessons that this involves. It demands, rightly, that it be given a place of honor in any system which claims to give us a rationalized scheme of the universe of experience.

The æsthetician tells the philosopher that he cannot but ask himself what significance æsthetic facts have for his pluralism, or for his monism. He claims that this question is too often overlooked entirely or too lightly considered; but that it must be satisfactorily answered if the system-maker is to find acceptance of his view. And in the attempt to answer this and kindred questions, the æsthetician is not without hope that no inconsiderable light may be thrown by the philosopher upon the solution of the problems of æsthetics itself.

Nor are the problems of æsthetics without relation to pure metaphysic. The existence of æsthetic standards must be considered by the metaphysician, and these standards, with those of logic and ethics, must be treated by him as data for the study of ontological problems.

But beyond this, æsthetics cries out for special aid from the ontologist. What, he asks, is the significance of our standards of æsthetic appreciation? What the inner nature of that which we call the real of beauty? What its relation with the real of goodness and the real of truth?

From a practical standpoint this last-mentioned question is of special import at this time. For the world of art has for centuries been torn asunder by the contention of the æsthetic realists and their opponents.

That, in its real essence, beauty is truth, and truth beauty, is a claim which has often been, and is still heard; and it is a claim which must finally be adjudicated by the metaphysician who deals with the nature of the real.

The practical importance of the solution of this problem is brought home forcibly to those who, like myself, seem to see marked æsthetic deterioration in the work of those artists who have been led to listen to the claims of æsthetic realism; who learn to strive for the expression of truth, thinking thus certainly to gain beauty.

That many great artists have announced themselves as æsthetic realists shows how powerfully the claims of the doctrine appeal to them. But one who studies the artistic work of Leonardo, for instance, cannot but believe that he was a great artist *notwithstanding* his theoretical belief, and cannot but believe that all others of his way of thinking, so far as they are artists, are such because in them genius has overridden their dogmatic thought.

It is clearly not without significance that the world of values is by common consent held to be covered by the categories of the True, the Good, and the Beautiful. This common consent seems surely to imply that each of the three is independent of the other two, although all are bound together in one group. And if this is true, then the claim of the æsthetic realist can surely not be correct.

But this claim will not be overthrown by any reference to such a generalization as that above mentioned. The claim of the æsthetic realist is based upon what he feels to be clear evidence founded upon experience; and he cannot be answered unless we are able to show him what is the basis for his ready conviction that truth and beauty are one and identical; and what is the true relation between the True, the Good, and the Beautiful. And these problems, which are in our day of vital importance to the artist, the philosopher alone can answer.

In my view some aid in the solution of this problem may be gained from the examination of the meaning of our terms. From this study I feel convinced that we must hold that when we speak of the True, and the Good, and the Beautiful, as mutually exclusive as above, we use the term "true" in a narrow sense. On the other hand, the True is often used in a broader sense, as equivalent to the Real.

This being so we may say

That the Beautiful is the Real as discovered in the world of impression; the relatively permanent pleasure which gives us the sense of beauty being the most stable characteristic of those parts of the field of impression which interest us we may also assent

That the Good is the Real as discovered in the world of expression, that is, of impulse, which is due to the inhibited capacity for expression, and the reaction of the self in its efforts to break down the inhibition. And in the same way we may conclude

That the True (using the term in the narrow sense) is the Real as discovered in the realm of experience exclusive of impression or expression.

THE REAL or THE TRUE (in the broad sense of the term)	α. The Real of Impression	— The Beautiful
	β. The Real of Expression	— The Good
	γ. The Real in realms exclusive of α and β	— The True (in the narrower sense of the term)

That the Beautiful is part of the REAL, that is, is always the TRUE, *using the term true in the broader sense,* is not questioned: and that, in my view, is the theoretical truth recognized by the æsthetic realists. But in practice the æsthetic realist maintains that the beautiful is always the true, *using the term true in the narrow sense,* and in this, in my view, lies his error.

And if the relation of the beautiful to the true demands the attention of the philosopher, equally so does the relation of the beautiful to the good. As I look upon it, all of the true (using the term as above explained in the narrow sense) and all of the good, so far as either involve relatively permanent pleasure of impression, are possible elements of beauty. But, on the other hand, it seems clear that neither the true (still using the term in the narrower sense), nor the good, is necessarily pleasing, but may be unpleasant, and therefore either of them may be an element of ugliness, and as such must lose all possibility of becoming an element in the beautiful.

One further word, in closing, upon the closely allied question as to the nature of worth-values. There is a worth-value involved in the Good, and a worth-value involved in the True, and a worth-value involved in the Beautiful: and each of these worth-values in itself seems to be involved with pleasure-getting. Now if this is the case, then, under the theory I uphold, any worth-value should be a possible æsthetic element, and this I think it will be granted is true. But the distinctions between these worth-values are on different planes, as it were. In the case of the worth-value of the Good, we appreciate the worth-pleasure within the realm of the Real of Expression, that is, of impulse. In the case of the worth-value of the True (in the narrow sense), we appreciate the worth-pleasure within the realm of the Real in other fields than that of expression or that of impression. In the case of the worth-value of the Beautiful, we appreciate the worth-pleasure within the realm of the Real of Impression; that is, we appreciate, with pleasure, the significance for life of the existence of relatively permanent pleasure in and for itself.

THE FUNDAMENTAL QUESTIONS OF CONTEMPORARY ÆSTHETICS

BY MAX DESSOIR

(*Translated from the German by Miss Ethel D. Puffer, Cambridge, Mass.*)

[**Max Dessoir**, Professor of Philosophy, University of Berlin, since 1897. b. 1867, Berlin, Germany. Ph.D. Berlin, 1889; M.D. Würzburg, 1892. Privat-docent, University of Berlin, 1892–97. Member German Psychological Society, Society for Psychical Research, London. **Author of** *The Double Ego; History of the New German Psychology; Philosophical Reader; Æsthetik u. Allgemeine Kunstwissenschaft*; and many other works and papers on philosophy.]

I

In the development which our science has undergone, from its inception up to the present day, one thought has held a central place, — that æsthetic enjoyment and production, beauty and art, are inseparably allied. The subject-matter of this science is held to be, though varied, of a unitary character. Art is considered as the representation of the beautiful, which comes to pass out of an æsthetic state or condition, and is experienced in a similar attitude; the science which deals with these two psychical states, with the beautiful and its modifications, and with art in its varieties, is, inasmuch as it constitutes a unity, designated by the single name of æsthetics.

The critical thought of the present day is, however, beginning to question whether the beautiful, the æsthetic, and art stand to one another in a relation that can be termed almost an identity. The undivided sway of the beautiful has already been assailed. Since art includes the tragic and the comic, the graceful and the sublime, and even the ugly, and since æsthetic pleasure can attach itself to all these categories, it is clear that by "the beautiful" something narrower must be meant than the artistically and æsthetically valuable. Yet beauty might still constitute the end and aim and central point of art, and it might be that the other categories but denote the way to beauty — beauty in a state of becoming, as it were.

But even this view, which sees in beauty the real content of art, and the central object of æsthetic experiences, is open to serious question. It is confronted with the fact, above all, that the beauty enjoyed in life and that enjoyed in art are not the same. The artist's copy of the beauty of nature takes on a quite new character. Solid objects in space become in painting flat pictures, the existent is in poetry transformed into matter of speech; and in every realm is a like metamorphosis. The subjective impression might indeed be supposed to remain the same, in spite of objective differentiations. But even that is not the case. Living human beauty — an acknowledged passport for its possessor — speaks to all our senses; it often stirs sex-feeling in however delicate and scarce conscious a way; it involuntarily influences our actions. On the other hand, there hangs about the marble statue of a naked human being an atmosphere of coolness in which we do not consider whether we are looking upon man or woman: even the most beauteous body is enjoyed as sexless shape, like the beauty of a landscape or a melody. To the æsthetic impression of the forest belongs its aromatic fragrance, to the impression of tropical vegetation its glowing heat, while from the enjoyment of art the sensations of the lower senses are barred. In return for what is lost, as it were, art-enjoyment involves pleasure in the personality of the artist, and in his power to overcome difficulties, and in the same way many other elements of pleasure, which are never produced by natural beauty. Accordingly, what we call beautiful in art must be distinguished from what goes by that name in life, both as regards the object and the subjective impression.

Another point, too, appears from our examples. Assuming that we may call the pure, pleasurable contemplation of actual things and events æsthetic, — and what reason against it could be adduced from common usage ? — it is thus clear that the circle of the æsthetic is wider than the field of art. Our admiring and adoring self-abandonment to nature-beauties bears all the marks of the æsthetic attitude, and needs for all that no connection with art. Further: in all intellectual and social spheres a part of the productive energy expresses itself in æsthetic forms; these products, which are not works of art, are yet æsthetically enjoyed. As numberless facts of daily experience show us that taste can develop and become effective independently of art, we must then concede to the sphere of the æsthetic a wider circumference than that of art.

This is not to maintain that the circle of art is a narrow section of a large field. On the contrary, the æsthetic principle does not by any means exhaust the content and purpose of that realm of human production which taken together we call "art." Every true work of art is extraordinarily complex in its motives and its effects; it arises not alone from the free play of æsthetic impulse, and aims at more than pure beauty — at more than æsthetic pleasure. The desires and energies in which art is grounded are in no way fulfilled by the serene satisfaction which is the traditional criterion of the æsthetic impression, as of the æsthetic object. In reality the arts have a function in intellectual and social life, through which they are closely bound up with all our knowing and willing.

It is, therefore, the duty of a general science of art to take account of the broad facts of art in all its relations. Æsthetics is not capable of this task, if it is to have a determined, self-complete, and clearly bounded content. We may no longer obliterate the differences between the two disciplines, but must rather so sharply separate them by ever finer distinctions that the really existent connections become clear. The first step thereto has been taken by Hugo Spitzer. The relation of earlier to current views is comparable to that between materialism and positivism. While materialism ventured on a pretty crude resolution of the spiritual into the corporeal, positivism set up a hierarchy of forces of nature, whose order was determined by the relation of dependence. Thus mechanical forces, physico-chemical processes, the biological and the social-historical groups of facts, are not traced back each to the preceding by an inner connection, but are so linked that the higher orders appear as dependent on the lower. In the same way is it now sought to link art methodologically with the æsthetic. Perhaps even more closely, indeed, since already æsthetics and the science of art often play into each other's hands, like the tunnel-workers who pierce a mountain from opposite points, to meet at its centre.

Often it so happens, but not invariably. In many cases research is carried to an end, quite irrespectively of what is going on in other quarters. The field is too great, and the interests are too various. Artists recount their experiences in the process of creation, connoisseurs enlighten us as to the technique of the special arts; sociologists investigate the social function, ethnologists the origin, of art; psychologists explore the æsthetic impression, partly by experiment, partly through conceptual analysis; philosophers expound æsthetic methods and principles; the historians of literature, music, and pictorial art have collected a vast deal of material — and the sum total of these scientific inquiries constitutes the most substantial though not the greatest part of the published discussions, which, written from every possible point of view, abound in newspapers and magazines. "There is left, then, for the serious student, naught but to resolve to fix a central point somewhere, and thence to find out a way to deal with all the rest as outlying territory" (Goethe).

Only by the mutual setting of bounds can a united effect be possible from the busy whirl of efforts. Contradictory and heterogeneous facts are still very numerous. He who should undertake to construct thereof a clear intelligible unity of concepts, would destroy the energy which now proves itself in the encounters, crossing of swords, and lively controversies of scholars, and would mutilate the fullness of experience which now expresses itself in the manifold special researches. System and method signify for us: to be free from *one* system and *one* method.

II

If we are to consider how we answer to-day the questions put for scientific consideration as to the facts of æsthetic life and of art, first of all we must examine the now prevailing theories of æsthetics. They fall in general into æsthetic objectivism and subjectivism. By the first collective name we denote the aggregate of all theories which find the characteristic of their field of inquiry essentially in the quality and conformation of the object, not in the attitude of the enjoying subject. This quality of the æsthetically valuable is most easily characterized by setting it off against reality. Of such theories, which explain "the beautiful" and art from their relation to what is given in nature, naturalism stands for the identity of reality and art, while the various types of idealism set forth art as more than reality, and *vice versa*, formalism, illusionism, sensualism make it less than reality.

Inasmuch as naturalism is still defended only by a handful of artists who write, it would appear almost superfluous to consider it. But the refutations of it which are still appearing indicate that it must have some life. And in fact it still exists, partly as a present-day phenomenon in literature and art, partly as the permanent conviction of many artists. The naturalistic style testifies to revolt against forms and notions which are dying out; it therefore only attains a pure æsthetic interest through the theoretic ground which is furnished to it. And this rests above all on the testimony of the artists, who are never weary of assuring us that they immediately reproduce what is given in perception. Some philosophical conceptions also play therein a certain rôle. The adherents of the doctrine that only the sense-world is real come as a matter of course to the demand that art shall hold itself strictly to the given. And what optimist, who explains the real world as the best of all possible worlds, can, without a logical weakening, admit a play of imagination different from the reality.

Æsthetic idealism, too, is borne on general philosophical premises. However various these are, in this they all agree, that the world is not exhausted by appearances, but has an ideal content and import, which finds in the æsthetic and in the field of art its expression to sense. Even H. Taine sets to art the task of showing the "dominant character" of things. The beautiful is therefore something higher than the chance reality, — the typical as over against the anomalous natural objects or events. It can then be objectively determined with reference to its typical and generic quality and in its various kinds.

Somewhat different is the case of formalism, which to-day scarcely anywhere sets up to be a complete system of æsthetics, but points the way for many special investigations. It seeks the æsthetically effective in the form, that is, in the relation of parts, which has in principle nothing to do with the content of the object. Every clearly perceptible unity in manifoldness is pleasing. As this arrangement is independent of the material, the æsthetic represents only a part of reality.

In contrast thereto, illusionism sets the world of art as a whole over against the whole of reality. Art, we are taught, presents neither a new aspect of the given nor hidden truth, nor pure form; it is, on the contrary, a world of appearance only, and is to be enjoyed without regard to connections in life or any consequences. While we otherwise consider objects as to how they serve our interests and as to their place in the actual connection of all things, in the æsthetic experience this twofold relation is disregarded. Neither what things do for us, nor what they do for each other, comes in question. Their reality disappears, and the beautiful semblance comes to its own. Konrad Lange has given to this theory — especially in the line of a subjective side, to be later mentioned — its modern form.

Of the nearly-related sensualism, the connoisseur Fiedler and the sculptor Hildebrand are the recent exponents; Rutgers Marshall and certain French scholars also lean that way. It is the arts which fix the transitory element of the sense-image, hold fast the fleeting, make immortal the perishable, and lend stability and permanence to all pleasure that is bound up with perception. What does painting accomplish? Arisen, as it has, out of the demands of the eye, its sole task is to gain for the undefined form- and color-impressions of reality a complete and stable existence. The same thing is true of the other arts, for their respective sense-impressions.

To sum up: If the transformation of reality is acknowledged as a fundamental principle of art, it is also to be granted that this takes place in two directions: — art is something at once more and less than nature. Inasmuch as art pushes on to the *vraie vérité*, and at the same time disregards all that is not of the nature of semblance or image, we take from it ideas whose quality enthralls and stimulates us quite independently of their meaning. Art shows us the hidden essence of the world and of life and at the same time the outsides of things created for our pleasure; that is, the objects' pure psychical value in the field of sense. It involves a lifting above nature, and at the same time the rounding out and fulfillment of sense. Through making of the object an image, it frees us from our surrounding, yet leaves us at rest in it.

We turn now to æsthetic subjectivism. Under this name we comprehend the essence of those theories which seek to solve the riddle of the beautiful by a general characterization of the æsthetic attitude. Many of these are near akin to the objectivistic theories; some, however, like the *Einfühlung*-theory, take an independent place. For the former, therefore, a mere indication will suffice. The principle of "semblance" or illusion, for instance, takes very easily a subjectivistic turn. The question then runs: Wherein consists the peculiarity of the conscious processes which are set up by the semblance? The answer as given by Meinong and Witasek starts from the fact that the totality of psychical processes falls into two divisions. Every process in one division has its counterpart in the other. To perception corresponds imagination, to judgment assumption, to real emotion ideal emotion, to earnest desire fancied desire. The æsthetic emotions attached to assumptions, the semblance-emotions, that is, are held to be scarcely distinguished, so far as feeling goes, from other emotions, at most, perhaps, by less intensity. The chief difference lies rather in the premise or basis of emotion; and this is but a mere assumption or fiction.

A critical treatment of the foregoing cannot be given here; nor of that view which explains the psychical condition in receiving an æsthetic impression as a conscious self-deception, a continued and intentional confusion of reality and semblance. The æsthetic pleasure, according to this, is a free and conscious hovering between reality and unreality; or, otherwise expressed, the never successful seeking for fusion of original and copy. The enjoyment of a good graphic representation of a globe would then depend on the spectator's now thinking he sees a real globe, now being sure he views a flat drawing.

While this theory has found but small acceptance, comparatively many modern æstheticians admit the doctrine of *Einfühlung*. Its leading exponent, Theodor Lipps, sees the decisive characteristic of æsthetic enjoyment in the fusion of an alien experience with one's own: as soon as something objectively given furnishes us the possibility of freely living ourselves into it, we feel æsthetic pleasure. In the example of the Doric column, rearing itself and gathering itself up to our view, Lipps has sought to show how given space-forms are interpreted first dynamically, then anthropomorphically. We read into the geometrical figure not only the expression of energy, but also free purposiveness. In so far as we look at it in the light of our own activity, and sympathize with it accordingly, in so far do we feel it as beautiful.

Could we enter upon a critical discussion at this point, it would appear that the *Einfühlung*-theory, like its fellows, is open to well-founded objections. The belief in an all-explaining formula is a delusion. In truth, every one of the enumerated principles is relatively justified. And as they all have points of similarity with one another, it is not hard for the past-master of terminology and the technique of concepts to epitomize the common element in a single phrase or thesis. Still, nothing is gained by such a general formula in presence of the richness of the reality; and just as little — as an exhaustive treatment would have to prove — by the concise exposition of a single method for our science.

The specially approved method of procedure at the present day is that of psychological description and explanation. It seems, indeed, very natural to see in psychical processes the real subject-matter of æsthetics, and in psychology, accordingly, the science to which it is subordinate. Some philosophers, however, — among whom I may instance Jonas Cohn, — wish to make of æsthetics a science of values, and demand that on the basis of this pretension the mutually contradictory judgments of taste and types of art be tried and tested. They will have no mere descriptive and explanatory æsthetics, but a normative, precept-giving science. In truth, the opposition of the schools is complete at every point; in the writings of Volkelt and Groos we have the proof of it.

III

The special research in the narrower field of æsthetics is at present almost entirely of the psychological type. Our survey can touch upon only the salient points.

The aim of the extended and highly detailed study consists in fixating by means of psychological analysis the course of development, the effective elements, and the various sub-species of the æsthetic experience. Certain philosophers seek a point of departure for this undertaking in the æsthetic object. Thus Volkelt's system of æsthetics finds, for the chief elements of the æsthetic enjoyment, corresponding features in the object; in the special field of poetry Dilthey has undertaken an analysis along the same lines. For the most part, however, such dissection is limited to the subjective side. In Wundt's psychology, for instance, the æsthetic state of mind is shown to be built up of sense-feelings, feelings from perceptions, intellectual and emotional excitements; the most important, that is to say, the pivotal feelings, which are bound up with space- and time-relations, become in turn the condition and support of the higher emotions, because they lead over from the field of sense to that of the logical and emotional.

If we limit ourselves to the psychological, we must first ask in what order the elements of the æsthetic impression are wont to follow each other. The phases of this development, however, are as yet not completely studied, although they are of great significance for the differences in enjoyment. The second problem concerns the constitution (taken as timeless) of the experience. All formulas which attempt to fix in two words the totality of the impression fail completely, — so extraordinarily various and manifold are the factors which enter here. What these are and how they are bound together is the question which is for the moment occupying the scholars with a leaning toward psychology.

The æsthetic impression is an emotion. According to the well-known sensualistic theory of the emotions, it must therefore, in so far as it is more than mere perception or idea, be composed of organic sensations. G. Sergi and Karl Lange see, in fact, the peculiar mark of the æsthetic experience in the general sensations which appear with changes in the circulation, breathing, etc. Unprejudiced observation must satisfy every one that much in all this is true. On the other hand, it is to be recalled that we do not reckon the organic sensations to the objective qualities of æsthetic things, and that we cannot explain in this way every artistic enjoyment. — In regard to the sensations of taste, smell, and touch, it is generally granted that they play a certain rôle, even if but as reproduced ideas and only corresponding to natural beauty. Among the most important are the attitudes and imitative movements, finely investigated by Karl Groos. — To this must be added the sensuous pleasantness of visual and auditory perceptions. Yet attempts to construct the æsthetic enjoyment in its entirety out of such pleasure-factors have so far failed. The undertaking is already wrecked by the fact that elements displeasing to sense are demonstrably present, not only as negligible admixtures, but also as necessary factors. The relations of similarity between the contents of a sense-field, and the spatial and temporal connections between them, are in any case incomparably more important; we devote to them, therefore, a closer consideration. Finally, alongside all these ideas and the emotions immediately attaching to them, there must be arrayed the great multitude of associated ideas and connecting judgments. While scientific interest in the associations is now greatly diminished, explanations of the part played by the element of really active thought are many. A universally satisfactory theory is still to appear, for the reason, above all, that here the higher principles referred to in the second section enter into the problems.

Elementary æsthetics, therefore, willingly turns aside from the shore of the very complex emotions, of association, *Einfühlung* and illusion in æsthetic experience, in order to become independent of general philosophical fundamental conceptions. Its own field lies in the general province of the perception-feelings determined immediately by the object: more exactly, of the feelings which are induced partly by the relations of similarity, partly by the outer connections of the content, partly by the linking of inner and outer reference. The qualitative relation of tones and colors arouses the so-called feelings of harmony; the arrangement in space and time awakes the so-called proportion-feelings; and from the coöperation of these two arise the so-called æsthetic complication-feelings.

As to the pleasurable tone- and color-combinations, the first are better known than the second, but even their theoretical interpretation is not well settled. More diligent and successful at the present time is the research into the proportion-feelings. So far as these bear upon space-relations, they attach either to the outlines or to the structure of the forms. The bounding-lines are then pleasing, one theory holds, when they correspond to the easiest eye-movements, and in general meet our desire for easy, effortless orientation. Another doctrine, already referred to, explains their æsthetic value from a coöperation of general bodily feelings, especially sensations of breathing and equilibrium. Accurate experiments have not succeeded in finding a real conformity to law in either the first or the second direction. In the matter of the structure of forms, symmetry in the horizontal position, and the proportion of the golden section in the vertical position, receive especial attention. All those space-shapes may be called symmetrical, whose halves are of equal value æsthetically. How these must be constituted, has been studied from the simplest examples by Münsterberg and his pupils. The explanation of the pleasing quality rests on the fact that the spectator feels the contents of the two halves — lines or colors — as light or heavy, according to the energy expended in the necessary eye-movements. In the vertical position a proportion pleases (as does also equality) which is only approximately that of the golden section. The numerical proportion is, therefore, not the ground of pleasure, for otherwise those forms which are thus divided would have to be the absolutely beautiful ones, and the more a division varies from the exact fraction, the more would it sacrifice in beauty. The ground of pleasure is rather descried in the fact that in the case of the pleasing divisions the two parts stand out as distinct and clearly characterized, while yet unified effect is secured through the larger division.

The temporal ordering of an æsthetic character is that of rhythm. Concerning the æsthetic object as such — that is, concerning the metrical forms in music and poetry, the views are still widely at variance; this is true to a startling degree of poetry, because here the element, that is to say, the word, is made up of accented and unaccented syllables, and because the tendency of the logical connections of the content to create unities cannot be done away with. This state of confusion is so much the more to be regretted as it is just to the art-forms that the most vivid rhythmical feelings attach. The psychological investigations of Neumann, Bolton, and others have nevertheless much advanced our scientific understanding of this subject. A new point of view has taken its rise from Souriau and Bücher: the connection of the art-rhythm with work and other aspects of life. But the collections of data do not yet render it possible to settle the question in what manner the rhythm of work, which runs on automatically, and is controlled by the idea of an end, goes over into æsthetic rhythm.

The æsthetic complication-feelings are bound up with the products of the fusion of rhythm and harmony, form and color, rhythm and form (in the dance). So long as all elements of association are neglected, three characteristics remain to be noted: an increasing valuation of the absolute quantity, the building-up of definite form-qualities (*Gestaltqualitäten*), and a reconciliation or harmony of differences, wherein the quantitative element is wont to be the unifying, the qualitative element the separating factor. I need not, however, go any further into investigations so subtle, and even now merely in their beginnings.

This entire fabric of experience, from which but a few threads have been drawn out to view, can now take on various shadings. These we refer to as the æsthetic moods, or by a less psychological name, as the æsthetic categories. The ideally beautiful and the sublime, the tragic and the ugly, the comic and the graceful, are the best known among them. Modern science has shown most interest in the study of the comic and the tragic. According to Lipps the specific emotion of the comic arises in the disappointing of a psychical preparation for a strong impression, by the appearance of a weak one. The pleasurable character of the experience would be explained by the fact that the surplus of psychical impulse, like every excess of inner energy, is felt as agreeable. The tragic mood is understood no longer as arising in fear and pity, but in pathos and wonder. Its objective correlate should not be forced to the standard of a narrow ethics. The demand for guilt and expiation is being given up by progressive thinkers in æsthetics; but the constituents of tragedy remain fast bound to the realm of harshness, cruelty, and dissonance.

IV

From a period more or less remote there have existed poetics, musical theory, and the science of art. To examine the presuppositious methods and aims of these disciplines from the epistemological point of view, and to sum up and compare their most important results, is the task of a general science of art; this has besides, in the problems of artistic creation and the origin of art, and of the classification of the arts and their social function, certain fields of inquiry that would otherwise have no definite place. They are worked, indeed, with remarkable diligence and productiveness. Most to be regretted, on the other hand, is that so little energy is applied to laying the epistemological foundation.

The theory of the development of art deals with it both in its individual and its generally human aspect. Concerning the genesis of the child's understanding of art and impulse to produce it, we learn most from the studies of his drawings at an early age. Here are to be noted well-established results of observation, even though as yet they are few in number. On the other hand, the unfolding of primitive feeling (and of the æsthetic sensibility in general) during the historical period can be only approximately reconstructed. The case is somewhat more favorable for our information in regard to the beginnings of art, especially since it has been systematically assembled by Ernst Grosse and Yrjö Hirn. If the conditions of the most primitive of the races now living in a state of nature can be taken as identical with those at the beginnings of civilization, the entire vast material of ethnology can be made use of. We gather therefrom how close-linked with the useful and the necessary beauty is, and see clearly that primitive art is thoroughly penetrated by the purpose of a common enjoyment, and is effective in a social way; but beyond such general principles one can go only with hesitation, inasmuch as it seems scarcely possible to us, creatures of civilization, to fix the boundaries of what is really art there.

There are three conjectures as to objective origin of art. It may be that the separate arts have developed through variation from one embryonic state. Or the main arts may have been separate from the very first, having arisen independently of each other. Finally, there are middle views, like that of Spencer, according to which poetry, music, and the dance on the one hand, and writing, painting, and sculpture on the other, have a common root; Möbius recognizes three primitive arts, to which the others are to be traced back. The solution of this question would be especially important, could one hope to find Darwin's maxim for all ætiological investigations valid for our field also — that is, the dictum: What is of like origin is of like character.

As psychological conditions, from which the artistic activity is likely first to have arisen, the following functions have been suggested and maintained, — the play-instinct, imitation, the need for expression and communication, the sense for order and arrangement, the impulse to attract others and the opposed impulse to startle others. Each of these theories of conditions must clearly connect itself with one or the other of the just-named three theories of art's origin; for had music, taken in our sense and independently, existed as the original art, one could hardly regard imitation as the psychological root of art. All in all, art and the play-instinct seem most closely linked; that is also true, moreover, of its development with the child.

I come now to the fundamental problems of artistic creation. It is they which present the most obstinate difficulties to a thorough and exact investigation, for experiment and the questionnaire —

which aims at least at objectivity — are but crude means to the end in view. At the present day, as earlier, there is no lack of very refined, penetrating, — nay, brilliant analyses. They have a very superior value; but this has no special significance for the present status of the science of æsthetics, and for this reason our survey may omit much which yet has an interest for individuals.

The influence of heredity and environment on the artist's talent offers rich material for research. It is conceded, though, that how the most material and the most spiritual of influences, inherited disposition and fortune, the chances of descent and of intercourse with one's fellows, — how all this is fused into a unified personality, can be established only in individual cases by the biographer. A second very productive source of material in this field has appeared in Lombroso's teaching. The days of the most violent controversies lie behind us. It is the general view that genius and madness are near allied in their expression, that greatness often breaks forth in questionable forms; yet the majority perceive an essential difference; the genius points onward, the mind diseased harks back; the one has purposive significance, the other not. After these more introductory inquiries, the real work begins. It has to show in what points every gift for art coincides with generally disseminated abilities, and just where the specific power sets in, which the inartistic person lacks. Take, for example, the memory. We retain this or that fact without, in principle, any selection; the remembrance of the artist, on the contrary, is dissociative — it favors what is needful for its own ends. The memory of the painter battens on forms and colors, the consciousness of the musician is filled with melodies, the fancy of the poet lives in verbal images. Also there is, especially with the poet, a peculiar understanding for human experience. In truth, the fanciful products of the imagination are but the starting-point for the soul-knowledge of the poet. Without going into details we may say that by such penetrating and delimiting analyses the superficial theory of inspiration is refuted. Out of date, too, is the notion that the artist creates by putting things together; on the contrary his fancy has the whole before the parts, it gives to the world an organism, within which the members gradually emerge. Finally, the old theory is no longer held, according to which the work of art is already complete in the inner man, and afterwards merely brought to light. More definite explanation is given by the doctrine of the way in which the artistic creation runs its course, which Eduard v. Hartmann has skillfully portrayed.

The distinction, differentiation, and comparison of the special arts offers opportunity and material for numberless special studies. Music is here the least fully represented, since it is only exceptionally that art-philosophers feel a drawing to it. So much the more, however, are they inclined to the study of poetry. They are even beginning to make use, for poetics, of the studies in the modern psychology of language, since it is acknowledged that language is the essential element, and thus more than the mere form of expression, of the poetic art. Th. A. Meyer has thrown an apple of discord into the question whether the poet's words must, in order to arouse pleasure, also awake an image. As a matter of fact, the æsthetic value does not depend on the chance-aroused sense-images, but on the language itself and the images which belong to it alone; for the most part the understanding of the words alone is enough to give the reader pleasure in the poetic treatment. In the general theory of the visually representative arts there are two opposed doctrines. The one emphasizes the common element, and believes to have found it in the so-called *Fernbild*, or distant image; the other seeks salvation in complete separations — as, for instance, of the so-called *Griffelkunst*, or graphic art, from painting. Only the future can decide between them.

The existence of the total field of art as an essential factor of human endeavors involves difficulties which must be removed partly in the philosophical consideration, partly in law and governmental practice. The last factor must also be taken account of in theory; for so long as we do not live in an ideal world, the state will claim regulation of all activities expressing themselves in it, and so also of art. In first line it is concerned for art's relation to morality. Secondly, the social problems arise: does art bind men together, or part them? does it reconcile or intensify oppositions? is it democratic or aristocratic? is it a necessity or a luxury? does it further or reject patriotic, ethical, pedagogical ends? The artistic education of youth and the race has become a burning question. Ruskin and Morris have developed from art-critics to critics of the social order, and Tolstoï has contracted the democratic point of view to the most extreme degree. With the desire to transform art from the privilege of the few to the possession of all is, finally, bound up the wish that art shall emerge from another seclusion — that it shall not be throned in museums and libraries, in theatres and concert-halls, but shall mingle with our daily domestic life, and direct and color every act of the scholar as of the peasant.

A satisfactory decision can be reached only by him who keeps in view that art presents something extremely complex, and by no means mere æsthetic form; that, on the other hand, the æsthetic life is not banished to the sacred circle of the independent arts. With this conclusion we return to the first words of our reflections herein presented.

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A short paper was contributed by Professor A. D. F. Hamlin, of Columbia University, on the "Sources of Savage Conventional Patterns." The speaker said that, in the exhibit of the Department of the Interior, two glass cases displayed side by side the handiwork of the American Indian of one hundred years ago and of to-day. In the Fine Arts palace the blankets and basketry of the Navahoes were shown beside the leather work and other handicrafts of white Americans. In both instances the contrast between the savage and the civilized work emphasizes the fact that civilization tends to stifle or destroy the decorative instinct. The savage art is spontaneous, instructive, unpremeditated. The work of the civilized artist is thoughtful, carefully elaborated, intellectual. Among these peoples both the crafts and the patterns are traditional, and there is little or no ambition to innovate. The forms and combinations we admire in their work are the result of long-continued processes of evolution and elimination in which, as in the world of living organisms, the fittest have survived. The structure of savage patterns is almost always extremely simple. There are three theories advanced to account for them: that they were invented out of hand; that they were evolved out of the technical processes, tools, and materials of primitive industry; that they are descended from fetish or animistic representations of natural forms. The first is the common view of laymen; the second was first expressed (though chiefly with reference to civilized art) by Semper; and the third is widely entertained by anthropologists.

The savage instinct for decoration has probably developed from primitive animism — from that fear of the powers of nature, and that confounding of the animate and inanimate world which is universally recognized as a primitive trait. But once awakened in even the slightest degree, it has found exercise in the operations of primitive industry, and given existence to a long series of repetitive forms produced in weaving, basketry, string-lashing, and carving. The two classes of patterns thus originated — those derived from the imitation of nature under fetish ideas, and those derived from technical processes — have invariably converged, overlapping at last in many forms of decorative art, so that the real origin of a given pattern may be dual. Myths have invariably arisen to explain the origin of the technical patterns, which have received magic significance and names, in accordance with savage tendency to assign magical powers to all visible or at least to all valued objects: all savage art is talismanic. One ought to be cautious about dogmatizing as to origins in dealing with savage art, because both the phenomenon of what I call convergence in ornament evolution, and that of the myths, poetic faculty, and habit among savages, tend to confuse and obscure the real origin of the patterns with which they deal. And finally, for the artist as distinguished from the archæologist and the theorist, the real lesson of savage art is not in its origins, but in its products; in the strength, simplicity, admirable distribution, and high decorative effects of poor and despised peoples. Savage all-over patterns and Greek carved ornament and decorative sculpture represent the opposed poles of decorative design, and both are of fundamental value as objects of study for the designer.

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OF HARVARD UNIVERSITY

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DEPARTMENT II — MATHEMATICS

DEPARTMENT II — MATHEMATICS

(*Hall 7, September 20, 11.15 a. m.*)

CHAIRMAN: PROFESSOR HENRY S. WHITE, Northwestern University.
SPEAKERS: PROFESSOR MAXIME BÔCHER, Harvard University.
PROFESSOR JAMES P. PIERPONT, Yale University.

THE Chairman of the Department of Mathematics was Professor Henry S. White, of Northwestern University. In opening the proceedings Professor White said:

"Influenced by patriotism and by pride in material progress, cities and whole nations meet and celebrate the building of bridges, the opening of long railways, the tunneling of difficult mountain passes, the acquisition of new territories, or commemorate with festivity the discovery of a continent. These things are real and significant to us all.

"In the realm of ideas also there are events of no less moment, discoveries and conquests that greatly enlarge the empire of human reason. In the lapse of a century there may be many such notable achievements, even in the domain of a single science.

"Mathematics is a science continually expanding; and its growth, unlike some political and industrial events, is attended by universal acclamation. We are wont to-day, as devotees of this noble and useful science, to pass in review the newest phases of her expansion, — I say *newest*, for in retrospect a century is but brief, — and to rejoice in the deeds of the past. At the same time, also, we turn an eye of aspiration and resolution towards the mountains, rivers, deserts, and the obstructing seas that are to test the mathematicians of the future."

THE FUNDAMENTAL CONCEPTIONS AND METHODS OF MATHEMATICS

BY PROFESSOR MAXIME BÔCHER

[**Maxime Bôcher,** Professor of Mathematics, Harvard University. **b.** August 28, 1867, Boston, Mass. A.B. Harvard, 1888; Ph.D. Göttingen, 1891. Instructor, Assistant Professor and Professor, Harvard University, 1891–. Fellow of the American Academy. **Author of** *Ueber die Reihenentwickelungen der Potentialtheorie;* and various papers on mathematics.]

I. *Old and New Definitions of Mathematics*

I AM going to ask you to spend a few minutes with me in considering the question: what is mathematics? In doing this I do not propose to lay down dogmatically a precise definition; but rather, after having pointed out the inadequacy of traditional views, to determine what characteristics are common to the most varied parts of mathematics but are not shared by other sciences, and to show how this opens the way to two or three definitions of mathematics, any one of which is fairly satisfactory. Although this is, after all, merely a discussion of the meaning to be attached to a name, I do not think that it is unfruitful, since its aim is to bring unity into the fundamental conceptions of the science with which we are concerned. If any of you, however, should regard such a discussion of the meaning of words as devoid of any deeper significance, I will ask you to regard this question as merely a bond by means of which I have found it convenient to unite what I have to say on the fundamental conceptions and methods of what, with or without definition, we all of us agree to call mathematics.

The old idea that mathematics is the science of quantity, or that it is the science of space and number, or indeed that it can be characterized by any enumeration of several more or less heterogeneous objects of study, has pretty well passed away among those mathematicians who have given any thought to the question of what mathematics really is. Such definitions, which might have been intelligently defended at the beginning of the nineteenth century, became obviously inadequate as subjects like projective geometry, the algebra of logic, and the theory of abstract groups were developed; for none of these has any necessary relation to quantity (at least in any ordinary understanding of that word), and the last two have no relation to space. It is true that such examples have had little effect on the more or less orthodox followers of Kant, who regard mathematics as concerned with those conceptions which are obtained by direct intuition of time and space without the aid of empirical observation. This view seems to have been held by such eminent mathematicians as Hamilton and DeMorgan; and it is a very difficult position to refute, resting as it does on a purely metaphysical foundation which regards it as certain that we can evolve out of our inner consciousness the properties of time and space. According to this view the idea of quantity is to be deduced from these intuitions; but one of the facts most vividly brought home to pure mathematicians during the last half-century is the fatal weakness of intuition when taken as the logical source of our knowledge of number and quantity.[1]

The objects of mathematical study, even when we confine our attention to what is ordinarily regarded as *pure* mathematics are, then, of the most varied description; so that, in order to reach a satisfactory conclusion as to what really characterizes mathematics, one of two methods is open to us. On the one hand we may seek some hidden resemblance in the various objects of mathematical investigation, and having found an aspect common to them all we may fix on this as the one true object of mathematical study. Or, on the other hand, we may abandon the attempt to characterize mathematics by means of its *objects of study*, and seek in its *methods* its distinguishing characteristic. Finally, there is the possibility of our combining these two points of view. The first of these methods is that of Kempe, the second will lead us to the definition of Benjamin Peirce, while the third has recently been elaborated at great length by Russell. Other mathematicians have naturally followed out more or less consistently the same ideas, but I shall nevertheless take the liberty of using the names Kempe, Peirce, and Russell as convenient designations for these three points of view. These different methods of approaching the question lead finally to results which, without being identical, still stand in the most intimate relation to one another, as we shall now see. Let us begin with the second method.

II. *Peirce's Definition*

More than a third of a century ago Benjamin Peirce wrote:[2] *Mathematics is the science which draws necessary conclusions.* According to this view there is a mathematical element involved in every inquiry in which exact reasoning is used. Thus, for instance,[3] a jury listening to the attempt of the counsel for the prisoner to prove an alibi in a criminal case might reason as follows: "If the witnesses are telling the truth when they say that the prisoner was in St. Louis at the moment the crime was committed in Chicago, and if it is true that a person cannot be in two places at the same time, it follows that the prisoner was not in Chicago when the crime was committed." This, according to Peirce, is a bit of mathematics; while the further reasoning by which the jury would decide whether or not to believe the witnesses, and the reasoning (if they thought any necessary) by which they would satisfy themselves that a person cannot be in two places at once, would be inductive reasoning, which can give merely a high degree of probability to the conclusion, but never certainty. This mathematical element may be, as the example just given shows, so slight as not to be worth noticing from a practical point of view. This is almost always the case in the transactions of daily life and in the observational sciences. If, however, we turn to such subjects as chemistry and mineralogy, we find the mathematical element of considerable importance, though still subordinate. In physics and astronomy its importance is much greater. Finally in geometry, to mention only one other science, the mathematical element predominates to such an extent that this science has been commonly rated a branch of pure mathematics, whereas, according to Peirce, it is as much a branch of applied mathematics as is, for instance, mathematical physics.

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[1] I refer here to such facts as that there exist continuous functions without derivatives, whereas the direct untutored intuition of space would lead any one to believe that every continuous curve has tangents.

[2] *Linear Associative Algebra.* Lithographed 1870. Reprinted in the *American Journal of Mathematics*, vol. IV.

[3] This illustration was suggested by the remarks by J. Richard, *Sur la philosophie des mathmèatiques.* Paris, Gauthier-Villars, 1903, p. 50.

It is clear from what has just been said that, from Peirce's point of view, mathematics does not necessarily concern itself with quantitative relations, and that any subject becomes capable of mathematical treatment as soon as it has secured data from which important consequences can be drawn by exact reasoning. Thus, for example, even though psychologists be right when they assure us that sensations and the other objects with which they have to deal cannot be measured, we need still not necessarily despair of one day seeing a mathematical psychology, just as we already have a mathematical logic.

I have said enough, I think, to show what relation Peirce's conception of mathematics has to the applications. Let us then turn to the definition itself and examine it a little more closely. You have doubtless already noticed that the phrase, "the science which draws necessary conclusions," contains a word which is very much in need of elucidation. What is a *necessary* conclusion? Some of you will perhaps think that the conception here involved is one about which, in a concrete case at least, there can be no practical diversity of opinion among men with well-trained minds; and in fact when I spoke a few minutes ago about the reasoning of the jurymen when listening to the lawyer trying to prove an alibi, I assumed tacitly that this is so. If this really were the case, no further discussion would be necessary, for it is not my purpose to enter into any purely philosophical speculations. But unfortunately we cannot dismiss the matter in this way; for it has happened not infrequently that the most eminent men, including mathematicians, have differed as to whether a given piece of reasoning was exact or not; and, what is worse, modes of reasoning which seem absolutely conclusive to one generation no longer satisfy the next, as is shown by the way in which the greatest mathematicians of the eighteenth century used geometric intuition as a means of drawing what they regarded as necessary conclusions.[1]

I do not wish here to raise the question whether there is such a thing as absolute logical rigor, or whether this whole conception of logical rigor is a purely psychological one bound to change with changes in the human mind. I content myself with expressing the belief, which I will try to justify a little more fully in a moment, that as we never have found an immutable standard of logical rigor in the past, so we are not likely to find it in the future. However this may be, so much we can say with tolerable confidence, as past experience shows, that no reasoning which claims to be exact can make any use of intuition, but that it must proceed from definitely and completely stated premises according to certain principles of formal logic. It is right here that modern mathematicians break sharply with the tradition of *a priori* synthetic judgments (that is, conclusions drawn from intuition) which, according to Kant, form an essential part of mathematical reasoning.

If then we agree that "necessary conclusions" must, in the present state of human knowledge, mean conclusions drawn according to certain logical principles from definitely and completely stated premises, we must face the question as to what these principles shall be. Here, fortunately, the mathematical logicians from Boole down to C. S. Peirce, Schröder, and Peano have prepared the field so well that of late years Peano and his followers [2] have been able to make a rather short list of logical conceptions and principles upon which it would seem that all exact reasoning depends.[3] We must remember, however, when we are tempted to put implicit confidence in certain fundamental logical principles, that, owing to their extreme generality and abstractness, no very great weight can be attached to the mere fact that these principles appeal to us as obviously true; for, as I have said, other modes of reasoning which are now universally recognized as faulty have appealed in just this way to the greatest minds of the past. Such confidence as we feel must, I think, come from the fact that those modes of reasoning which we trust have withstood the test of use in an immense number of cases and in very many fields. This is the severest test to which any theory can be put, and if it does not break down under it we may feel the greatest confidence that, at least in cognate fields, it will prove serviceable. But we can never be sure. The accepted modes of exact reasoning may any day lead to a contradiction which would show that what we regard as universally applicable principles are in reality applicable only under certain restrictions.[1]

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[1] All writers on elementary geometry from Euclid down almost to the close of the nineteenth century use intuition freely, though usually unconsciously, in obtaining results which they are unable to deduce from their axioms. The first few demonstrations of Euclid are criticised from this point of view by Russell in his *Principles of Mathematics*, vol. I, 404–407. Gauss's first proof (1799) that every algebraic equation has a root gives a striking example of the use of intuition in what was intended as an absolutely rigorous proof by one of the greatest and at the same time most critical mathematical minds the world has ever seen.

[2] And, independently, Frege.

[3] It is not intended to assert that a single list has been fixed upon. Different writers naturally use different lists.

To show that the danger which I here point out is not a purely fanciful one, it is sufficient to refer to a very recent example. Independently of one another, Frege and Russell have built up the theory of arithmetic from its logical foundations. Each starts with a definite list of apparently self-evident logical principles, and builds up a seemingly flawless theory. Then Russell discovers that his logical principles when applied to a very general kind of logical *class* lead to an absurdity; and both Frege and Russell have to admit that something is wrong with the foundations which looked so secure. Now there is no doubt that these logical foundations will be somehow recast to meet this difficulty, and that they will then be stronger than ever before.[2] But who shall say that the same thing will not happen again?

It is commonly considered that mathematics owes its certainty to its reliance on the immutable principles of formal logic. This, as we have seen, is only half the truth imperfectly expressed. The other half would be that the principles of formal logic owe such degree of permanence as they have largely to the fact that they have been tempered by long and varied use by mathematicians. "A vicious circle!" you will perhaps say. I should rather describe it as an example of the process known to mathematicians as the method of successive approximations. Let us hope that in this case it is really a convergent process, as it has every appearance of being.

But to return to Peirce's definition. From what are these necessary conclusions to be drawn? The answer clearly implied is, from any premises sufficiently precise to make it possible to draw necessary conclusions from them. In geometry, for instance, we have a large number of intuitions and fixed beliefs concerning the nature of space: it is homogeneous and isotropic, infinite in extent in every direction, etc.; but none of these ideas, however clearly defined they may at first sight seem to be, gives any hold for exact reasoning. This was clearly perceived by Euclid, who therefore proceeded to lay down a list of axioms and postulates, that is, specific facts which he assumes to be true, and from which it was his object to deduce all geometric propositions. That his success here was not complete is now well known, for he frequently assumes unconsciously further data which he derives from intuition; but his attempt was a monumental one.

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[1] If the view which I here maintain is correct, it follows that if the term "absolute logical rigor" has a meaning, and if we should some time arrive at this absolute standard, the only indication we should ever have of the fact would be that for a long period, several thousand years let us say, the logical principles in question had stood the test of use. But this state of affairs might equally well mean that during that time the human mind had degenerated, at least with regard to some of its functions. Consider, for instance, the twenty centuries following Euclid when, without doubt, the high tide of exact thinking attained during Euclid's generation had receded.

[2] Cf. Poincaré's view in *La Science et l'Hypothèse*, p. 179, according to which a theory never renders a greater service to science than when it breaks down.

III. *The Abstract Nature of Mathematics*

Now a further self-evident point, but one to which attention seems to have been drawn only during the last few years, is this: since we are to make no use of intuition, but only of a certain number of explicitly stated premises, it is not necessary that we should have any idea what the nature of the objects and relations involved in these premises is.[1] I will try to make this clear by a simple example. In plane geometry we have to consider, among other things, points and straight lines. A point may have a peculiar relation to a straight line which we express by the words, the point lies on the line. Now one of the fundamental facts of plane geometry is that two points determine a line, that is, if two points are given, there exists one and only one line on which both points lie. All the facts that I have just stated correspond to clear intuitions. Let us, however, eliminate our intuition of what is meant by a point, a line, a point lying on a line. A slight change of language will make it easy for us to do this. Instead of points and lines, let us speak of two different kinds of objects, say A-objects and B-objects; and instead of saying that a point lies on a line we will simply say that an A-object bears a certain relation R to a B-object. Then the fact that two points determine a line will be expressed by saying: If any two A-objects are given, there exists one and only one B-object to which they both bear the relation R. This statement, while it does not force on us any specific intuitions, will serve as a basis for mathematical reasoning[2] just as well as the more familiar statement where the terms *points* and *lines* are used. But more than this. Our A-objects, our B-objects, and our relation R may be given an interpretation, if we choose, very different from that we had at first intended.

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[1] This was essentially Kempe's point of view in the papers to be referred to presently. In the geometric example which follows it was clearly brought out by H. Wiener: *Jahresbericht d. deutschen Mathematiker-Vereinigung*, vol. I (1891), p. 45.

[2] In conjunction, of course, with further postulates with which we need not here concern ourselves.

We may, for instance, regard the A-objects as the straight lines in a plane, the B-objects as the points in the same plane (either finite or at infinity), and when an A-object stands in the relation R to a B-object, this may be taken to mean that the line passes through the point. Our statement would then become: Any two lines being given, there exists one and only one point through which they both pass. Or we may regard the A-objects as the men in a certain community, the B-objects as the women, and the relation of an A-object to a B-object as friendship. Then our statement would be: In this community any two men have one, and only one, woman friend in common.

These examples are, I think, sufficient to show what is meant when I say that we are not concerned in mathematics with the nature of the objects and relations involved in our premises, except in so far as their nature is exhibited in the premises themselves. Accordingly mathematicians of a critical turn of mind, during the last few years, have adopted more and more a purely nominalistic attitude towards the objects and relations involved in mathematical investigation. This is, of course, not the crude mixture of nominalism and empiricism of the philosopher Hobbes, whose claim to mathematical fame, it may be said in passing, is that of a circle-squarer.[1] The nominalism of the present-day mathematician consists in treating the objects of his investigation and the relations between them as mere symbols. He then states his propositions, in effect, in the following form: If there exist any objects in the physical or mental world with relations among themselves which satisfy the conditions which I have laid down for my symbols, then such and such facts will be true concerning them.

It will be seen that, according to Peirce's view, the mathematician *as such* is in no wise concerned with the source of his premises or with their harmony or lack of harmony with any part of the external world. He does not even assert that any objects really exist which correspond to his symbols. Mathematics may therefore be truly said to be the most abstract of all sciences, since it does not deal directly with reality.[2]

This, then, is Peirce's definition of mathematics. Its advantages in the direction of unifying our conception of mathematics and of assigning to it a definite place among the other sciences are clear.

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[1] Hobbes practically obtains as the ratio of a circumference to its diameter the value $\sqrt{10}$. Cf. for instance Molesworth's edition of Hobbes's English Works, vol. VII, p. 431.

[2] Cf. the very interesting remarks along this line of C. S. Peirce in *The Monist*, vol. VII, pp. 23–24.

What are its disadvantages? I can see only two. First that, as has been already remarked, the idea of drawing necessary conclusions is a slightly vague and shifting one. Secondly, that it lays exclusive stress on the rigorous logical element in mathematics and ignores the intuitional and other non-rigorous tendencies which form an important element in the great bulk of mathematical work concerning which I shall speak in greater detail later.

IV. *Geometry an Experimental Science*

Some of you will also regard it as an objection that there are subjects which have almost universally been regarded as branches of mathematics but are excluded by this definition. A striking example of this is geometry, I mean the science of the actual space we live in; for though geometry is, according to Peirce's definition, preëminently a mathematical science, it is not exclusively so. Until a system of axioms is established mathematics cannot begin its work. Moreover, the actual perception of spatial relations, not merely in simple cases but in the appreciation of complicated theorems, is an essential element in geometry which has no relation to mathematics as Peirce understands the term. The same is true, to a considerable extent, of such subjects as mechanical drawing and model-making, which involve, besides small amounts of physics and mathematics, mainly non-mathematical geometry. Moreover, although the mathematical method is the traditional one for arriving at the truth concerning geometric facts, it is not the only one. Direct appeal to the intuition is often a short and fairly safe cut to geometric results; and on the other hand experiments may be used in geometry, just as they are used every day in physics, to test the truth of a proposition or to determine the value of some geometric magnitude.[1]

We must, then, admit, if we hold to Peirce's view, that there is an independent science of geometry just as there is an independent science of physics, and that either of these *may* be treated by mathematical methods. Thus geometry becomes the simplest of the natural sciences, and its axioms are of the nature of physical laws, to be tested by experience and to be regarded as true only within the limits of error of observation. This view, while it has not yet gained universal recognition, should, I believe, prevail, and geometry be recognized as a science independent of mathematics, just as psychology is gradually being recognized as an independent science and not as a branch of philosophy.

The view here set forth, according to which geometry is an experimental science like physics or chemistry, has been held ever since Gauss's time by almost all the leading mathematicians who have been conversant with non-Euclidean geometry.[1] Recently, however, Poincaré has thrown the weight of his great authority against this view,[2] claiming that the experiments by which it is sought to test the truth of geometric axioms are really not geometrical experiments at all but physical ones, and that any failure of these experiments to agree with the ordinary geometrical axioms could be explained by the inaccuracy of the *physical* laws ordinarily assumed. There is undoubtedly an important element of truth here. Every experiment depends for its results not merely on the law we wish to test, but also on other laws which for the moment we assume to be true. But, if we prefer, we may, in many cases, assume as true the law we were before testing and our experiment will then serve to test some of the remaining laws. If, then, we choose to stick to the ordinary Euclidean axioms of geometry in spite of what any future experiments may possibly show, we can do so, but at the cost, perhaps, of our present simple physical laws, not merely in one branch of physics but in several. Poincaré's view [3] is that it will always be expedient to preserve simple geometric laws at all costs, an opinion for which I fail to see sufficient reason.

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[1] I am thinking of measurements and observations made on accurately constructed drawings and models. A famous example is Galileo's determination of the area of a cycloid by cutting out a cycloid from a metallic sheet and weighing it.

V. *Kempe's Definition*

Let us now turn from Peirce's method of defining mathematics to Kempe's, which, however, I shall present to you in a somewhat modified form.[4] The point of view adopted here is to try to define mathematics, as other sciences are defined, by describing the objects with which it deals. The diversity of the objects with which mathematics is ordinarily supposed to deal being so great, the first step must be to divest them of what is unessential for the mathematical treatment, and to try in this way to discover their common and characteristic element.

The first point on which Kempe insists is that the objects of mathematical discussion, whether they be the points and lines of geometry, the numbers real or complex of algebra or analysis, the elements of groups or anything else, are always individuals, infinite in number perhaps, but still distinct individuals. In a particular mathematical investigation we may, and usually do, have several different kinds of individuals; as for instance, in elementary plane geometry, points, straight lines, and circles. Furthermore, we have to deal with certain relations of these objects to one another. For instance, in the example just cited, a given point may or may not lie on a given line; a given line may or may not touch a given circle; three or more points may or may not be collinear, etc. This example shows how in a single mathematical problem a large number of relations may be involved, relations some of which connect two objects, others three, etc. Moreover these relations may connect like or they may connect unlike objects; and finally the order in which the objects are taken is not by any means immaterial in general, as is shown by the relation between three points which states that the third is collinear with and lies between the first two.

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[1] Gauss, Riemann, Helmholtz are the names which will carry perhaps the greatest weight.

[2] Cf. *La Science et l'Hypothèse.* Paris, 1903.

[3] L. c., chapter v. In particular, p. 93.

[4] Kempe has set forth his ideas in rather popular form in the *Proceedings of the London Mathematical Society*, vol. XXVI (1894), p. 5; and in *Nature*, vol. XLIII (1890), p. 156, where references to his more technical writings will be found.

But even this is not all; for, besides these objects and relations of various kinds, we often have *operations* by which objects can be combined to yield another object, as, for instance, addition or multiplication of numbers. Here the objects combined and the resulting object are all of the same kind, but this is by no means necessary. We may, for instance, consider the operation of combining two points and getting the perpendicular bisector of the line connecting them; or we may combine a point and a line and get the perpendicular dropped from the point on the line.

These few examples show how diverse the relations and operations, as well as the objects of mathematics, seem at first sight to be. Out of this apparent diversity it is not difficult to obtain a very great uniformity by simply restating the facts in a little different language. We shall find it convenient to indicate that the objects $a, b, c, \ldots,$ taken in the order named, satisfy a relation R by simply writing $R(a, b, c, \ldots)$, where it should be understood that among the objects $a, b, c, \ldots$ the same object may occur a number of times. On the other hand, if two objects a and b are combined to yield a third object c, we may write $a \circ b = c$,[1] where the symbol $\circ$ is characteristic of the special operation with which we are concerned.

Let us first notice that the equation $a \circ b = c$ denotes merely that the three objects a, b, c bear a certain relation to one another, say $R(a, b, c)$. In other words the idea of an operation or law of combination between the objects we deal with, however convenient and useful it may be as a matter of notation, is essentially merely a way of expressing the fact that the objects combined bear a certain relation to the object resulting from their combination. Accordingly, in a purely abstract discussion like the present, where questions of practical convenience are not involved, we need not consider such rules of combination.[2]

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[1] I speak here merely of dyadic operations, — *i. e.*, of operations by which two objects are combined to yield a third, — these being by far the most important as well as the simplest. What is said, however, obviously applies to operations by which any number of objects are combined.

[2] Even from the point of view of the technical mathematician it may sometimes be desirable to adopt the point of view of a relation rather than that of an operation. This is seen, for instance, in laying down a system of postulates for the

Furthermore, it is easy to see that when we speak of objects of different *kinds*, as, for instance, the points and lines of geometry, we are introducing a notion which can very readily be expressed in our relational notation. For this purpose we need merely to introduce a further relation which is satisfied by two or more objects when and only when they are of the same "kind."

Let us turn finally to the relations themselves. It is customary to distinguish here between dyadic relations, triadic relations, etc., according as the relation in question connects two objects, three objects, etc. There are, however, relations which may connect any number of objects, as, for instance, the relation of collinearity which may hold between any number of points. Any relation holds for certain ordered groups of objects but not for others, and it is in no way *necessary* for us to fix our attention on the fact, if it be true, that the number of objects in all the groups for which a particular relation holds is the same. This is the point of view we shall adopt, and we shall relegate the property that a relation is dyadic, triadic, etc., to the background along with the various other properties relations may have,[1] all of which must be taken account of in the proper place.

We are thus concerned in any mathematical investigation, from our present point of view, with just two conceptions: first a set, or as the logicians say, a *class* of objects $a, b, c, \ldots$; and secondly a class of relations $R, S, T, \ldots$. We may suppose these objects divested of any qualitative, quantitative, spatial, or other attributes which they may have had, and regard them merely as satisfying or not satisfying the relations in question, where, again, we are wholly indifferent to the nature which these relations originally had. And now we are in a position to state what I conceive to be really the essential point in Kempe's definition of mathematics, although I have omitted one of the points on which he insists most strongly,[2] by saying:

If we have a certain class of objects and a certain class of relations, and if the only questions which we investigate are whether ordered groups of these objects do or do not satisfy the relations, the results of the investigation are called mathematics.

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theory of abstract groups (cf., for example, Huntington, *Bulletin of the American Mathematical Society*, June, 1902), where the postulate:

If a and b belong to the class, $a \circ b$ belongs to the class,

which in this form looks indecomposable, immediately breaks up, when stated in the relational form, into the following two:

1. If a and b belong to the class, there exists an element c of the class such that $R(a, b, c)$.
2. If a, b, c, d belong to the class, and if $R(a, b, c)$ and $R(a, b, d)$, then $c = d$.

[1] For instance, the property of symmetry. A relation is said to be symmetrical if it holds or fails to hold independently of the *order* in which the objects are taken.

[2] Namely, that the only relation that need be considered is that of being "indistinguishable," *i. e.*, a symmetrical and transitive relation between two groups of objects.

It is convenient to have a term to designate a class of objects associated with a class of relations between these objects. Such an aggregate we will speak of as a *mathematical system*. If now we have two different mathematical systems, and if a one-to-one correspondence can be set up between the two classes of objects, and also between the two classes of relations in such a way that whenever a certain ordered set of objects of the first system satisfies a relation of that system, the set consisting of the corresponding objects of the second system satisfies the corresponding relation of that system, and *vice versa*, then it is clear that the two systems are, from our present point of view, mathematically equivalent, however different the nature of the objects and relations may be in the two cases.[1] To use a technical term, the two systems are *simply isomorphic*.[2]

It will be noticed that in the definition of mathematics just given nothing is said as to the method by which we are to ascertain whether or not a given relation holds between the objects of a given set. The method used may be a purely empirical one, or it may be partly or wholly deductive. Thus, to take a very simple case, suppose our class of objects to consist of a large number of points in a plane and suppose the only relation between them with which we are concerned is that of collinearity. Then, if the points are given us by being marked in ink on a piece of white paper, we can begin by taking three pins, sticking them into the paper at three of the points; then, by sighting along them, we can determine whether or not these points are collinear. We can do the same with other groups of three points, then with all groups of four points, etc. The same result can be obtained with much less labor if we make use of certain simple properties which the relation of collinearity satisfies, properties which are expressed by such propositions as:

$R(a, b, c)$ implies $R(b, a, c)$,

$R(a, b, c, d)$ implies $R(a, b, c)$,

$R(a, b, c)$ and $R(a, b, d)$ together imply $R(a, b, c, d)$, etc.

By means of a small number of propositions of this sort it is easy to show that no empirical observations as to the collinearity of groups of more than three points need be made, and that it may not be necessary to examine even all groups of three points. Having made this relatively small number of observations, the remaining results would be obtained deductively. Finally, we may suppose the points given by their coördinates, in which case the complete answer to our question may be obtained by the purely deductive method of analytic geometry.

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[1] The point of view here brought out, including the term isomorphism, was first developed in a special case, — the theory of groups.

[2] Inasmuch as the relations in a mathematical system are themselves objects, we may, if we choose, take our class of objects so as to include these relations as well as what we called objects before, some of which, we may remark in passing, may themselves be relations. Looked at from this point of view, we need one additional relation which is now the only one which we explicitly call a relation. If we denote this relation by inclosing the objects which satisfy it in parentheses, then if the relation denoted before by $R(a, b)$ is satisfied, we should now write (R, a, b), whereas we should *not* have (a, R, b) (S, R, a, b), etc. Thus we see that any mathematical system may be regarded as consisting of a class of objects and a *single* relation between them.

According to the modified form of Kempe's definition which I have just stated, mathematics is not necessarily a deductive science. This view, while not in accord with the prevailing ideas of mathematicians, undoubtedly has its advantages as well as its dangers. The non-deductive processes, of which I shall have more to say presently, play too important a part in the life of mathematics to be ignored, and the definition just given has the merit of not excluding them. It would seem, however, that the definition in the form just given is too broad. It would include, for instance, the determination by experimental methods of what pairs of chemical compounds of the known elements react on one another when mixed under given conditions.

VI. *Axioms and Postulates. Existence Theorems*

If, however, we restrict ourselves to exact or deductive mathematics, it will be seen that Kempe's definition becomes coextensive with Peirce's. Here, in order to have a starting-point for deductive reasoning, we must assume a certain number of facts or *primitive propositions* concerning any mathematical system we wish to study, of which all other propositions will be necessary consequences.[1] We touch here on a subject whose origin goes back to Euclid and which has of late years received great development, primarily at the hands of Italian mathematicians.[2]

It is important for us to notice at this point that not merely these primitive propositions but all the propositions of mathematics may be divided into two great classes. On the one hand, we have propositions which state that certain specified objects satisfy certain specified relations. On the other hand are the *existence theorems*, which state that there exist objects satisfying, along with certain specified objects, certain specified relations.[3] These two classes of propositions are well known to logicians and are designated by them

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[1] These primitive propositions may be spoken of as *axioms* or *postulates*, according to the point of view we wish to take concerning their source, the word axiom, which has been much misused of late, indicating an intuitional or empirical source.

[2] Peano, Pieri, Padoa, Burali-Forti. We may mention here also Hilbert, who, apparently without knowing of the important work of his Italian predecessors, has also done valuable work along these lines.

[3] Or we might conceivably have existence theorems which state that there exist relations which are satisfied by certain specified objects; or these two kinds of existence theorems might be combined. If we take the point of view explained in the second footnote on p. 467, all existence theorems will be of the type mentioned in the text.

universal and *particular* propositions respectively.[1] It is only during the last fifty years or so that mathematicians have become conscious of the fundamental importance in their science of existence theorems, which until then they had frequently assumed tacitly as they needed them, without always being conscious of what they were doing.

It is sometimes held by non-mathematicians that if mathematics were really a purely deductive science, it could not have gained anything like the extent which it has without losing itself in trivialities and becoming, as Poincaré puts it, a vast tautology.[2] This view would doubtless be correct if all primitive propositions were universal propositions. One of the most characteristic features of mathematical reasoning, however, is the use which it makes of auxiliary elements. I refer to the auxiliary points and lines in proofs by elementary geometry, the quantities formed by combining in various ways the numbers which enter into the theorems to be proved in algebra, etc. Without the use of such auxiliary elements mathematicians would be incapable of advancing a step; and whenever we make use of such an element in a proof, we are in reality using an existence theorem.[3] These existence theorems need not, to be sure, be among the primitive propositions; but if not, they must be deduced from primitive propositions some of which are existence theorems, for it is clear that an existence theorem cannot be deduced from universal propositions alone.[4] Thus it may fairly be said that existence theorems form the vital principle of mathematics, but these in turn, it must be remembered, would be impotent without the material basis of universal propositions to work upon.

VII. *Russell's Definition*

We have so far arrived at the view that exact mathematics is the study by deductive methods of what we have called a mathematical system, that is, a class of objects and a class of relations between them. If we elaborate this position in two directions we shall reach the standpoint of Russell.[5]

In the first place Russell makes precise the term *deductive method* by laying down explicitly a list of logical conceptions and principles which alone are to be used; and, secondly, he insists,[1] on the contrary, that no mathematical system, to use again the technical term introduced above, be studied in pure mathematics whose existence cannot be established solely from the logical principles on which all mathematics is based. Inasmuch as the development of mathematics during the last fifty years has shown that the existence of most, if not all the mathematical systems which have proved to be important can be deduced when once the existence of positive integers is granted, the point about which interest must centre here is the proof, which Russell attempts, of the existence of this latter system.[2] This proof will necessarily require that, among the logical principles assumed, existence theorems be found. Such theorems do not seem to be explicitly stated by Russell, the existence theorems which make their appearance further on being evolved out of somewhat vague philosophical reasoning. There are also other reasons, into which I cannot enter here, why I am not able to regard the attempt made in this direction by Russell as completely successful.[3] Nevertheless, in view of the fact that the system of finite positive integers is necessary in almost all branches of mathematics (we cannot speak of a triangle or a hexagon without having the numbers three and six at our disposal), it seems extremely desirable that the system of logical principles which we lay at the foundation of all mathematics be assumed, if possible, broad enough so that the existence of positive integers — at least finite integers — follows from it; and there seems little doubt that this can be done in a satisfactory manner. When this has been done we shall perhaps be able to regard, with Russell, pure mathematics as consisting exclusively of deductions "by logical principles from logical principles."

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[1] "All men are mortals" is a standard example of a universal proposition; while as an illustration of a particular proposition is often given: "Some men are Greeks." That this is really an existence theorem is seen more clearly when we state it in the form: "There exists at least one man who is a Greek."

[2] Cf. *La Science et l'Hypothèse*, p. 10.

[3] Even when in algebra we consider the sum of two numbers $a+b$, we are using the existence theorem which says that, any two numbers a and b being given, there exists a number c which stands to them in the relation which we indicate in ordinary language by saying that c is the sum of a and b.

[4] The power which resides in the method of mathematical induction, so called, comes from the fact that this method depends on an existence theorem. It is, however, not the only fertile principle in mathematics as Poincaré would have us believe (cf. *La Science et l'Hypothèse*). In fact there are great branches of mathematics, like elementary geometry, in which it takes little or no part.

[5] *The Principles of Mathematics*, Cambridge, England, 1903.

VIII. *The Non-Deductive Elements in Mathematics*

I fear that many of you will think that what I have been saying is of an extremely one-sided character, for I have insisted merely on the rigidly deductive form of reasoning used and the purely abstract character of the objects considered in mathematics. These, to the great majority of mathematicians, are only the dry bones of the science. Or, to change the simile, it may perhaps be said that instead of inviting you to a feast I have merely shown you the empty dishes and explained how the feast would be served if only the dishes were filled.[1] I fully agree with this opinion, and can only plead in excuse that my subject was the *fundamental* conceptions and methods of mathematics, not the infinite variety of detail and application which give our science its real vitality. In fact I should like to subscribe most heartily to the view that in mathematics, as elsewhere, the discussion of such fundamental matters derives its interest mainly from the importance of the theory of which they are the so-called foundations.[2] I like to look at mathematics almost more as an art than as a science; for the activity of the mathematician, constantly creating as he is, guided though not controlled by the external world of the senses, bears a resemblance, not fanciful I believe but real, to the activity of an artist, of a painter let us say. Rigorous deductive reasoning on the part of the mathematician may be likened here to technical skill in drawing on the part of the painter. Just as no one can become a good painter without a certain amount of this skill, so no one can become a mathematician without the power to reason accurately up to a certain point. Yet these qualities, fundamental though they are, do not make a painter or a mathematician worthy of the name, nor indeed are they the most important factors in the case. Other qualities of a far more subtle sort, chief among which in both cases is imagination, go to the making of the good artist or good mathematician. I must content myself merely by recalling to you this somewhat vague and difficult though interesting field of speculation which arises when we attempt to attach *value* to mathematical work, a field which is familiar enough to us all in the analogous case of artistic or literary criticism.

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[1] In the formal definition of mathematics at the beginning of the book this is not stated or in any way implied; and yet it comes out so clearly throughout the book that this is a point of view which the author regards as essential, that I have not hesitated to include it as a part of his definition.

[2] Cf. also Burali-Forti, *Congrès internationale de philosophie.* Paris, vol. III, p. 289.

[3] Russell's unequivocal repudiation of nominalism in mathematics seems to me a serious if not an insurmountable barrier to progress.

We are in the habit of speaking of logical rigor and the consideration of axioms and postulates as the foundations on which the superb structure of modern mathematics rests; and it is often a matter of wonder how such a great edifice can rest securely on such a small foundation. Moreover, these foundations have not always seemed so secure as they do at present. During the first half of the nineteenth century certain mathematicians of a critical turn of mind — Cauchy, Abel, Weierstrass, to mention the greatest of them — perceived to their dismay that these foundations were not sound, and some of the best efforts of their lives were devoted to strengthening and improving them. And yet I doubt whether the great results of mathematics seemed less certain to any of them because of the weakness they perceived in the foundations on which these results are built up. The fact is that what we call mathematical rigor is merely one of the foundation stones of the science; an important and essential one surely, yet not the only thing upon which we can rely. A science which has developed along such broad lines as mathematics, with such numerous relations of its parts both to one another and to other sciences, could not long contain serious error without detection. This explains how, again and again, it has come about, that the most important mathematical developments have taken place by methods which cannot be wholly justified by our present canons of mathematical rigor, the logical "foundation" having been supplied only long after the superstructure had been raised. A discussion and analysis of the non-deductive methods which the creative mathematician really uses would be both interesting and instructive. Here I must content myself with the enumeration of a few of them.

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[1] Notice that just as the empty dishes could be filled by a great variety of viands, so the empty symbols of mathematics can be given meanings of the most varied sorts.

[2] Cf. the following remark by Study, *Jahresbericht der deutschen Mathematiker-Vereinigung*, vol. xi (1902), p. 313:

"So wertvoll auch Untersuchungen über die systematische Stellung der mathematischen Grundbegriffe sind . . . *wertvoller* ist doch noch der materielle Inhalt der einzelnen Disciplinen, um dessentwillen allein ja derartige Untersuchungen überhaupt Zweck haben. . . ."

First and foremost there is the use of intuition, whether geometrical, mechanical, or physical. The great service which this method has rendered and is still rendering to mathematics both pure and applied is so well known that a mere mention is sufficient.

Then there is the method of experiment; not merely the physical experiments of the laboratory or the geometrical experiments I had occasion to speak of a few minutes ago, but also arithmetical experiments, numerous examples of which are found in the theory of numbers and in analysis. The mathematicians of the past frequently used this method in their printed works. That this is now seldom done must not be taken to indicate that the method itself is not used as much as ever.

Closely allied to this method of experiment is the method of analogy, which assumes that something true of a considerable number of cases will probably be true in analogous cases. This is, of course, nothing but the ordinary method of induction. But in mathematics induction may be employed not merely in connection with the experimental method, but also to extend results won by deductive methods to other analogous cases. This use of induction has often been unconscious and sometimes overbold, as, for instance, when the operations of ordinary algebra were extended without scruple to infinite series.

Finally there is what may perhaps be called the method of optimism, which leads us either willfully or instinctively to shut our eyes to the possibility of evil. Thus the optimist who treats a problem in algebra or analytic geometry will say, if he stops to reflect on what he is doing: "I know that I have no right to divide by zero; but there are so many other values which the expression by which I am dividing might have that I will assume that the Evil One has not thrown a zero in my denominator this time." This method, if a proceeding often unconscious can be called a method, has been of great service in the rapid development of many branches of mathematics, though it may well be doubted whether in a subject as highly developed as is ordinary algebra it has not now survived its usefulness.[1]

While no one of these methods can in any way compare with that of rigorous deductive reasoning as a method upon which to base mathematical results, it would be merely shutting one's eyes to the facts to deny them their place in the life of the mathematical world, not merely of the past but of to-day. There is now, and there always will be room in the world for good mathematicians of every grade of logical precision. It is almost equally important that the small band whose chief interest lies in accuracy and rigor should not make the mistake of despising the broader though less accurate work of the great mass of their colleagues; as that the latter should not attempt to shake themselves wholly free from the restraint the former would put upon them. The union of these two tendencies in the same individuals, as it was found, for instance, in Gauss and Cauchy, seems the only sure way of avoiding complete estrangement between mathematicians of these two types.

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[1] Cf. the very suggestive remarks by Study, *Jahresbericht d. Deutschen Mathematiker-Vereinigung*, vol. XI (1902), p. 100, footnote, in which it is pointed out how rigor, in cases of this sort, may not merely serve to increase the correctness of the result, but actually to suggest new fields for mathematical investigation.

THE HISTORY OF MATHEMATICS IN THE NINETEENTH CENTURY

BY PROFESSOR JAMES P. PIERPONT OF YALE UNIVERSITY

THE extraordinary development of mathematics in the last century is quite unparalleled in the long history of this most ancient of sciences. Not only have those branches of mathematics which were taken over from the eighteenth century steadily grown, but entirely new ones have sprung up in almost bewildering profusion, and many of these have promptly assumed proportions of vast extent.

As it is obviously impossible to trace in the short time allotted to me the history of mathematics in the nineteenth century even in merest outline, I shall restrict myself to the consideration of some of its leading theories.

Theory of Functions of a Complex Variable

Without doubt one of the most characteristic features of mathematics in the last century is the systematic and universal use of the complex variable. Most of its great theories received invaluable aid from it, and many owe their very existence to it. What would the theory of differential equations or elliptic functions be to-day without it, and is it probable that Poncelet, Steiner, Chasles, and von Staudt would have developed synthetic geometry with such elegance and perfection without its powerful stimulus?

The necessities of elementary algebra kept complex numbers persistently before the eyes of every mathematician. In the eighteenth century the more daring, as Euler and Lagrange, used them sparingly; in general one avoided them when possible. Three events, however, early in the nineteenth century changed the attitude of mathematicians toward this mysterious guest. In 1813 Argand published his geometric interpretation of complex numbers. In 1824 came the discovery by Abel of the imaginary period of the elliptic function. Finally Gauss in his second memoir on biquadratic residues (1832) proclaims them a legitimate and necessary element of analysis.

The theory of function of a complex variable may be said to have had its birth when Cauchy discovered his integral theorem

$$\int f(x)dx = 0$$

published in 1825. In a long series of publications beginning with the *Cours d'Analyse* (1821), Cauchy gradually developed his theory of functions and applied it to problems of the most diverse nature;

for example, existence theorems for implicit functions and the solutions of certain differential equations, the development of functions in infinite series and products, and the periods of integrals of one and many valued functions.

Meanwhile Germany is not idle; Weierstrass and Riemann develop Cauchy's theory along two distinct and original paths. Weierstrass starts with an explicit analytical expression, a power series, and defines his function as the totality of its analytical continuations. No appeal is made to geometric intuition, his entire theory is strictly arithmetical. Riemann growing up under Gauss and Dirichlet not only relies largely on geometric intuition, but he also does not hesitate to impress mathematical physics into his service. Two noteworthy features of his theory are the many leaved surfaces named after him, and the extensive use of conformal representation.

The history of functions as first developed is largely a theory of algebraic functions and their integrals. A general theory of functions is only slowly evolved. For a long time the methods of Cauchy, Riemann, and Weierstrass were cultivated along distinct lines by their respective pupils. The schools of Cauchy and Riemann were the first to coalesce. The entire rigor which has recently been imparted to their methods has removed all reason for founding, as Weierstrass and his school have urged, the theory of functions on a single algorithm, namely, the power series. We may therefore say that at the close of the century there is only one theory of functions in which the ideas of its three great creators are harmoniously united.

Let us note briefly some of its lines of advance. Weierstrass early observed that an analytic expression might represent different analytic functions in different regions. Associated with this is the phenomenon of natural boundaries. The question therefore arose, What is the most general domain of definition of an analytic function? Runge has shown that any connected region may serve this purpose. An important line of investigation relates to the analytic expression of a function by means of infinite series, products, and fractions. Here may be mentioned Weierstrass's discovery of prime factors; the theorems of Mittag-Leffler and Hilbert; Poincaré's uniformization of algebraic and analytic functions by means of a third variable, and the work of Stieljes, Padé, and Van Vleck on infinite fractions. Since an analytic function is determined by a single power series, which in general has a finite circle of convergence, two problems present themselves: determine, first, the singular points of the analytic function so defined, and, second, an analytic expression valid for its whole domain of definition. The celebrated memoir of Hadamard inaugurated a long series of investigations on the first problem; while Mittag-Leffler's star theorem is the most important result yet obtained relating to the second.

Another line of investigation relates to the work of Poincaré, Borel, Padé, *et al.*, on divergent series. It is, indeed, a strange vicissitude of our science that these series which early in the century were supposed to be banished once and for all from rigorous mathematics should at its close be knocking at the door for readmission.

Let us finally note an important series of memoirs on integral transcendental functions, beginning with Weierstrass, Laguerre, and Poincaré.

Algebraic Functions and their Integrals

A branch of the theory of functions has been developed to such an extent that it may be regarded as an independent theory; we mean the theory of algebraic functions and their integrals. The brilliant discoveries of Abel and Jacobi in the elliptic functions from 1824 to 1829 prepared the way for a similar treatment of the hyperelliptic case. Here a difficulty of gravest nature was met. The corresponding integrals have $2p$ linearly independent periods; but as Jacobi had shown, a one valued function having more than two periods admits a period as small as we choose. It therefore looked as if the elliptic functions admitted no further generalization. Guided by Abel's theorem, Jacobi at last discovered the solution to the difficulty (1832); to get functions analogous to the elliptic functions we must consider functions not of one but of p independent variables, namely, the p independent integrals of the first species. The great problem now before mathematicians, known as Jacobi's Problem of Inversion, was to extend this *aperçu* to the case of any algebraic configuration and develop the consequences. The first to take up this immense task were Weierstrass and Riemann, whose results belong to the most brilliant achievements of the century. Among the important notions hereby introduced we note the following: the birational transformation, rank of an algebraic configuration, class invariants, prime functions, the theta and multiply periodic functions in several variables. Of great importance is Riemann's method of proving existence theorems, as also his representation of algebraic functions by means of integrals of the second species.

A new direction was given to research in this field by Clebsch, who considered the fundamental algebraic configuration as defining a curve. His aim was to bring about a union of Riemann's ideas and the theory of algebraic curves for their mutual benefit. Clebsch's labors were continued by Brill and Nöther; in their work the transcendental methods of Riemann are placed quite in the background. More recently Klein and his school have sought to unite the transcendental methods of Riemann with the geometric direction begun by Clebsch, making systematic use of homogeneous coördinates and the invariant theory. Noteworthy, also, is his use of normal curves in (p—1) way space. to represent the given algebraic configuration. Dedekind and Weber, Hensel and Landsberg, have made use of the ideal theory with marked success. Many of the difficulties of the older theory, *e. g.*, the resolution of singularities of the algebraic configuration, are treated with a truly remarkable ease and generality.

In the theory of multiply periodic functions and the general θ functions we mention, besides Weierstrass, the researches of Prym, Krazer, Frobenius, Poincaré, and Wirtinger.

Automorphic Functions

Closely connected with the elliptic functions is a class of functions which has come into great prominence in the last quarter of a century, namely, the elliptic modular and automorphic functions. Let us consider first the modular functions of which the modulus κ and the absolute invariant J are the simplest types.

The transformation theory of Jacobi gave algebraic relations between such functions in endless number. Hermite, Fuchs, Dedekind, and Schwarz are forerunners, but the theory of modular functions as it stands to-day is principally due to Klein and his school. Its goal is briefly stated thus: Determine all sub-groups of the linear group

$$(1) \qquad x^1=\frac{\alpha x+\beta}{\gamma x+\delta}$$

where α, β, γ, δ are integers and $\alpha\delta-\beta\gamma=1$; determine for each such group associate modular functions and investigate their relation to one another and especially to J. Important features in this theory are the congruence groups of (1); the fundamental polygon belonging to a given sub-group, and its use as substitute for a Riemann surface; the principle of reflection over a circle, the modular forms.

The theory of automorphic functions is due to Klein and Poincaré. It is a generalization of the modular functions; the coefficients in (1) being any real or imaginary numbers, with non-vanishing determinant, such that the group is discontinuous. Both authors have recourse to non-Euclidean geometry to interpret the substitutions (1). Their manner of showing the existence of functions belonging to a given group is quite different. Poincaré by a brilliant stroke of genius actually writes down their arithmetic expressions in terms of his celebrated θ series. Klein employs the existence methods of Riemann. The relation of automorphic functions to differential equations is studied by Poincaré in detail. In particular, he shows that both variables of a linear differential equation with algebraic coefficients can be expressed uniformly by their means.

Differential Equations

Let us turn now to another great field of mathematical activity, the theory of differential equations. The introduction of the theory of functions has completely revolutionized this subject. At the beginning of the nineteenth century many important results had indeed been established, particularly by Euler and Lagrange; but the methods employed were artificial, and broad comprehensive principles were lacking. By various devices one tried to express the solution in terms of the elementary functions and quadratures — a vain attempt; for as we know now, the goal they strove so laboriously to reach was in general unattainable.

A new epoch began with Cauchy, who by means of his new theory of functions first rigorously established the existence of the solution of certain classes of equations in the vicinity of regular points. He also showed that many of the properties of the elliptic functions might be deduced directly from their differential equations. Ere long, the problem of integrating a differential equation changed its base. Instead of seeking to express its solution in terms of the elementary functions and quadratures, one asked what is the nature of the functions defined by a given equation. To answer this question we must first know what are the singular points of the integral function and how does it behave in their vicinity. The number of memoirs on this fundamental and often difficult question is enormous; but this is not strange if we consider the great variety of interesting and important classes of equations which have to be studied.

One of the first to open up this new path was Fuchs, whose classic memoirs (1866-68) gave the theory of linear differential equations its birth. These equations enjoy a property which renders them particularly accessible, namely, the absence of movable singular points. They may, however, possess points of indetermination, to use Fuchs's terminology, and little progress has been made in this case. Noteworthy in this connection is the introduction by v. Koch of infinite determinants, first considered by our distinguished countryman Hill; also the use of divergent series — that invention of the Devil, as Abel called them — by Poincaré. A particular class of linear differential equations of great importance is the hypergeometric equation; the results obtained by Gauss, Kummer, Riemann, and Schwarz relating to this equation have had the greatest influence on the development of the general theory. The vast extent and importance of the theory of linear differential equations may be estimated when we recall that within its borders it embraces not only almost all the elementary functions, but also the modular and automorphic functions.

Too important to pass over in silence is the subject of algebraic differential equations with uniform solutions. The brilliant researches of Poinlevé deserve especial mention.

Another field of great importance, especially in mathematical physics, relates to the determination of the solution of differential equations with assigned boundary conditions. The literature of this subject is enormous; we may therefore be pardoned if mention is made only of the investigation of our countrymen Bôcher, Van Vleck, and Porter.

Since 1870 the theory of differential equations has been greatly advanced by Lie's theory of groups. Assuming that an equation or a system of equations admits one or more infinitesimal transformations, Lie has shown how they may be employed to simplify the problem of integration. In many cases they give us exact information how to conduct the solution and upon what system of auxiliary equations the solution depends. One of the most striking illustrations of this is the theory of ordinary linear differential equations which Picard and Vessiot have developed, analogous to Galois's theory for algebraic equations. An interesting result of this theory is a criterion for the solution of such equations by quadratures. As an application, we find that Ricatti's equation cannot be solved by quadratures. The attempts to effect such a solution of this celebrated equation in the century before were therefore necessarily in vain.

A characteristic feature of Lie's theories is the prominence given to the geometrical aspects of the questions involved. Lie thinks in geometrical images, the analytical formulation comes afterwards. Already Morge had shown how much might be gained in geometrizing the problem of integration. Lie has gone much farther in this direction. Besides employing all the geometrical notions of his predecessors extended to n-way space, he has introduced a variety of new conceptions, chief of which are his surface element and contact transformations.

He has also used with great effect Plücker's line geometry, and his own sphere geometry in the study of certain types of partial differential equations of the first and second orders which are of great geometrical interest, for example, equations whose characteristic curves are lines of curvature, geodesics, etc. Let us close by remarking that Lie's theories not only afford new and valuable points of view for attacking old problems, but also give rise to a host of new ones of great interest and importance.

Groups

We turn now to the second dominant idea of the century, the group concept.

Groups first became objects of study in algebra when Lagrange (1770), Ruffini (1799), and Abel (1826) employed substitution groups with great advantage in their work on the quintic. The enormous importance of groups in algebra was, however, first made clear by Galois, whose theory of the solution of algebraic equations is one of the great achievements of the century. Its influence has stretched far beyond the narrow bounds of algebra.

With an arbitrary but fixed domain of rationality, Galois observed that every algebraic equation has attached to it a certain group of substitutions. The nature of the auxiliary equations required to solve the given equation is completely revealed by an inspection of this group.

Galois's theory showed the importance of determining the subgroups of a given substitution group, and this problem was studied by Cauchy, Serret, Matthieu, Kirkmann, and others. The publication of Jordan's great treatise in 1870 is a noteworthy event. It collects and unifies the results of his predecessors and contains an immense amount of new matter.

A new direction was given to the theory of groups by the introduction by Cayley of abstract groups (1854, 1878). The work of Sylow, Hölder and Frobenius, Burnside and Miller, deserve especial notice.

Another line of research relates to the determination of the finite groups in the linear group of any number of variables. These groups are important in the theory of linear differential equations with algebraic solutions, in the study of certain geometrical problems as the points of inflection of a cubic, the twenty-seven lines on a surface of the third order, in crystallography, etc. They also enter prominently into Klein's Formen-problem. An especially important class of finite linear groups are the congruence groups first considered by Galois. Among the laborers in the field of linear groups, we note Jordan, Klein, Moore, Maschke, Dickson, Frobenius, and Wiman.

Up to the present we have considered only groups of finite order. About 1870 entirely new ideas coming from geometry and differential equations give the theory of groups an unexpected development. Foremost in this field are Lie and Klein.

Lie discovers and gradually perfects his theory of continuous transformation groups and shows their relations to many different branches of mathematics. In 1872 Klein publishes his *Erlanger Programme* and in 1877 begins his investigations on elliptic modular functions, in which infinite *discontinuous* groups are of primary importance, as we have already seen. In the now famous *Programme*, Klein asks what is the principle which underlies and unifies the heterogeneous geometrical methods then in vogue, as, for example, the geometry of the ancients, whose figures are rigid and invariable; the modern projective geometry, whose figures are in ceaseless flux passing from one form to another; the geometries of Plücker and Lie, in which the elements of space are no longer points, but line spheres, or other configurations at pleasure, the geometry of birational transformation, the analysis situs, etc., etc. Klein finds this answer: In each geometry we have a system of objects and a group which transforms these objects one into another. We seek the invariants of this group. In each case it is the abstract group and not the concrete objects which is essential. The fundamental rôle of a group in geometrical research is thus made obvious. Its importance is the solution of algebraic equation, in the theory of differential equations in the automorphic functions we have already seen. The immense theory of algebraic invariants developed by Cayley and Sylvester, Aronhold, Clebsch, Gordan, Hermite, Brioschi, and a host of zealous workers in the middle of the century, also finds its place in the far more general invariant theory of Lie's theory of groups. The same is true of the theory of surfaces, so far as it rests on the theory of differential forms. In the theory of numbers, groups have many important applications, for example, in the composition of quadratic forms and the cyclotomic bodies. Finally, let us note the relation between hypercomplex numbers and continuous groups discovered by Poincaré.

In résumé, we may thus say that the group concept, hardly noticeable at the beginning of the century, has at its close become one of the fundamental and most fruitful notions in the whole range of our science.

Infinite Aggregates

Leaving the subject of groups, we consider now briefly another fundamental concept, namely, infinite aggregates. In the most diverse mathematical investigations we are confronted with such aggregates. In geometry the conceptions of curves, surface, region, frontier, etc., when examined carefully, lead us to a rich variety of aggregates. In analysis they also appear, for example, the domain of definition of an analytic function, the points where a function of a real variable ceases to be continuous or to have a differential coefficient, the points where a series of functions ceases to be uniformly convergent, etc.

To say an aggregate (not necessarily a point aggregate) is infinite is often an important step; but often again only the first step. To penetrate farther into the problem may require us to state *how* infinite. This requires us to make distinctions in infinite aggregates, to discover fruitful principles of classification, and to investigate the properties of such classes.

The honor of having done this belongs to George Cantor. The theory of aggregates is for the most part his creation; it has enriched mathematical science with fundamental and far-reaching notions and results.

The theory falls into two parts; a theory of aggregates in general, and a theory of point aggregates. In the theory of point aggregates the notion of limiting points gives rise to important classes of aggregates as discrete, dense, everywhere dense, complete, perfect, connected, etc., which are so important in the function theory.

In the general theory two notions are especially important, namely, the one to one correspondence of the elements of two aggregates, and well-ordered aggregates. The first leads to cardinal numbers and the idea of enumerable aggregates, the second to transfinite or ordinal numbers.

Two striking results of Cantor's theory are these: the algebraic and therefore the rational numbers, although everywhere dense, are enumerable; and secondly, one-way and n-way space have the same cardinal number.

Cantor's theory has already found many applications, especially in the function theory, where it is to-day an indispensable instrument of research.

Functions of Real Variables — The Critical Movement

One of the most conspicuous and distinctive features of mathematical thought in the nineteenth century is its critical spirit. Beginning with the calculus, it soon permeates all analysis, and toward the close of the century it overhauls and recasts the foundation of geometry and aspires to further conquests in mechanics and in the immense domains of mathematical physics.

Ushered in with Lagrange and Gauss just at the close of the eighteenth century, the critical movement receives its first decisive impulse from the teachings of Cauchy, who in particular introduces our modern definition of limit and makes it the foundation of the calculus. We must also mention in this connection Abel, Bolzano, and Dirichlet. Especially Abel adopted the reform ideas of Cauchy with enthusiasm, and made important contributions in infinite series.

The figure, however, which towers above all others in this movement, whose name has become an epithet of rigor, is Weierstrass. Beginning at the very foundations, he creates an arithmetic of real and complex numbers, assuming the theory of positive integers to be given. The necessity of this is manifest when we recall that until then the simplest properties of radicals and logarithms were utterly devoid of a rigorous foundation; so, for example,

$$\sqrt{2}\,\sqrt{5}=\sqrt{10} \qquad\qquad \log 2+\log 5=\log 10$$

Characteristic of the pre-Weierstrassean era is the loose way in which geometrical and other intuitional ideas were employed in the demonstration of analytical theorems. Even Gauss is open to this criticism. The mathematical world received a great shock when Weierstrass showed them an example of a continuous function without a derivative, and Hankel and Cantor, by means of their principle of condensation of singularities, could construct analytic expressions for functions having in any interval however small an infinity of points of oscillation, an infinity of points in which the differential coefficient is altogether indeterminate, or an infinity of points of discontinuity. Another rude surprise was Cantor's discovery of the one to one correspondence between the points of a unit segment and a unit square, followed up by Peano's example of a space-filling curve.

These examples and many others made it very clear that the ideas of a curve, a surface region, motion, etc., instead of being clear and simple, were extremely vague and complex. Until these notions had been cleared up, their admission in the demonstration of an analytical theorem was therefore not to be tolerated. On a purely arithmetical basis, with no appeal to our intuition, Weierstrass develops his stately theory of functions which culminates in the theory of Abelian and multiply periodic functions.

But the notion of rigor is relative and depends on what we are willing to admit either tacitly or explicitly. As we observed, Gauss, whose rigor was the admiration of his contemporaries, freely admitted geometrical notions. This Weierstrass would criticise. On the other hand, Weierstrass has made a grave oversight: he nowhere shows that his definitions relative to the number he introduces do not involve mutual contradictions. If he replied that such contradictions would involve contradictions in the theory of positive integers, one might ask what assurance have we that such contradictions may not actually exist. A flourishing young school of mathematical logic has recently grown up under the influence of Peano. They have investigated with marked success the foundations of analysis and geometry, and in particular have attempted to show the non-contradictoriness of the axioms of our number-system by making them depend on the axioms of logic, which axioms we must admit, to reason at all.

The critical spirit, which in the first half of the century was to be found in the writings of only a few of the foremost mathematicians, has in the last quarter of the century become almost universal, at least in analysis. A searching examination of the foundation of arithmetic and the calculus has brought to light the insufficiency of much of the reasoning formerly considered as conclusive. It became necessary to build up these subjects anew. The theory of irrational numbers invented by Weierstrass has been supplanted by the more flexible theories of Dedekind and Cantor. Stolz has given us a systematic and rigorous treatment of arithmetic. The calculus has been completely overhauled and arithmetized by Thomae, Harnack, Peano, Stolz, Jordan, and Vallée-Poussin.

Leaving the calculus, let us notice briefly the theory of functions of real variables. The line of demarcation between these two subjects is extremely arbitrary. We might properly place in the latter all those finer and deeper questions relating to the number-system; the study of our curve, surface, and other geometrical notions, the peculiarities that functions present with reference to discontinuity, oscillation, differentiation, and integration; as well as a very extensive class of investigations whose object is the greatest possible extension of the processes, concepts, and results of the calculus. Among the many not yet mentioned who have made important contributions to this subject we note: Fourier, Riemann, Stokes, Dini, Tannery, Pringsheim, Arzelà, Osgood, Broden, Ascoli, Borel, Baire, Kopke, Hölder, Volterra, and Lebesgue.

Closely related with the differential calculus is the calculus of variations; in the former the variables are given infinitesimal variations, in the latter the functions. Developed in a purely formal manner by Jacobi, Hamilton, Clebsch, and others in the first part of the century, a new epoch began with Weierstrass, who, having subjected the labors of his predecessors to an annihilating criticism, placed the theory on a new and secure foundation and so opened the path for further research by Schwarz, A. Mayer, Scheffers, v. Escherich, Kneser, Osgood, Bolza, Kobb, Zermelo, and others. At the very close of the century Hilbert has given the theory a fresh impulse by the introduction of new and powerful methods, which enable us in certain cases to neglect the second variation and simplifies the consideration of the first. As application he gives the first direct and yet simple demonstration of Dirichlet's celebrated Principle.

Theory of Numbers — Algebraic Bodies

The theory of numbers as left by Fermat, Euler, and Legendre was for the most part concerned with the solution of Diophantine equations, that is, given an equation $f(x, y, z, \ldots)=0$ whose coefficients are integers, find all rational, and especially all integral solutions. In this problem Lagrange had shown the importance of considering the theory of forms. A new era begins with the appearance of Gauss's *Disquisitiones arithmeticae* in 1801. This great work is remarkable for three things: (1) The notion of divisibility in the form of congruences is shown to be an instrument of wonderful power; (2) the Diophantine problem is thrown in the background and the theory of forms is given a dominant rôle; (3) the introduction of algebraic numbers, namely, the roots of unity.

The theory of forms has been further developed along the lines of the *Disquisitiones* by Dirichlet, Eisenstein, Hermite, H. Smith, and Minkowski.

Another part of the theory of numbers also goes back to Gauss, namely, algebraic numerical bodies. The Law of Reciprocity of Quadratic Residues, one of the gems of the higher arithmetic, was first rigorously proved by Gauss. His attempts to extend this theorem to cubic and biquadratic residues showed that the elegant simplicity which prevailed in quadratic residues was altogether missing in these higher residues, until one passed from the domain of real integers to the domain formed of the third and fourth roots of unity. In these domains, as Gauss remarked, algebraic integers have essentially the same properties as ordinary integers. Further exploration in this new and promising field by Jacobi, Eisenstein, and others soon brought to light the fact that already in the domain formed of the twenty-third roots of unity the laws of divisibility were altogether different from those of ordinary integers; in particular, a number could be expressed as the product of prime factors in more than one way. Further progress in this direction was therefore apparently impossible.

It is Kummer's immortal achievement to make further progress possible by the invention of his ideals. These he applied to Fermat's celebrated Last Theorem and the Law of Reciprocity of Higher Residues.

The next step in this direction was taken by Dedekind and Kronecker, who developed the ideal theory for any algebraic domain. So arose the theory of algebraic numerical bodies, which has come into such prominence in the last decades of the century through the researches of Hensel, Hurwitz, Minkowski, Weber, and, above all, Hilbert.

Kronecker has gone farther, and in his classic *Grundzüge* he has shown that similar ideas and methods enable us to develop a theory of algebraic bodies in any number of variables. The notion of divisibility so important in the preceding theories is generalized by Kronecker still farther in the shape of his system of moduli.

Another noteworthy field of research opened up by Kronecker is the relation between quadratic forms with negative determinant and complex multiplication of elliptic functions. H. Smith, Gierster, Hurwitz, and especially Weber have made important contributions.

A method of great power in certain investigations has been created by Minkowski, which he called the *Geometrie der Zahlen*. Introducing a generalization of the distance function, he is led to the conception of a fundamental body (*Aichkörper*). Minkowski shows that every fundamental body is nowhere concave, and conversely to each such body belongs a distance function. A theorem of great importance is now the following: The minimum value which each distance function has at the lattice points is not greater than a certain number depending on the function chosen.

We wish finally to mention a line of investigation which makes use of the infinitesimal calculus and even the theory of functions. Here belong the brilliant researches of Dirichlet relating to the number of classes of binary forms for a given determinant, the number of primes in a given arithmetic progression; and Riemann's remarkable memoir on the number of primes in a given interval.

In this analytical side of the theory of numbers we notice also the researches of Mertens. Weber, and Hadamard.

Projective Geometry

The tendencies of the eighteenth century were predominantly analytical. Mathematicians were absorbed for the most part in developing the wonderful instrument of the calculus with its countless applications. Geometry made relatively little progress. A new era begins with Monge. His numerous and valuable contributions to analytical descriptive and differential geometry, and especially his brilliant and inspiring lectures at the Ecole Polytechnique (1795, 1809), put fresh life into geometry and prepared it for a new and glorious development in the nineteenth century.

When one passes in review the great achievements which have made the nineteenth century memorable in the annals of our science, certainly projective geometry will occupy a foremost place. Pascal, De la Hire, Monge, and Carnot are forerunners, but Poncelet, a pupil of Monge, is its real creator. The appearance of his *Traité des propriétés projectives des figures*, in 1822, gives modern geometry its birth. In it we find the line at infinity, the introduction of imaginaries, the circular points at infinity, polar reciprocation, a discussion of homology, the systematic use of projection, section, and anharmonic ratio.

While the countrymen of Poncelet, especially Chasles, do not fail to make numerous and valuable contributions to the new geometry, the next great steps in advance are made on German soil. In 1827 Möbius publishes the *Barycentrische Calcul;* Plücker's *Analytisch-geometrische Entwickelungen* appears in 1828–31 and Steiner's *Entwickelung der Abhängigkeit geometrischer Gestalten von einander* in 1832. In the ten years which embrace the publication of these immortal works of Poncelet, Plücker, and Steiner, geometry has made more real progress than in the two thousand years which had elapsed since the time of Appolonius. The ideas which had been slowly taking shape since the time of Descartes suddenly crystallized and almost overwhelmed geometry with an abundance of new ideas and principles.

To Möbius we owe the introduction of homogeneous coördinates, and the far-reaching conception of geometric transformation, including collineation and duality as special cases. To Plücker we owe the use of the abbreviate notation which permits us to study the properties of geometric figures without the intervention of the coördinates, the introduction of line and plane coördinates, and the notion of generalized space elements. Steiner, who has been called the greatest geometer since Appolonius, besides enriching geometry in countless ways, was the first to employ systematically the method of generating geometrical figures by means of projective pencils.

Other noteworthy works belonging to this period are Plücker's *System der analytischen Geometrie* (1835), and Chasles's classic *Aperçu* (1837).

Already at this stage we notice a bifurcation in geometrical methods. Steiner and Chasles become eloquent champions of the synthetic school of geometry, while Plücker, and later Hesse and Cayley, are leaders in the analytical movement. The astonishing fruitfulness and beauty of synthetic methods threatened for a short time to drive the analytic school out of existence. The tendency of the synthetic school was to banish more and more metrical methods. In effecting this the anharmonic ratio became constantly more prominent. To define this fundamental ratio without reference to measurement, and so free projective geometry from the galling bondage of metric relations, was thus a problem of fundamental importance. The glory of this achievement, which has, as we shall see, a far wider significance, belongs to v. Staudt. Another equally important contribution of v. Staudt to synthetic geometry is his theory of imaginaries. Poncelet, Steiner, Chasles operate with imaginary elements as if they were real. Their only justification is recourse to the so-called principles of continuity or to some other equally vague principle. V. Staudt gives this theory a rigorous foundation, defining the imaginary points, lines, and planes by means of involutions without ordinal elements.

The next great advance made is the advent of the theory of algebraic invariants. Since projective geometry is the study of those properties of geometric figures which remain unaltered by projective transformations, and since the theory of invariants is the study of those forms which remain unaltered (except possibly for a numerical factor) by the group of linear substitutions, these two subjects are inseparably related and in many respects only different aspects of the same thing. It is no wonder, then, that geometers speedily applied the new theory of invariants to geometrical problems. Among the pioneers in this direction were Cayley, Salmon, Aronhold, Hesse, and especially Clebsch.

Finally we must mention the introduction of the line as a space element. Forerunners are Grassmann (1844) and Cayley (1859), but Plücker in his memoirs of 1865, and his work *Neue Geometrie des Raumes* (1868–69), was the first to show its great value by studying complexes of the first and second order and calling attention to their application to mechanics and optics.

The most important advance over Plücker has been made by Klein, who takes as coördinates six-line complexes in involution. Klein also observed that line geometry may be regarded as a point geometry on a quadric in five-way space. Other laborers in this field are Clebsch, Reye, Segre, Sturm, and Königs.

Differential Geometry

During the first quarter of the century this important branch of geometry was cultivated chiefly by the French. Monge and his school study with great success the generation of surfaces in various ways, the properties of envelopes, evolutes, lines of curvature, asymptotic lines, skew curves, orthogonal systems, and especially the relation between the surface theory and partial differential equations.

The appearance of Gauss's *Disquisitiones generales circa superficies curvas*, in 1828, marks a new epoch. Its wealth of new ideas has furnished material for countless memoirs, and given geometry a new direction. We find here the parametric representation of a surface, the introduction of curvilinear coördinates, the notion of spherical image, the Gaussian measure of curvature, and a study of geodesics. But by far the most important contributions that Gauss makes in this work is the consideration of a surface as a flexible, inextensible film or membrane, and the importance given quadratic differential forms.

We consider now some of the lines along which differential geometry has advanced. The most important is perhaps the theory of differential quadratic forms with their associate invariants and parameters. We mention here Lamé, Beltrami, Menardi, Codazzi, Christoffel, and Weingarten.

An especially beautiful application of this theory is the immense subject of applicability and deformation of surfaces, in which Minding, Bauer, Beltrami, Weingarten, and Voss have made important contributions.

Intimately related with the theory of applicability of two surfaces is the theory of surfaces of constant curvature which play so important a part in non-Euclidean geometry. We mention here the work of Minding, Beltrami, Dini, Backlund, and Lie.

The theory of rectilinear congruences has also been the subject of important researches from the standpoint of differential geometry. First studied by Monge as a system of normals to a surface and then in connection with optics by Malus, Dupin, and Hamilton, the general theory has since been developed by Kummer, Ribaucour, Guichard, Darboux, Voss, and Weingarten. An important application of this theory is the infinitesimal deformation of a surface.

Minimum surfaces have been studied by Monge, Bonnet, and Enneper. The subject owes its present extensive development principally to Weierstrass, Riemann, Schwarz, and Lie. In it we find harmoniously united the theory of surfaces, the theory of functions, the calculus of variations, the theory of groups, and mathematical physics.

Another extensive division of differential geometry is the theory of orthogonal systems, of such importance in physics. We note especially the investigations of Dupin, Jacobi, Darboux, Combescure, and Bianchi.

Other Branches of Geometry

Under this head we group a number of subjects too important to pass over in silence, yet which cannot be considered at length for lack of time.

In the first place is the immense subject of algebraic curves and surfaces. To develop adequately all the important and elegant properties of curves and surfaces of the second order alone would require a bulky volume. In this line of ideas would follow curves and surfaces of higher order and class. Their theory is far less complete, but this lack it amply makes good by offering an almost bewildering variety of configurations to classify and explore. No single geometer has contributed more to this subject than Cayley.

A theory of great importance is the geometry on a curve or surface inaugurated by Clebsch in 1863.

Expressing the coördinates of a plane cubic by means of elliptic functions and employing their addition theorems, he deduced with hardly any calculation Steiner's theorem relating to the inscribed polygons and various theorems concerning conics touching the curve. Encouraged by such successes, Clebsch proposed to make use of Riemann's theory of Abelian functions in the study of algebraic curves of any order. The most important result was a new classification of such curves. Instead of the linear transformation, Clebsch in harmony with Riemann's ideas employs the birational transformation as a principle of classification. From this standpoint we ask what are the properties of algebraic curves which remain invariant for such transformation.

Brill and Nöther follow Clebsch. Their method is, however, algebraical, and rests on their celebrated Residual theorem which in their hands takes the place of Abel's theorem. We mention further the investigation of Castelnuovo, Weber, Krause, and Segre. An important division of this subject is the theory of correspondences. First studied by Chasles for curves of deficiency 0 in 1864, Cayley, and, immediately after, Brill extended the theory to the case of any p. The most important advance made in later years has been made by Hurwitz, who considers the totality of possible correspondences on an algebraic curve, making use of the corresponding integrals of the first species.

Alongside the geometry on a curve is the vastly more difficult and complicated geometry on a surface, or more generally, on any algebraic spread in n-way space. Starting from a remark of Clebsch (1868), Nöther made the first great step in his famous memoir of 1868–74. Further progress has been due to the French and Italian mathematicians. Picard, Poincaré, and Humbert make use of transcendental methods, in which figure prominently double integrals which remain finite on the surface and single integrals of total differentials. On the other hand, Enriques and Castelnuovo have attacked the subject from a more algebraic-geometric standpoint by means of linear systems of algebraic curves on the surface.

The first invariants of a surface were discovered by Clebsch and Nöther; still others have been found by Castelnuovo and Enriques in connection with irregular surfaces.

Leaving this subject, let us consider briefly the geometry of n dimensions. A characteristic of nineteenth-century mathematics is the generality of its methods and results. When such has been impossible with the elements in hand, fresh ones have been invented; witness the introduction of imaginary numbers in algebra and the function theory, the ideals of Kummer in the theory of numbers, the line and plane at infinity in projective geometry. The benefit that analysis derived from geometry was too great not to tempt mathematicians to free the latter from the narrow limits of three dimensions, and so give it the generality that the former has long enjoyed. The first pioneer in this abstract field was Grassmann (1844); we must, however, consider Cayley as the real founder of n-dimensional geometry (1869). Notable contributions have been made by the Italian school, Veronese, Segre, etc.

Non-Euclidean Geometry

Each century takes over as a heritage from its predecessor a number of problems whose solution previous generations of mathematicians have arduously but vainly sought. It is a signal achievement of the nineteenth century to have triumphed over some of the most celebrated of these problems.

The most ancient of them is the Quadrature of the Circle, which already appears in our oldest mathematical document, the *Papyrus Rhind*, B.C. 2000. Its impossibility was finally shown by Lindemann (1882).

Another famous problem relates to the solution of the quintic, which had engaged the attention of mathematicians since the middle of the sixteenth century. The impossibility of expressing its roots by radicals was finally shown by the youthful Abel (1824), while Hermite and Kroneker (1858) showed how they might be expressed by the elliptic modular functions, and Klein (1875) by means of the icosahedral irrationality.

But of all problems which have come down from the past, by far the most celebrated and important relates to Euclid's parallel axiom. Its solution has profoundly affected our views of space, and given rise to questions even deeper and more far-reaching which embrace the entire foundation of geometry and our space conception. Let us pass in rapid review the principal events of this great movement. Wallis in the seventeenth, Seccheri, Lambert, and Legendre in the eighteenth, are the first to make any noteworthy progress before the nineteenth century. The really profound investigations of Seccheri and Lambert, strangely enough, were entirely overlooked by later writers and have only recently come to light.

In the nineteenth century non-Euclidean geometry develops along four directions, which roughly follow each other chronologically. Let us consider them in order.

The naïve-synthetic direction.—The methods employed are similar to those of Euclid. His axioms are assumed with the exception of the parallel axiom; the resulting geometry is what is now called hyperbolic or Lobatschewski's geometry. Its principal properties are deduced, in particular its trigonometry, which is shown to be that of a sphere with imaginary radius as Lambert had divined. As a specific result of these investigations the long-debated question relating to the independence of the parallel axiom was finally settled. The great names in this group are Lobatschewski, Bolyai, and Gauss. The first publications of Lobatschewski are his *Exposition succinct des principes de la géométrie* (1829), and the *Geometrische Untersuchungen*, in 1840. Bolyai's *Appendix* was published in 1832. As to the extent of Gauss's investigations, we can only judge from scattered remarks in private letters and his reviews of books relating to the parallel axioms. His dread of the *Geschrei der Boötier*, that is, the followers of Kant, prevented him from publishing his extensive speculations.

The metric-differential direction.—This is inaugurated by three great memoirs by Riemann, Helmholtz, and Beltrami, all published in the same year, 1868.

Beltrami, making use of results of Gauss and Minding relating to the applicability of two surfaces, shows that the hyperbolic geometry of a plane may be interpreted on a surface of constant negative curvature, the pseudosphere. By means of this discovery the purely logical and hypothetical system of Lobatschewski and Bolyai takes on a form as concrete and tangible as the geometry of a plane.

The work of Riemann is as original as profound. He considers space as an n-dimensional continuous numerical multiplicity, which is distinguished from the infinity of other such multiplicities by certain well-defined characters. Chief of them are (1) the quadratic differential expression which defines the length of an elementary arc, and (2) a property relative to the displacements of this multiplicity about a point. There are an infinity of space multiplicities which satisfy Riemann's axioms. By extending Gauss's definition of a curvature k, of a surface at a point to curvature of *space* at a point, by considering the geodesic surfaces passing through that point, Riemann finds that all these spaces fall into three classes according as k is equal to, greater, or less than 0. For $n=3$ and $k=0$ we have Euclidean space; when $k<0$ we have the space found by Gauss, Lobatschewski, and Bolyai; when $k>0$ we have the space first considered in the long-forgotten writings of Seccheri and Lambert, in which the right line is finite.

Helmholtz, like Riemann, considers space as a numerical multiplicity. To characterize it further, Helmholtz makes use of the notions of rigid bodies and free mobility. His work has been revised and materially extended by Lie from the standpoint of the theory of groups.

In the present category also belong important papers by Newcomb and Killing.

The projective direction. — We have already noticed the efforts of the synthetic school to express metric properties by means of projective relations. In this the circular points at infinity were especially serviceable. An immense step in this direction was taken by Laguerre, who showed, in 1853, that all angles might be expressed as an anharmonic ratio with reference to these points, that is, with reference to a certain fixed conic. The next advance is made by Cayley in his famous sixth memoir on quantics, in 1859. Taking any fixed conic (or quadric, for space) which he calls the absolute, Cayley introduces two expressions depending on the anharmonic ratio with reference to the absolute. When this degenerates into the circular points at infinity, these expressions go over into the ordinary expressions for the distance between two points and the angle between two lines. Thus all metric relations may be considered as projective relations with respect to the absolute. Cayley does not seem to be aware of the relation of his work to non-Euclidean geometry. This was discovered by Klein, in 1871. In fact, according to the nature of the absolute, three geometries are possible; these are precisely the three already mentioned. Klein has made many important contributions to non-Euclidean geometry. We mention his modification of v. Staudt's definition of anharmonic ratio so as to be independent of the parallel axiom, his discovery of the two forms of Riemann's space, and finally his contributions to a class of geometries first noticed by Clifford and which are characterized by the fact that only certain of its motions affect space as a whole.

As a result of all these investigations, both in the projective as also in the metric differential direction, we are led irresistibly to the same conclusion, namely: The facts of experience can be explained by all three geometries when the constant k is taken small enough. It is, therefore, merely a question of convenience whether we adopt the parabolic, hyperbolic, or elliptic geometry.

The critical synthetic direction represents a return to the old synthetic methods of Euclid, Lobatschewski, and Bolyai, with the added feature of a refined and exacting logic. Its principal object is no longer a study of non-Euclidean but of Euclidean geometry. Its aim is to establish a system of axioms for our ordinary space which is complete, compatible, and irreducible. The fundamental terms point, line, plane, between, congruent, etc., are introduced as abstract marks whose properties are determined by inter-relations in the form of axioms. Geometric intuition has no place in this order of ideas which regards geometry as a mere division of pure logic. The efforts of this school have already been crowned with eminent success, and much may be expected from it in the future. Its leaders are Peano, Veronese, Pieri, Padoa, Burali-Forti, and Levi-Civittà, in Italy, Pasch and Hilbert in Germany, and Moore in America.

Closing at this point our hasty and imperfect survey of mathematics in the last century, let us endeavor to sum up its main characteristics. What strikes us at once is its colossal proportions and rapid growth in nearly all directions, the great variety of its branches, the generality and complexity of its methods; an inexhaustible creative imagination, the fearless introduction and employment of ideal elements, and an appreciation for a refined and logical development of all its parts.

We who stand on the threshold of a new century can look back on an era of unparalleled progress. Looking into the future, an equally bright prospect greets our eyes; on all sides fruitful fields of research invite our labor and promise easy and rich returns.

Surely this is the golden age of mathematics.

SECTION A—ALGEBRA AND ANALYSIS

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(*Hall* 9, *September* 22, 10 *a. m.*)

CHAIRMAN: PROFESSOR E. H. MOORE, University of Chicago.
SPEAKERS: PROFESSOR CHARLES EMILE PICARD, The Sorbonne; Member of the Institute of France.
PROFESSOR HEINRICH MASCHKE, University of Chicago.
SECRETARY: PROFESSOR A. G. BLISS, University of Chicago.

ON THE DEVELOPMENT OF MATHEMATICAL ANALYSIS AND ITS RELATIONS TO SOME OTHER SCIENCES

BY CHARLES EMILE PICARD

(*Translated from the French by Professor George Bruce Halsted, Kenyon College*)

[**Charles Emile Picard**, Professor of Higher Algebra and Higher Analysis, University of Paris; also Professor of General Mechanics, l'Ecole Centrale des Arts et Manufactures, Paris. **b.** Paris, France, July 24, 1856. LL.D. Clark University, Glasgow University, University of Christiania. Member of Institute of France; Academy of Science, Berlin, St. Petersburg, Bologna, Boston, Turin, Copenhagen, Washington, and many others; Mathematical Society of London. Former President of Mathematical Society of France, Mathematical Societies of London and Kharkow, and many other mathematical societies. **Author and editor of** *Memoirs, Traits and Discussions of Mathematics; Theory of Algebraic Functions of Two Variables.*]

IT is one of the objects of a congress such as this which now brings us together, to show the bonds between the diverse parts of science taken in its most extended acceptation. So the organizers of this meeting have insisted that the relations between different sections should be put in evidence.

To undertake a study of this sort, somewhat indeterminate in character, it is necessary to forget that all is in all; in what concerns algebra and analysis, a Pythagorean would be dismayed at the extent of his task, remembering the celebrated formula of the school: "Things are numbers." From this point of view my subject would be inexhaustible.

But I, for the best of reasons, will make no such pretensions.

In casting merely a glance over the development of our science through the ages, and particularly in the last century, I hope to be able to characterize sufficiently the rôle of mathematical analysis in its relations to certain other sciences.

I

It would appear natural to commence by speaking of the concept itself of whole number; but this subject is not alone of logical order, it is also of order historic and psychologic, and would draw us away into too many discussions.

Since the concept of number has been sifted, in it have been found unfathomable depths; thus, it is a question still pending to know, between the two forms, the cardinal number and the ordinal number, under which the idea of number presents itself, which of the two is anterior to the other, that is to say, whether the idea of number properly so called is anterior to that of order, or if it is the inverse.

It seems that the geometer-logician neglects too much in these questions psychology and the lessons uncivilized races give us; it would seem to result from these studies that the priority is with the cardinal number.

It may also be there is no general response to the question, the response varying according to races and according to mentalities.

I have sometimes thought, on this subject, of the distinction between auditives and visuals, auditives favoring the ordinal theory, visuals the cardinal.

But I will not linger on this ground full of snares; I fear that our modern school of logicians with difficulty comes to agreement with the ethnologists and biologists; these latter in questions of origin are always dominated by the evolution idea, and, for more than one of them, logic is only the résumé of ancestral experience. Mathematicians are even reproached with postulating in principle that there is a human mind in some way exterior to things, and that it has its logic. We must, however, submit to this, on pain of constructing nothing. We need this point of departure, and certainly, supposing it to have evolved during the course of prehistoric time, this logic of the human mind was perfectly fixed at the time of the oldest geometric schools, those of Greece; their works appear to have been its first code, as is expressed by the story of Plato writing over the door of his school, "Let no one not a geometer enter here."

Long before the bizarre word algebra was derived from the Arabic, expressing, it would seem, the operation by which equalities are reduced to a certain canonic form, the Greeks had made algebra without knowing it; relations more intimate could not be imagined than those binding together their algebra and their geometry, or rather, one would be embarrassed to classify, if there were occasion, their geometric algebra, in which they reason not on numbers but on magnitudes.

Among the Greeks also we find a geometric arithmetic, and one of the most interesting phases of its development is the conflict which, among the Pythagoreans, arose in this subject between number and magnitude, apropos of irrationals.

Though the Greeks cultivated the abstract study of numbers, called by them arithmetic, their speculative spirit showed little taste for practical calculation, which they called logistic.

In remote antiquity, the Egyptians and the Chaldeans, and later the Hindus and the Arabs, carried far the science of calculation.

They were led on by practical needs; logistic preceded arithmetic, as land-surveying and geodesy opened the way to geometry; in the same way trigonometry developed under the influence of the increasing needs of astronomy.

The history of science at its beginnings shows a close relation between pure and applied mathematics; this we shall meet again constantly in the course of this study.

We have remained up to this point in the domain which ordinary language calls elementary algebra and arithmetic.

In fact, from the time that the incommensurability of certain magnitudes had been recognized, the infinite had made its appearance, and, from the time of the sophisms of Zeno on the impossibility of motion, the summation of geometric progressions must have been considered.

The procedures of exhaustion which are found in Eudoxus and in Euclid appertain already to the integral calculus, and Archimedes calculates definite integrals.

Mechanics also appeared in his treatise on the quadrature of the parabola, since he first finds the surface of the segment bounded by an arc of a parabola and its chord with the help of the theorem of moments; this is the first example of the relations between mechanics and analysis, which since have not ceased developing.,

The infinitesimal method of the Greek geometers for the measure of volumes raised questions whose interest is even to-day not exhausted.

In plane geometry, two polygons of the same area are either equivalent or equivalent-by-completion, that is to say, can be decomposed into a finite number of triangles congruent in pairs, or may be regarded as differences of polygons susceptible of such a partition.

It is not the same for the geometry of space, and we have lately learned that stereometry cannot, like planimetry, get on without recourse to procedures of exhaustion or of limit, which require the axiom of continuity or the Archimedes assumption.

Without insisting further, this hasty glance at antiquity shows how completely then were amalgamated algebra, arithmetic, geometry, and the first endeavors at integral calculus and mechanics, to the point of its being impossible to recall separately their history.

In the Middle Ages and the Renaissance, the geometric algebra of the ancients separated from geometry. Little by little algebra properly so called arrived at independence, with its symbolism and its notation more and more perfected; thus was created this language so admirably clear, which brings about for thought a veritable economy and renders further progress possible.

This is also the moment when distinct divisions are organized.

Trigonometry, which, in antiquity, had been only an auxiliary of astronomy, is developed independently; toward the same time the logarithm appears, and essential elements are thus put in evidence.

II

In the seventeenth century, the analytic geometry of Descartes, distinct from what I have just called the geometric algebra of the Greeks by the general and systematic ideas which are at its base, and the new-born dynamic were the origin of the greatest progress of analysis.

When Galileo, starting from the hypothesis that the velocity of heavy bodies in their fall is proportional to the time, from this deduced the law of the distances passed over, to verify it afterward by experiment, he took up again the road upon which Archimedes had formerly entered and on which would follow after him Cavalieri, Fermat, and others still, even to Newton and Leibnitz. The integral calculus of the Greek geometers was born again in the kinematic of the great Florentine physicist.

As to the calculus of derivatives or of differentials, it was founded with precision apropos of the drawing of tangents.

In reality, the origin of the notion of derivative is in the confused sense of the mobility of things and of the rapidity more or less great with which phenomena happen; this is well expressed by the words *fluents* and *fluxions*, which Newton used, and which one might suppose borrowed from old Heraclitus.

The points of view taken by the founders of the science of motion, Galileo, Huygens, and Newton, had an enormous influence on the orientation of mathematical analysis.

It was with Galileo an intuition of genius to discover that, in natural phenomena, the determining circumstances of the motion produce accelerations: this must have conducted to the statement of the principle that the rapidity with which the dynamic state of a system changes depends in a determinate manner on its static state alone. In a more general way we reach the postulate that the infinitesimal changes, of whatever nature they may be, occurring in a system of bodies, depend uniquely on the actual state of this system.

In what degree are the exceptions apparent or real? This is a question which was raised only later and which I put aside for the moment.

From the principles enunciated becomes clear a point of capital importance for the analyst: Phenomena are ruled by differential equations which can be formed when observation and experiment have made known for each category of phenomena certain physical laws.

We understand the unlimited hopes conceived from these results. As Bertrand says in the preface of his treatise, "The early successes were at first such that one might suppose all the difficulties of science surmounted in advance, and believe that the geometers, without being longer distracted by the elaboration of pure mathematics, could turn their meditations exclusively toward the study of the natural laws."

This was to admit gratuitously that the problems of analysis, to which one was led, would not present very grave difficulties.

Despite the disillusions the future was to bring, this capital point remained, that the problems had taken a precise form, and that a classification could be established in the difficulties to be surmounted.

There was, therefore, an immense advance, one of the greatest ever made by the human mind. We understand also why the theory of differential equations acquired a considerable importance.

I have anticipated somewhat, in presenting things under a form so analytic. Geometry was intermingled in all this progress. Huygens, for example, followed always by preference the ancients, and his *Horologium oscillatorium* rests at the same time on infinitesimal geometry and mechanics; in the same way, in the *Principia* of Newton, the methods followed are synthetic.

It is, above all, with Leibnitz that science takes the paths which were to lead to what we call mathematical analysis; it is he who, for the first time, in the latter years of the seventeenth century, pronounces the word function.

By his systematic spirit, by the numerous problems he treated, even as his disciples James and John Bernoulli, he established in a final way the power of the doctrines to the edification of which had successively contributed a long series of thinkers from the distant times of Eudoxus and of Archimedes.

The eighteenth century showed the extreme fecundity of the new methods. That was a strange time, the era of mathematical duels where geometers hurled defiance, combats not always without acrimony, when Leibnitzians and Newtonians encountered in the lists.

From the purely analytic point of view, the classification and study of simple functions is particularly interesting; the function idea, on which analysis rests, is thus developed little by little.

The celebrated works of Euler hold then a considerable place. However, the numerous problems which present themselves to the mathematicians leave no time for a scrutiny of principles; the foundations themselves of the doctrine are elucidated slowly, and the *mot* attributed to d'Alembert, "Allez en avant et la foi vous viendra," is very characteristic of this epoch.

Of all the problems started at the end of the seventeenth century or during the first half of the eighteenth, it will suffice for me to recall those isoperimetric problems which gave birth to the calculus of variations.

I prefer to insist on the interpenetration still more intimate between analysis and mechanics when, after the inductive period of the first age of dynamics, the deductive period was reached where one strove to give a final form to the principles. The mathematical and formal development played then the essential rôle, and the analytic language was indispensable to the greatest extension of these principles.

There are moments in the history of the sciences and, perhaps, of society, when the spirit is sustained and carried forward by the words and the symbols it has created, and when generalizations present themselves with the least effort. Such was particularly the rôle of analysis in the formal development of mechanics.

Allow me a remark just here. It is often said an equation contains only what one has put into it. It is easy to answer, first, that the new form under which one finds the things constitutes often of itself an important discovery.

But sometimes there is more; analysis, by the simple play of its symbols, may suggest generalizations far surpassing the primitive outline. Is it not so with the principle of virtual velocities, of which the first idea comes from the simplest mechanisms; the analytic form which translates it will suggest extensions leading far from the point of departure.

In the same sense, it is not just to say analysis has created nothing, since these more general conceptions are its work. Still another example is furnished us by Lagrange's system of equations; here calculus transformations have given the type of differential equations to which one tends to carry back to-day the notion of mechanical explanation.

There are in science few examples comparable to this, of the importance of the form of an analytic relation and of the power of generalization of which it may be capable.

It is very clear that, in each case, the generalizations suggested should be made precise by an appeal to observation and experiment, then it is still the calculus which searches out distant consequences for checks, but this is an order of ideas which I need not broach here.

Under the impulse of the problems set by geometry, mechanics, and physics, we see develop or take birth almost all the great divisions of analysis. First were met equations with a single independent variable. Soon appear partial differential equations, with vibrating cords, the mechanics of fluids and the infinitesimal geometry of surfaces.

This was a wholly new analytic world; the origin itself of the problems treated was an aid which from the first steps permits no wandering, and in the hands of Monge geometry rendered useful services to the new-born theories.

But of all the applications of analysis, none had then more renown than the problems of celestial mechanics set by the knowledge of the law of gravitation and to which the greatest geometers gave their names.

Theory never had a more beautiful triumph; perhaps one might add that it was too complete, because it was at this moment above all that were conceived for natural philosophy the hopes at least premature of which I spoke above.

In all this period, especially in the second half of the eighteenth century, what strikes us with admiration and is also somewhat confusing, is the extreme importance of the applications realized, while the pure theory appeared still so ill assured. One perceives it when certain questions are raised like the degree of arbitrariness in the integral of vibrating cords, which gives place to an interminable and inconclusive discussion.

Lagrange appreciated these insufficiencies when he published his theory of analytic functions, where he strove to give a precise foundation to analysis.

One cannot too much admire the marvelous presentiment he had of the rôle which the functions, which with him we call analytic, were to play; but we may confess that we stand astonished before the demonstration he believed to have given of the possibility of the development of a function in Taylor's series.

The exigencies in questions of pure analysis were less at this epoch. Confiding in intuition, one was content with certain probabilities, and agreed implicitly about certain hypotheses that it seemed useless to formulate in an explicit way; in reality, one had confidence in the ideas which so many times had shown themselves fecund, which is very nearly the *mot* of d'Alembert.

The demand for rigor in mathematics has had its successive approximations, and in this regard our sciences have not the absolute character so many people attribute to them.

III

We have now reached the first years of the nineteenth century. As we have explained, the great majority of the analytic researches had, in the eighteenth century, for occasion a problem of geometry, and especially of mechanics and of physics, and we have scarcely found the logical and æsthetic preoccupations which are to give a physiognomy so different to so many mathematical works, above all in the latter two thirds of the nineteenth century.

Not to anticipate, however, after so many examples of the influences of physics on the developments of analysis, we meet still a new one, and one of the most memorable, in Fourier's theory of heat. He commences by forming the partial differential equations which govern temperature.

What are for a partial differential equation the conditions at the limits permitting the determination of a solution?

For Fourier, the conditions are suggested by the physical problem, and the methods that he followed have served as models to the physicist-geometers of the first half of the last century.

One of these consists in forming a series with certain simple solutions. Fourier thus obtained the first types of developments more general than the trigonometric developments, as in the problem of the cooling of a sphere, where he applies his theory to the terrestrial globe, and investigates the law which governs the variations of temperature in the ground, trying to go even as far as numerical applications.

In the face of so many beautiful results, we understand the enthusiasm of Fourier which scintillates from every line of his preliminary discourse. Speaking of mathematical analysis, he says, "There could not be a language more universal, more simple, more exempt from errors and from obscurities, that is to say, more worthy to express the invariable relations of natural things. Considered under this point of view, it is as extended as nature herself; it defines all sensible relations, measures times, spaces, forces, temperatures. This difficult science forms slowly, but it retains all the principles once acquired. It grows and strengthens without cease in the midst of so many errors of the human mind."

The eulogy is magnificent, but permeating it we see the tendency which makes all analysis uniquely an auxiliary, however incomparable, of the natural sciences, a tendency, in conformity, as we have seen, with the development of science during the preceding two centuries; but we reach just here an epoch where new tendencies appear.

Poisson having in a report on the Fundamenta recalled the reproach made by Fourier to Abel and Jacobi of not having occupied themselves preferably with the movement of heat, Jacobi wrote to Legendre: "It is true that Monsieur Fourier held the view that the principal aim of mathematics was public utility, and the explanation of natural phenomena; but a philosopher such as he should have known that the unique aim of science is the honor of the human spirit, and that from this point of view a question about numbers is as important as a question about the system of the world." This was without doubt also the opinion of the grand geometer of Goettingen, who called mathematics the queen of the sciences, and arithmetic the queen of mathematics.

It would be ridiculous to oppose one to the other these two tendencies; the harmony of our science is in their synthesis.

The time was about to arrive when one would feel the need of inspecting the foundations of the edifice, and of making the inventory of accumulated wealth, using more of the critical spirit. Mathematical thought was about to gather more force by retiring into itself; the problems were exhausted for a time, and it is not well for all seekers to stay on the same road. Moreover, difficulties and paradoxes remaining unexplained made necessary the progress of pure theory. The path on which this should move was traced in its large outlines, and there it could move with independence without necessarily losing contact with the problems set by geometry, mechanics, and physics. At the same time more interest was to attach to the philosophic and artistic side of mathematics, confiding in a sort of preëstablished harmony between our logical and æsthetic satisfactions and the necessities of future applications.

Let us recall rapidly certain points in the history of the revision of principles where Gauss, Cauchy, and Abel likewise were laborers of the first hour. Precise definitions of continuous functions, and their most immediate properties, simple rules on the convergence of series, were formulated; and soon was established, under very general conditions, the possibility of trigonometric developments, legitimatizing thus the boldness of Fourier.

Certain geometric intuitions relative to areas and to arcs give place to rigorous demonstration. The geometers of the eighteenth century had necessarily sought to give account of the degree of the generality of the solution of ordinary differential equations. Their likeness to equations of finite differences led easily to the result; but the demonstration so conducted must not be pressed very close.

Lagrange, in his lessons on the calculus of functions, had introduced greater precision, and starting from Taylor's series, he saw that the equation of order m leaves indeterminate the function, and its $m - 1$ first derivatives for the initial value of the variable; we are not surprised that Lagrange did not set himself the question of convergence.

In twenty or thirty years the exigencies in the rigor of proofs had grown. One knew that the two preceding modes of demonstration are susceptible of all the precision necessary.

For the first, there was need of no new principle; for the second it was necessary that the theory should develop in a new way. Up to this point, the functions and the variables had remained real. The consideration of complex variables comes to extend the field of analysis. The functions of a complex variable with unique derivative are necessarily developable in Taylor's series; we come back thus to the mode of development of which the author of the theory of analytic functions had understood the interest, but of which the importance could not be put fully in evidence in limiting one's self to real variables. They also owe the grand rôle that they have not ceased to play to the facility with which we can manage them, and to their convenience in calculation.

The general theorems of the theory of analytic functions permitted to reply with precision to questions remaining up to that time undecided, such as the degree of generality of the integrals of differential equations. It became possible to push even to the end the demonstration sketched by Lagrange for an ordinary differential equation. For a partial differential equation or a system of such equations, precise theorems were established. It is not that on this latter point the results obtained, however important they may be, resolve completely the diverse questions that may be set; because in mathematical physics, and often in geometry, the conditions at the limits are susceptible of forms so varied that the problem called Cauchy's appears often under very severe form. I will shortly return to this capital point.

IV

Without restricting ourselves to the historic order, we will follow the development of mathematical physics during the last century, in so far as it interests analysis.

The problems of calorific equilibrium lead to the equation already encountered by Laplace in the study of attraction. Few equations have been the object of so many works as this celebrated equation. The conditions at the limits may be of divers forms. The simplest case is that of the calorific equilibrium of a body of which we maintain the elements of the surface at given temperatures; from the physical point of view, it may be regarded as evident that the temperature, continuous within the interior since no source of heat is there, is determined when it is given at the surface.

A more general case is that where, the state remaining permanent, there might be radiation toward the outside with an emissive power varying on the surface in accordance with a given law; in particular the temperature may be given on one portion, while there is radiation on another portion.

These questions, which are not yet resolved in their greatest generality, have greatly contributed to the orientation of the theory of partial differential equations. They have called attention to types of determinations of integrals, which would not have presented themselves in remaining at a point of view purely abstract.

Laplace's equation had been met already in hydrodynamics and in the study of attraction inversely as the square of the distance. This latter theory has led to putting in evidence the most essential elements, such as the potentials of simple strata and of double strata. Analytic combinations of the highest importance were there met, which since have been notably generalized, such as Green's formula.

The fundamental problems of static electricity belong to the same order of ideas, and that was surely a beautiful triumph for theory, the discovery of the celebrated theorem on electric phenomena in the interior of hollow conductors, which later Faraday rediscovered experimentally, without having known of Green's memoir.

All this magnificent *ensemble* has remained the type of the theories already old of mathematical physics, which seem to us almost to have attained perfection, and which exercise still so happy an influence on the progress of pure analysis in suggesting to it the most beautiful problems. The theory of functions offers us another memorable affiliation.

There the analytic transformations which come into play are not distinct from those we have met in the permanent movement of heat. Certain fundamental problems of the theory of functions of a complex variable lost then their abstract enunciation to take a physical form, such as that of the distribution of temperature on a closed surface of any connection and not radiating, in calorific equilibrium with two sources of heat which necessarily correspond to flows equal and of contrary signs. Transposing, we face a question relative to Abelian integrals of the third species in the theory of algebraic curves.

The examples which precede, where we have envisaged only the equations of heat and of attraction, show that the influence of physical theories has been exercised not only on the general nature of the problems to be solved, but even in the details of the analytic transformations. Thus is currently designated in recent memoirs on partial differential equations under the name of Green's formula, a formula inspired by the primitive formula of the English physicist. The theory of dynamic electricity and that of magnetism, with Ampère and Gauss, have been the origin of important progress; the study of curvilinear integrals and that of the integrals of surfaces have taken thence all their developments, and formulas, such as that of Stokes which might also be called Ampère's formula, have appeared for the first time in memoirs on physics. The equations of the propagation of electricity, to which are attached the names of Ohm and Kirchoff, while presenting a great analogy with those of heat, offer often conditions at the limits a little different; we know all that telegraphy by cables owes to the profound discussion of a Fourier's equation carried over into electricity.

The equations long ago written of hydrodynamics, the equations of the theory of electricity, those of Maxwell and of Hertz in electromagnetism, have offered problems analogous to those recalled above, but under conditions still more varied. Many unsurmounted difficulties are there met with; but how many beautiful results we owe to the study of particular cases, whose number one would wish to see increase. To be noted also as interesting at once to analysis and physics are the profound differences which the propagation may present according to the phenomena studied. With equations such as those of sound, we have propagation by waves; with the equation of heat, each variation is felt instantly at every distance, but very little at a very great distance, and we cannot then speak of velocity of propagation.

In other cases of which Kirchoff's equation relative to the propagation of electricity with induction and capacity offers the simplest type, there is a wave front with a velocity determined but with a remainder behind which does not vanish.

These diverse circumstances reveal very different properties of integrals; their study has been delved into only in a few particular cases, and it raises questions into which enter the most profound notions of modern analysis.

V

I will enter into certain analytic details especially interesting for mathematical physics.

The question of the generality of the solution of a partial differential equation has presented some apparent paradoxes. For the same equation, the number of arbitrary functions figuring in the general integral was not always the same, following the form of the integral envisaged. Thus Fourier, studying the equation of heat in an indefinite medium, considers as evident that a solution will be determined if its value for $t=0$ is given, that is to say one arbitrary function of the three coördinates x, y, z; from the point of view of Cauchy, we may consider, on the contrary, that in the general solution there are two arbitrary functions of the three variables. In reality, the question, as it has long been stated, has not a precise signification.

In the first place, when it is a question only of analytic functions, any finite number of functions of any number of independent variables presents, from the arithmetical point of view, no greater generality than a single function of a single variable, since in the one case and in the other the *ensemble* of coefficients of the development forms an enumerable series. But there is something more. In reality, beyond the conditions which are translated by given functions, an integral is subjected to conditions of continuity, or is to become infinite in a determined manner for certain elements; one may so be led to regard as equivalent to an arbitrary function the condition of continuity in a given space, and then we clearly see how badly formulated is the question of giving the number of the arbitrary functions. It is at times a delicate matter to demonstrate that conditions determine in a unique manner a solution, when we do not wish to be contented with probabilities; it is then necessary to make precise the manner in which the function and certain of its derivatives conduct themselves.

Thus in Fourier's problem relative to an indefinite medium certain hypotheses must be made about the function and its first derivatives at infinity, if we wish to establish that the solution is unique.

Formulas analogous to Green's render great services, but the demonstrations one deduces from them are not always entirely rigorous, implicitly supposing fulfilled for the limits conditions which are not, *a priori* at least, necessary. This is, after so many others, a new example of the evolution of exigencies in the rigor of proofs.

We remark, moreover, that the new study, rendered necessary, has often led to a better account of the nature of integrals.

True rigor is fecund, thus distinguishing itself from another purely formal and tedious, which spreads a shadow over the problems it touches.

The difficulties in the demonstration of the unity of a solution may be very different according as it is question of equations of which all the integrals are or are not analytic. This is an important point, and shows that even when we wish to put them aside, it is necessary sometimes to consider non-analytic functions.

Thus we cannot affirm that Cauchy's problem determines in a unique manner one solution, the data of the problem being general, that is to say not being characteristic.

This is surely the case, if we envisage only analytic integrals, but with non-analytic integrals there may be contacts of order infinite. And theory here does not outstrip applications; on the contrary, as the following example shows:

Does the celebrated theorem of Lagrange on the potentials of velocity in a perfect fluid hold good in a viscid fluid? Examples have been given where the coördinates of different points of a viscous fluid starting from rest are not expressible as analytic functions of the time starting from the initial instant of the motion, and where the nul rotations as well as all their derivatives with respect to the time at this instant are, however, not identically nul; Lagrange's theorem, therefore, does not hold true.

These considerations sufficiently show the interest it may have to be assured that all the integrals of a system of partial differential equations continuous as well as all their derivatives up to a determined order in a certain field of real variables are analytic functions; it is understood, we suppose, there are in the equations only analytic elements. We have for linear equations precise theorems, all the integrals being analytic, if the characteristics are imaginary, and very general propositions have also been obtained in other cases.

The conditions at the limits that one is led to assume are very different according as it is question of an equation of which the integrals are or are not analytic. A type of the first case is given by the problem generalized by Dirichlet; conditions of continuity there play an essential part, and, in general, the solution cannot be prolonged from the two sides of the continuum which serves as support to the data; it is no longer the same in the second case, where the disposition of this support in relation to the characteristics plays the principal rôle, and where the field of existence of the solution presents itself under wholly different conditions.

All these notions, difficult to make precise in ordinary language and fundamental for mathematical physics, are not of less interest for infinitesimal geometry.

It will suffice to recall that all the surfaces of constant positive curvature are analytic, while there exist surfaces of constant negative curvature not analytic.

From antiquity has been felt the confused sentiment of a certain economy in natural phenomena; one of the first precise examples is furnished by Fermat's principle relative to the economy of time in the transmission of light.

Then we came to recognize that the general equations of mechanics correspond to a problem of minimum, or more exactly of variation, and thus we obtained the principle of virtual velocities, then Hamilton's principle, and that of least action. A great number of problems appeared then as corresponding to minima of certain definite integrals.

This was a very important advance, because the existence of a minimum could in many cases be regarded as evident, and consequently the demonstration of the existence of the solution was effected.

This reasoning has rendered immense services; the greatest geometers, Gauss in the problem of the distribution of an attracting mass corresponding to a given potential, Riemann in his theory of Abelian functions, have been satisfied with it. To-day our attention has been called to the dangers of this sort of demonstration; it is possible for the minima to be simply limits and not to be actually attained by veritable functions possessing the necessary properties of continuity. We are, therefore, no longer content with the probabilities offered by the reasoning long classic.

Whether we proceed indirectly or whether we seek to give a rigorous proof of the existence of a function corresponding to the minimum, the route is long and arduous.

Further, not the less will it be always useful to connect a question of mechanics or of mathematical physics with a problem of minimum; in this first of all is a source of fecund analytic transformations, and besides in the very calculations of the investigation of variations useful indications may appear, relative to the conditions at the limits; a beautiful example of it was given by Kirchoff in the delicate investigation of the conditions at the limits of the equilibrium of flexure of plates.

VI

I have been led to expand particularly on partial differential equations.

Examples chosen in rational mechanics and in celestial mechanics would readily show the rôle which ordinary differential equations play in the progress of these sciences whose history, as we have seen, has been so narrowly bound to that of analysis.

When the hope of integrating with simple functions was lost, one strove to find developments permitting to follow a phenomenon as long as possible, or at least to obtain information of its qualitative bearing.

For practice, the methods of approximation form an extremely important part of mathematics, and it is thus that the highest parts of theoretic arithmetic find themselves connected with the applied sciences. As to series, the demonstrations themselves of the existence of integrals furnish them from the very first; thus Cauchy's first method gives developments convergent as long as the integrals and the differential coefficients remain continuous.

When any circumstance permits our foreseeing that such is always the case, we obtain developments always convergent. In the problem of n bodies, we can in this way obtain developments valid so long as there are no shocks.

If the bodies, instead of attracting, repel each other, this contingency need not be feared and we should obtain developments valid indefinitely; unhappily, as Fresnel said one day to Laplace, nature is not concerned about analytic difficulties and the celestial bodies attract instead of repelling each other.

One would even be tempted at times to go further than the great physicist and say that nature has sown difficulties in the paths of the analysts.

Thus, to take another example, we can generally decide, given a system of differential equations of the first order, whether the general solution is stable or not about a point, and to find developments in series valid for stable solutions it is only necessary that certain inequalities be verified.

But if we apply these results to the equations of dynamics to discuss stability, we find ourselves exactly in the particular case which is unfavorable. Nay, in general, here it is not possible to decide on the stability; in the case of a function of forces having a maximum, reasoning classic, but indirect, establishes the stability which cannot be deduced from any development valid for every value of the time.

Do not lament these difficulties; they will be the source of future progress.

Such are also the difficulties which still present to us, in spite of so many works, the equations of celestial mechanics; the astronomers have almost drawn from them, since Newton, by means of series practically convergent and approximations happily conducted, all that is necessary for the foretelling of the motions of the heavenly bodies.

The analysts would ask more, but they no longer hope to attain the integration by means of simple functions or developments always convergent.

What admirable recent researches have best taught them is the immense difficulty of the problem; a new way has, however, been opened by the study of particular solutions, such as the periodic solutions and the asymptotic solutions which have already been utilized. It is not perhaps so much because of the needs of practice as in order not to avow itself vanquished, that analysis will never resign itself to abandon, without a decisive victory, a subject where it has met so many brilliant triumphs; and again, what more beautiful field could the theories new-born or rejuvenated of the modern doctrine of functions find, to essay their forces, than this classic problem of n bodies?

It is a joy for the analyst to encounter in applications equations that he can integrate with known functions, with transcendents already classed.

Such encounters are unhapily rare; the problem of the pendulum, the classic cases of the motion of a solid body around a fixed point, are examples where the elliptic functions have permitted us to effect the integration.

It would also be extremely interesting to encounter a question of mechanics which might be the origin of the discovery of a new transcendent possessing some remarkable property; I should be embarrassed to give an example of it unless in carrying back to the pendulum the début of the theory of elliptic functions.

The interpenetration between theory and applications is here much less than in the questions of mathematical physics. Thus is explained that, since forty years, the works on ordinary differential equations attached to analytic functions have had in great part a theoretic character altogether abstract.

The pure theory has notably taken the advance; we have had occasion to say that it was well it should be so, but evidently there is here a question of measure, and we may hope to see the old problems profit by the progress accomplished.

It would not be over-difficult to give some examples, and I will recall only those linear differential equations, where figure arbitrary parameters whose singular values are roots of entire transcendent functions; which in particular makes the successive harmonics of a vibrating membrane correspond to the poles of a meromorphic function.

It happens also that the theory may be an element of classification in leading to seek conditions for which the solution falls under a determined type, as for example that the integral may be uniform. There have been and there yet will be many interesting discoveries in this way, the case of the motion of a solid heavy body treated by Madame de Kovalevski and where the Abelian functions were utilized is a remarkable example.

VII

In studying the reciprocal relations of analysis with mechanics and mathematical physics, we have on our way more than once encountered the infinitesimal geometry, which has proposed so many celebrated problems; in many difficult questions, the happy combination of calculus and synthetic reasonings has realized considerable progress, as is shown by the theories of applicable surfaces and systems triply orthogonal.

It is another part of geometry which plays a grand rôle in certain analytic researches, I mean the geometry of situation or *analysis situs*. We know that Riemann made from this point of view a complete study of the continuum of two dimensions, on which rests his theory of algebraic functions and their integrals.

When this number of dimensions augments, the questions of *analysis situs* become necessarily complicated; the geometric intuition ceases, and the study becomes purely analytic, the mind being guided solely by analogies which may be misleading and need to be discussed very closely. The theory of algebraic functions of two variables, which transports us into a space of four dimensions, without getting from *analysis situs* an aid so fruitful as does the theory of functions of one variable, owes it, however, useful orientations.

There is also another order of questions where the geometry of situation intervenes; in the study of curves traced on a surface and defined by differential equations, the connection of this surface plays an important rôle; this happens for geodesic lines.

The notion of connexity, moreover, presented itself long ago in analysis, when the study of electric currents and magnetism led to non-uniform potentials; in a more general manner certain multiform integrals of some partial differential equations are met in difficult theories, such as that of diffraction, and varied researches must continue in this direction.

From a different point of view, I must yet recall the relations of algebraic analysis with geometry, which manifest themselves so elegantly in the theory of groups of finite order.

A regular polyhedron, say an icosahedron, is on the one hand the solid that all the world knows; it is also, for the analyst, a group of finite order, corresponding to the divers ways of making the polyhedron coincide with itself.

The investigation of all the types of groups of motion of finite order interests not alone the geometers, but also the crystallographers; it goes back essentially to the study of groups of ternary linear substitutions of determinant +1, and leads to the thirty-two systems of symmetry of the crystallographers for the particular complex.

The grouping in systems of polyhedra corresponding so as to fill space exhausts all the possibilities in the investigation of the structure of crystals.

Since the epoch when the notion of group was introduced into algebra by Galois, it has taken, in divers ways, considerable development, so that to-day it is met in all parts of mathematics. In the applications, it appears to us above all as an admirable instrument of classification. Whether it is a question of substitution groups or of Sophus Lie's transformation groups, whether it is a question of algebraic equations or of differential equations, this comprehensive doctrine permits explanation of the degree of difficulty of the problems treated and teaches how to utilize the special circumstances which present themselves; thus it should interest as well mechanics and mathematical physics as pure analysis.

The degree of development of mechanics and physics has permitted giving to almost all their theories a mathematical form; certain hypotheses, the knowledge of elementary laws, have led to differential relations which constitute the last form under which these theories settle down, at least for a time. These latter have seen little by little their field enlarge with the principles of thermodynamics; to-day chemistry tends to take in its turn a mathematical form.

I will take as witness of it only the celebrated memoir of Gibbs on the equilibrium of chemical systems, so analytic in character, and where it needed some effort on the part of the chemists to recognize, under their algebraic mantle, laws of high importance.

It seems that chemistry has to-day gotten out of the premathematic period, by which every science begins, and that a day must come when will be systematized grand theories, analogous to those of our present mathematical physics, but far more vast, and comprising the *ensemble* of physicochemic phenomena.

It would be premature to ask if analysis will find in their developments the source of new progress; we do not even know beforehand what analytic types one might find.

I have constantly spoken of differential equations ruling phenomena; will this always be the final form which condenses a theory? Of this I know nothing certain, but we should, however, remember that many hypotheses have been made of more or less experimental nature; among them, one is what has been called the principle of *non-heredity*, which postulates that the future of a system depends only on its present state and its state at an instant infinitely near, or, more briefly, that accelerations depend only on positions and velocities.

We know that in certain cases this hypothesis is not admissible, at least with the magnitudes directly envisaged; one has sometimes misemployed on this subject the memory of matter, which recalls its past, and has spoken in affected terms of the life of a morsel of steel. Different attempts have been made to give a theory of these phenomena, where a distant past seems to interfere; of them I need not speak here. An analyst may think that in cases so complex it is necessary to abandon the form of differential equations, and resign one's self to envisage *functional equations*, where figure definite integrals which will be the witness of a sort of heredity.

To see the interest which is attached at this moment to functional equations, one might believe in a presentiment of the future needs of science.

VIII

After having spoken of non-heredity, I scarcely dare touch the question of the applications of analysis to biology.

It will be some time, no doubt, before one forms the functional equations of biologic phenomena; the attempts so far made are in a very modest order of ideas; yet efforts are being made to get out of the purely qualitative field, to introduce quantitative measures. In the question of the variation of certain characteristics, mensuration has been engaged in, and statistic measures which are translated by curves of frequency. The modifications of these curves with successive generations, their decompositions into distinct curves, may give the measure of the stability of species or of the rapidity of mutations, and we know what interest attaches itself to these questions in recent botanic researches. In all this so great is the number of parameters that one questions whether the infinitesimal method itself could be of any service. Some laws of a simple arithmetic character like those of Mendel come occasionally to give renewed confidence in the old aphorism which I cited in the beginning, that all things are explained by numbers; but, in spite of legitimate hopes, it is clear that, in its totality, biology is still far from entering upon a period truly mathematical.

It is not so, according to certain economists, with potential economy. After Cournot, the Lausanne school made an effort extremely interesting to introduce mathematical analysis into political economy.

Under certain hypotheses, which fit at least limiting cases, we find in learned treatises an equation between the quantities of merchandise and their prices, which recalls the equation of virtual velocities in mechanics: this is the equation of economic equilibrium. A function of quantities plays in this theory an essential rôle recalling that of the potential function. Moreover, the best authorized representatives of the school insist on the analogy of economic phenomena with mechanical phenomena. "As rational mechanics," says one of them, "considers material points, pure economy considers the *homo oeconomicus*."

Naturally, we find there also the analogues of Lagrange's equations, indispensable matrix of all mechanics.

While admiring these bold works, we fear lest the authors have neglected certain hidden masses, as Helmholtz and Hertz would have said. But although that may happen, there is in these doctrines a curious application of mathematics, which, at least, in well-circumscribed cases, has already rendered great services.

I have terminated, messieurs, this summary history of some of the applications of analysis, with the reflections which it has at moments suggested to me. It is far from being complete; thus I have omitted to speak of the calculus of probabilities, which demands so much subtlety of mind, and of which Pascal refused to explain the niceties to the Chevalier de Méré because he was not a geometer.

Its practical utility is of the first rank, its theoretic interest has always been great; it is further augmented to-day, thanks to the importance taken by the researches that Maxwell called *statistical* and which tend to envisage mechanics under a wholly new light.

I hope, however, to have shown, in this sketch, the origin and the reason of the bonds so profound which unite analysis to geometry and physics, more generally to every science bearing on quantities numerically measurable.

The reciprocal influence of analysis and physical theories has been in this regard particularly instructive.

What does the future hold?

Problems more difficult, corresponding to an approximation of higher order, will introduce complications which we can only vaguely forecast, in speaking, as I have just done, of functional equations replacing systematically our actual differential equations, or further of integrations of equations infinite in number with an infinity of unknown functions. But even though that happens, mathematical analysis will always remain that language which, according to the *mot* of Fourier, has no symbols to express confused notions, a language endowed with an admirable power of transformation and capable of condensing in its formulas an immense number of results.

ON PRESENT PROBLEMS OF ALGEBRA AND ANALYSIS

BY HEINRICH MASCHKE

[**Heinrich Maschke**, Associate Professor of Mathematics, University of Chicago. b. Breslau, Germany, October 24, 1853. A.B. Magdalenen Gymnasium, Breslau, 1872; Ph.D. Göttingen, 1880. Post-graduate Heidelberg, Breslau, Berlin, and Göttingen. Professor Mathematics Lvisenstädt. Gymnasium, Berlin, 1880–90; Electric Engineer at Weston Electric Company, Newark, New Jersey, 1890–92; Assistant Professor of Mathematics, University of Chicago, 1892–96.]

As set forth by the Committee directing the affairs of this International Congress, the address which I have the distinguished privilege of delivering to-day shall be on "Present Problems in Algebra and Analysis," — but it is *not* provided by the Committee how many of these problems shall be treated.

The different branches of algebra and analysis which have been investigated are so numerous that it would be quite impossible to give an approximately exhaustive representation even only of the most important problems, within the limits of the time allowed to me. I, therefore, have confined myself to the minimum admissible number, namely *one*, or rather one *group* of problems.

Of this one problem, however, this Section of Algebra and Analysis has the right to expect that it is neither purely algebraic nor purely analytic, but one which touches both fields; and at least in this respect I hope that my selection has been fortunate.

I purpose to speak to-day on the Theory of Invariants of Quadratic Differential Quantics. Invariants suggest at once algebra, differential quantics: analysis. At the same time the subject also leads into geometry, — it contains, for instance, a great part of differential geometry and of geometry of hyperspace. But is there, indeed, any algebraic or analytic problem which does not allow geometrical interpretation in some way or other? And when it comes to geometry of hyperspace, — it is then only geometrical language that we are using, — what we are actually considering are analytic or algebraic forms. Moreover, rigorous definitions and discussions of geometrical propositions of an invariant character in particular can only be given by tracing them back to their analytic origin.

In the following exposition I shall first speak on the various invariant expressions of differential quadratics as they occur in geometry of two and more dimensions, and then take up the purely analytic representation in the second part of the paper.

This corresponds also to the historical development of the subject: geometry has here as well as in many other branches of mathematics indicated the problems which in their later development turned out to be of paramount interest in pure analysis.

A few preliminary remarks concerning the nomenclature of the different types of invariant expressions will be necessary.

To a given differential quadratic form

$$A = \sum_{i,k=1}^{n} a_{ik}dx_idx_k, (a_{ki}=a_{ik})$$

where the a_{ik}'s are functions of the n independent variables $x_1, x_2, \ldots x_n$, we apply a general point transformation of the variables x,

$$x_i = x_i(y_1, y_2, \ldots y_n).$$

We observe that the differentials dx are then transformed into linear expressions of the differentials dy with the Jacobian of the x's with respect to the y's as the substitution-determinant which we shall call r.

By this transformation A goes into

$$A' = \Sigma a'_{ik}dy_idy_k.$$

Let now Φ be a function

(a) of the coefficients a_{ik} and their first, second, . . . derivatives,

(b) of $U, V, \ldots$ and their derivatives, where $U, V, \ldots$ are any arbitrary functions of $x_1, x_2, \ldots x_n$.

If then Φ remains the same whether formed for the new or for the old quantities, that is, if

$$\Phi(a'_{ik}, \frac{\partial a'_{ik}}{\partial y\lambda}, \ldots, U', \frac{\partial U'}{\partial y\lambda}, \ldots, V', \ldots) = \Phi(a_{ik}, \frac{\partial a_{ik}}{\partial x\lambda}, \ldots, U, \frac{\partial U}{\partial x\lambda}, \ldots V, \ldots)$$

we say that Φ is an invariant (in the wider sense) of A.

If Φ contains *only* the a_{ik}'s and their derivatives, we call it an *invariant proper*, and its order the order of the highest derivative occurring in it. If Φ contains also one or more arbitrary functions $U, V, \ldots$ we call it a *differential parameter*, the definition of order being the same as before.

If more than *one* differential quadratic is given it is easily understood what is meant by simultaneous invariants and simultaneous differential parameters.

In strict analogy with the algebraic theory of invariants we call *covariants* expressions Φ of the above invariantive nature, provided that we also allow the differentials dx to enter into Φ.

The first and the most important example of a differential quadratic quantic is the square of the arc-element on a surface

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2.$$

It was Gauss who made (1827), in his *Disquisitiones generales circa superficies curvas*, this expression the fundamental object of investigation. He also gave, in what has been called after him the Gaussian Curvature

$$K=(E,F,G,\frac{\delta E}{\delta u},\dots),$$

the first example of an invariant. Gauss defines this curvature geometrically and finds for it the analytic expression

$$\frac{LN-M^2}{EG-F^2}$$

which is a simultaneous invariant of two differential quantics, namely, of ds^2 and of $\frac{ds^2}{\rho}=Ldu^2+2Mdudv+Ndv^2$.

This shows that K is independent of the u,v-system on the surface. And now Gauss expresses K in terms of E, F, G and the first and second derivatives of these quantities alone. A direct demonstration that K is an invariant proper of the differential quantic ds^2 *alone*, — that is, without passing through the second differential quantic $\frac{ds^2}{\rho}$, — is of course desirable.[1] Each one of the general methods of treating the theory of invariants, which will be discussed in the latter part of this paper, furnishes such a direct proof. In particular, the aspect of the formula for K, on p. 528, deduced by the symbolic method, shows immediately the invariant character of K.

Differential parameters were introduced into differential geometry by Beltrami in 1863. These are the well-known expressions

$$\varDelta_1\varphi=\frac{E\left(\frac{\delta\varphi}{\delta v}\right)-2F\frac{\delta\varphi}{\delta u}\frac{\delta\varphi}{\delta v}+G\left(\frac{\delta\varphi}{\delta u}\right)^2}{EG-F^2},$$

$$\Gamma(\phi,\psi)=\frac{E\frac{\delta\varphi}{\delta v}\frac{\delta\psi}{\delta v}-F\left(\frac{\delta\varphi}{\delta u}\frac{\delta\psi}{\delta v}+\frac{\delta\varphi}{\delta v}\frac{\delta\psi}{\delta u}\right)+G\frac{\delta\varphi}{\delta u}\frac{\delta\psi}{\delta u}}{EG-F^2},$$

$$\varDelta_2\varphi=\frac{1}{\sqrt{EG-F^2}}\left\{\frac{\delta}{\delta u}\left[\frac{G\frac{\delta\varphi}{\delta u}-F\frac{\delta\varphi}{\delta v}}{\sqrt{EG-F^2}}\right]+\frac{\delta}{\delta v}\left[\frac{E\frac{\delta\varphi}{\delta v}-F\frac{\delta\varphi}{\delta u}}{\sqrt{EG-F^2}}\right]\right\},$$

where φ and ψ are the arbitrary functions which take the place of U, V in our general definition of differential parameters. Beltrami adopted the name "differential parameters" and also the notation

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[1] Cf. on this subject the interesting paper by Knoblauch: "Der Gauss'sche Satz vom Krümmungsmasse," *Sitzungsberichte der Berliner Mathem. Gesellschaft.* April 27, 1904.

Δ from Lamé, who, in his *Leçons sur les coördonnées curvilignes*, defined in 1859 his differential parameters

$$(\Delta_1\varphi)^2=\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2$$

$$\Delta_2\varphi=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}$$

for the three-dimensional case where the arc-element is of the form

$$ds^2=dx^2+dy^2+dz^2.$$

Lamé recognized the fundamental importance of these quantities and made a systematical use of them on account of their invariance with respect to any point-transformation preserving the form ds^2.

The general theory of invariants defines the differential parameters Δ_1 and Δ_2 for the case of n variables. From these general expressions Beltrami's differential parameters are directly obtained for $n=2$, Lamé's quantities $(\Delta_1)^2$ and Δ_2 for the special form of ds^2 in the case $n=3$.

The number of differential parameters is of course infinite, but Darboux in his *Leçons sur la théorie générale des surfaces* has proved that all of them are expressible by means of Δ_1, Δ_2, ∇ and the evident differential parameter

$$\theta(\varphi,\psi)=\frac{\dfrac{\partial\varphi}{\partial u}\dfrac{\partial\psi}{\partial v}-\dfrac{\partial\varphi}{\partial v}\dfrac{\partial\psi}{\partial u}}{\sqrt{EG-F^2}}$$

(by forming, for instance, $\Delta_1(\Delta_2\varphi)$ etc.) — an important theorem which has later been extended by Staeckel to an analogous theorem for the case of n variables.

The expression $\Delta_1\varphi$ occurs already in Gauss's *Disquisitiones*. By taking as parameter curves a singly infinite system of geodesics and its orthogonal trajectories he transforms the arc-element into the form

$$ds^2=dr^2+m^2d\varphi^2$$

and shows that r satisfies the differential equation

$$\Delta_1 r=1.$$

An important differential parameter is the geodesic curvature. Its expression was thrown by Bonnet into a form which is easily recognized as a differential parameter (of the second order). Its numerator $=0$ represents the differential equation of geodesic lines in an invariant form.

Since a transformation of the two independent variables u, v which preserves the same value of ds^2 can also be considered as a transformation of two surfaces which are applicable to each other, it follows that all invariants of ds^2 are also invariants of a surface with respect to the process of bending. From this reason these invariants have been called by Weingarten and Knoblauch, who were among the first writers emphasizing and developing to a certain extent the invariantive side of differential geometry, in the case of invariants proper, "Biegungsinvarianten," in the case of differential parameters, "Biegungscovarianten," and this notation has been more or less generally adopted. The notation "Biegungscovarianten" does not agree with the definition of a covariant given above, but a differential parameter of ds^2 can easily be modified into a covariantive form by replacing according to the differential equation of the curve

$$U(u,v)=const.$$

the derivatives $\frac{\partial U}{\partial u}$ and $\frac{\partial U}{\partial v}$ by μdv and $-\mu du$.

A surface is completely defined, apart from its location in space, when in addition to the quadratic form ds^2 also

$$\frac{ds^2}{\rho}=Ldu^2+2Mdudv+Ndv^2$$

is given, where ρ denotes the radius of curvature along ds, — a theorem which was proved (1867) by Bonnet.

With these two differential quantics given, we can now at once form simultaneous invariants and differential parameters. The six coefficients, E, F, G, L, M, N are, however, not independent; they are related by three partial differential equations, — the Gaussian relation and the two Codazzi-Mainardi equations. These three relations are expressible in an invariantive form. The Gaussian relation is

$$\frac{LN-M^2}{EG-F^2}=(E,F,G,\frac{\partial E}{\partial u},\cdots),$$

while the two Codazzi formulas are given by the identical vanishing of one simultaneous linear covariant.

As examples of simultaneous differential parameters and covariants I mention the expressions which, when set equal to zero, represent the differential equations of conjugate lines, asymptotic lines, and lines of curvature. The differential equation of lines of curvature, for instance, if written in terms of du, dv represents a linear simultaneous covariant; if written as a partial differential equation derived from

$$U(u,v)=const.$$

it represents a simultaneous differential parameter involving the arbitrary function U. The differential equation of conjugate lines, if written in two sets of differentials du, dv and δu, δv represents a bilinear simultaneous covariant; if written as a partial differential equation it represents a differential parameter involving two arbitrary functions U and V.

The theory of invariants of the above two differential quadratics, together with the condition of the vanishing of one simultaneous invariant proper and one simultaneous covariant, dominates then, in a certain sense, the whole of differential geometry.

Passing now to the case of n variables we may consider the differential quadratic form

$$\sum_{i,k=1}^{n} a_{ik} dx_i dx_k = ds^2$$

as the square of the arc in a hyperspace of n dimensions.

The fundamental rôle which the Gaussian curvature plays in the case $n=2$ is here represented by an invariant expresson of ds^2 which — in a certain sense — might be regarded as a generalization of the Gaussian curvature, namely, the Riemann curvature of the hyperspace. Riemann's investigations on this subject are found in his paper, *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen*, and in the mathematical supplement to it *Commentatio mathematica*, etc. in the prize-problem of the Parisian Academy, 1861.

The geometrical definition of the Riemann curvature is briefly the following: Starting from any point P with the coördinates x_i we consider two linear directions defined by the increments dx_i and δx_i. If we remain in the vicinity of P these two directions define a plane of two dimensions and the determinants

$$dx_i \delta x_k - dx_k \delta x_i$$

may be considered as the coördinates of this plane. If now we draw geodesic lines from the point P whose initial arc-elements lie all in this plane, then these geodesics define a surface of two dimensions and the Gaussian curvature of this geodesic surface at the point P is the Riemann curvature. The analytic expression for it is

$$R = -\tfrac{1}{2} \frac{\Sigma (ikrs)(dx_i \delta x_s - dx_s \delta x_i)(dx_k \delta x_r - dx_r \delta x_k)}{\Sigma (a_{ik} a_{rs} - a_{ir} a_{ks})(dx_i \delta x_s - dx_s \delta x_i)(dx_k \delta x_r - dx_r \delta x_k,}$$

where the sum is to be taken over all values of i, k, r, s from 1 to n with the exception of those for which $i=k$ or $r=s$.

The coefficients $(ikrs)$ are certain quantities depending on the coefficients a_{ik}, their first and second derivatives; they occur in the literature mostly under the name of the "Christoffel quadruple index symbols." A better, certainly shorter, notation would be the one used by Ricci, namely, "Riemann symbols."

The Riemann curvature R is an invariant expression, and as its form shows it is a covariant of two sets of differentials. For $n=2$ it is identical with the Gaussian curvature. For greater numbers n the value of R depends, at a given point, on the plane-direction at that point and in general varies with the plane. If it should be constant for all plane-directions through one point, and if this is so for all the points, then R is, as Schur has shown, altogether constant that is, for every point.

Spaces of constant Riemann curvature have been the object of numerous interesting investigations, but these are more or less of a specific geometric character.

If in particular R is zero, then all the Riemann symbols vanish and it can easily be shown that ds^2 can be transformed into the sum of n squares

$$ds^2 = \sum_{i=1}^{n} dy^2_i$$

The converse is true. In this case the hyperspace of n dimensions is called a *flat* or also *Euclidean* space.

In every case the quadratic

$$\sum_{i,k=1}^{n} a_{ik} dx_i dx_k$$

can be transformed into

$$\sum_{i=1}^{n+r} dy^2_i$$

where r has the maximum value $\frac{n(n-1)}{2}$. We might say then that the given hyperspace of n dimensions is always contained in an Euclidean space of $n+r$ dimensions, where r is one of the numbers, $0, 1, \cdots \frac{n(n-1)}{2}$.

The number r is evidently characteristic for the hyperspace the square of the arc-element of which is the given quadratic. This number r has been called by Ricci the *class* of the given differential quadratic quantic. It is evident that this class is an invariant number, and the condition that a given differential quadratic be of class r must certainly be an invariantive condition. For $r=0$ we have just seen that the condition is $R=0$. For higher values of r no attempt has yet been made, so far as I know, to establish this invariantive condition though this problem is certainly one of fundamental interest.

Beltrami, in his paper, *Teoria generale dei parametri differenziali*, has extended the definition of his differential parameters to the case of n variables. The definition, for instance, of the first differential parameters is

$$\Delta_1 \varphi = \frac{1}{a} \sum_{i,k=1}^{n} A_{ik} \frac{\partial \varphi}{\partial x_i} \frac{\partial \varphi}{\partial x_k}$$

where A_{ik} denotes the minor of the element a_{ik} in the determinant

$$|a_{ik}| = a.$$

Beltrami shows that by means of the geodesics emanating from one point and of the hypersurfaces orthogonal to them he can choose his parameters such that ds^2 is transformed into

$$ds^2 = dr^2 + \sum_{i,k=1}^{n-1} b_{ik} dy_i dy_k,$$

where r satisfies the equation $\Delta_1 r = 1$, and that thus Gauss's theorems on geodesic polar coördinates for $n=2$ admit a perfect analogon in hyperspace. Also in hyperspace then the determination of systems of geodesics amounts to the integration of the partial differential equation

$$\Delta_1\varphi=1.$$

This leads now to the application of differential quadratics to analytic mechanics. If we write down the expression of the vis viva of a (holonomous) material system in terms of generalized coördinates $q_1, q_2, \cdots q_n$

$$T=\tfrac{1}{2}\Sigma a_{ik}\frac{dq_i}{dt}\frac{dq_k}{dt}$$

we have at once in

$$2Tdt^2=ds^2$$

a differential quadratic before us.

If no external forces act on the system, then a geodesic line of ds^2 represents at once, as also Beltrami has shown, a path of the system. Thus the mechanical problem is practically reduced to the integration of the equation $\Delta_1\varphi=1$.

In the case of the existence of external forces having a potential U, the above differential quantic has to be replaced by

$$\Sigma(U+h)a_{ik}dq_idq_k$$

and the mechanical problem is equivalent to the integration of the equation

$$\Delta_1\varphi=U+h$$

where $\Delta_1\varphi$ is the differential parameter of the quadratic form denoted before by ds^2.

A detailed exposition of the above-mentioned researches of Beltrami, as well as this application to mechanics, is given in the second volume of Darboux's *Leçons sur la théorie des surfaces.*

Passing now to the second part of my address, the purely analytic theory of invariants of differential quadratics, I have first to discuss that paper which forms the foundation of almost all later literature on the subject: Christoffel's article in *Crelle's Journal,* vol. LXX (1870), "Ueber die Transformation der homogenen Differentialausdrücke des zweiten Grades."

Christoffel puts his problem in this form: Given two differential quadratics

$$A=\Sigma a_{ik}dx_idx_k \qquad \text{and } A'=\Sigma a'_{ik}dy_idy_k,$$

what are the necessary and sufficient conditions for the equivalence of the two quadratics, that is, for the existence of a transformation of one quantic into the other; and if these conditions are established how can the required transformation be determined? (I should mention that Lamé in his already quoted work, *Leçons sur les coör-*

données curvilignes, treats and solves the analogous problem for the case $A=dx^2+dy^2+dz^2$).

Since the differentials dx are substituted linearly in terms of the dy there exists one and only one *algebraic* condition for the transformation, namely,

$$|a'_{ik}|=r^2|a_{ik}|.$$

This condition would be sufficient if the coefficients a_{ik} and the elements of the determinant r were constants. In our case, however, other conditions must be satisfied, namely, the conditions of integrability in order that the expressions for the dx's are complete differentials. This is the way in which Christoffel introduces his problem to the reader.

The difficulty lies in the fact that the integrability conditions lead at once to a great number of partial differential equations of an apparently highly complex character. But Christoffel succeeds in substituting for all these partial differential equations a purely *algebraic* problem: The equivalence of two finite systems of algebraic forms in the sense of the algebraic theory of invariants. If this equivalence is satisfied, — which is merely a question of algebra, — no further discussion of the integrability conditions is required; they are all taken care of by the equivalence of the two systems.

For the following it will be necessary to sketch briefly the character of these forms.

The first is the quadratic form A itself. The next form is a quadrilinear covariant G_4 in four sets of differentials dx^1, dx^2, dx^3, dx^4, the coefficients of which are precisely the quantities $(i\ k\ r\ s)$ — the "Christoffel quadruple index symbols" or the "Riemann symbols" — which occur in the expression for the Riemann curvature:

$$G_4=\Sigma(ikrs)\ d'x_i d^2x_k d^3x_r d^4x_s.$$

It is highly interesting to observe how the quantities $(\ i\ k\ r\ s\ )$ have entered into the theory from two so apparently different standpoints. Christoffel found these expressions quite independently. Though Riemann's paper was written in 1861, that is, before Christoffel's article which appeared in 1870, it was only published in 1876, ten years after Riemann's death, by Weber-Dedekind.

For the deduction of the following forms $G_5, G_6,$ — . . . — these forms are covariants linear in resp. $_5, _6,$ . . . sets of differentials — Christoffel uses a certain reduction process. The coefficients $(\lambda\ i\ k\ r\ s)$ for instance of G_5 are obtained from $(i\ k\ r\ s)$ first by differentiating $(i\ k\ r\ s)$ with respect to x_λ and then by the addition of a sum of $5n$ terms which are linear in the different symbols $(\ i\ k\ r\ s\ )$ with coefficients depending on the so-called Christoffel triple index symbols of the second kind — expressions involving the quantities a_{ik} and their first derivatives.

Continuing in this way Christoffel obtains a well-defined set of covariants G_5, G_6, . . . , and this is his final result: the necessary and sufficient condition for the equivalence of the two differential quadratics is the algebraic equivalence — in the sense of the algebraic theory of invariants — of the forms $A, G_4, G_5, \ldots G_\mu$, and A', $G'_4, G'_5 \ldots G'_\mu$, where μ is a certain finite number.

In several papers covering the period from about 1884 up to the present time Ricci has worked out in a systematical way the fundamental principles of Christoffel's investigation, and has applied his theory to many problems in analysis, geometry, mechanics, and mathematical physics. He recognized in particular the importance of Christoffel's deduction of the covariants $G_{\lambda+1}$ from G_λ. He found that this process of deduction can be applied with a proper modification to any functions of the x's and the a_{ik}'s and that whenever invariantive relations with respect to the fundamental differential quadratic A come into question, this process is always of vital importance. He calls this process *covariantive differentiation* with respect to the fundamental quadratic A. On the systematical use of this covariantive differentiation Ricci based a calculus which he called *Calcolo differenziale assoluto.*

A collection of all his various investigations is given in two places:

(1) In a paper published, together with Levi-Civittà in the *Math. Annalen*, vol. LIV.

(2) In his *Lezioni sulla teoria delle superficie*, Verona, Padua, 1898.

In the introduction of these autographed lectures he presents a complete exposition of his absolute differential calculus. Characteristic is the way in which he treats in his *Lezioni* the differential geometry. He divides it into two parts:

(1) Properties of surfaces depending on the one differential quadratic ds^2.

(2) Properties of surfaces depending on the two quadratics

$$ds^2 \text{ and } \frac{ds^2}{\rho}.$$

We are here chiefly interested in his applications to the theory of differential invariants. This is the result in his language: In order to obtain all invariants proper and differential parameters of order μ, it is sufficient to determine the algebraic invariants of the system of the following forms:

(1) The fundamental differential quantic A.

(2) The covariantive derivatives of the arbitrary functions $U, V, \ldots$ up to the order μ.

(3) (for $\mu > 1$) the quadrilinear covariant G_4 and its covariantive derivatives up to the order $\mu - 2$.

Another treatment of the invariant theory of differential quantics was given by myself. I applied a symbolic method to the theory which consists chiefly in identifying the fundamental quadratic

$$\Sigma a_{ik} dx_i dx_k$$

with the square of a linear expression

$$(\Sigma f_i dx_i)^2$$

by setting $f_i f_k = a_{ik}$. This is strictly analogous to the introduction of symbols in the algebraic theory. The difference, of course, comes in at once when we have to consider also the derivatives of a_{ik}.

A systematic development leads to expressions and formulas which with respect to simplicity and shortness are as superior to the formulas of the ordinary notation as the formulas of the so-called symbolic notation in the algebraic theory are superior to the non-symbolic expressions.

As examples I give the most important invariant expressions for the case $n=2$.

Let us introduce the abbreviation

$$\frac{1}{\sqrt{a_{11}a_{22}-a^2_{12}}}(P_1 Q_2 - P_2 Q_1) = (PQ), \text{ where } P_k = \frac{\delta P}{\delta x_k} \text{ etc.;}$$

let further f, φ, ψ . . . be symbols of A, so that

$$f_i f_k = \varphi_i \varphi_k = \psi_i \psi_k = \cdot\ \cdot\ \cdot = a_{ik}$$

and let U, V . . . be arbitrary functions of x_1, x_2.

Then we have

$$(fU)^2 = \Delta_1 U,$$

$$(fU)\,(fV) = \nabla(UV),$$

$$(f(fU)) = \Delta_2 U,$$

$$(\varphi\psi)\ ((f\varphi)\ (f\psi)) = 2K \text{ (Gaussian curvature),}$$

$$(f\varphi)\ (\varphi U)\ ((fU)U) : (\Delta_1 U)^3 = \text{Geodesic curvature of curve } U = \mathit{const}.$$

To give also some examples of simultaneous invariant expressions let $f, \varphi,$ . . . be as before symbols of

$$Edu^2 + 2Fdudv + Gdv^2$$

and F, Φ . . . symbols of

$$Ldu^2 + 2Mdudv + Ndv^2,$$

Then:

$$(F\Phi)^2 = 2K,$$

$$(fF)^2 = \text{mean curvature.}$$

The differential equations

of asymptotic curves $U=c$ are $(FU)^2=0$,

of conjugate curves $U=c$, $V=c$: $(FU)(FV)=0$,

of lines of curvature $U=c$: $(fF)(fU)(FU)=0$.

The equation $(f\varphi)(\varphi F)((f\varphi)U)=0$ gives the two Cadazzi formulas by setting the coefficients of U_1 and U_2 separately equal to zero.

In these examples the invariant expressions always appear as products of factors of the type (PR). The general theorem holds that any product of factors of this type represents always an invariant expression provided that the symbols $f, \varphi, \ldots, F, \Phi, \ldots$ occur in such a connection as to permit actual meaning.

The symbolic representation of invariant expressions suggested by the case $n=2$ can without essential difficulty be extended to the general case of n variables. In this treatment of the subject all the essential quantities entering into the theory present themselves quite naturally; they lie, so to say, on the surface; so, for instance, all the Christoffel symbols of the different kinds including the Riemann symbols and in particular also the process of covariantive differentiation.

The results of my investigation are chiefly laid down in the paper "A symbolic treatment of the theory of invariants of quadratic differential quantics of n variables," *Transactions of the American Mathematical Society*, vol. IV.

A third method of investigation of our theory of invariants is based on Lie's theory of continuous groups. The general point transformation by which A is transformed into A' defines a so-called "infinite" continuous group. In order to obtain the invariants of A, this group must first be "extended" in Lie's sense to include the coefficients a_{ik} of A and also the arbitrary functions involved in the differential parameters.

Lie himself developed a short outline of the determination of invariants in the second volume of the *Mathematische Annalen* for the case $n=2$, and indicated in particular how the Gaussian curvature and the parameter $\Delta_1\varphi$ could be found. The general plan of investigation was taken up in the sixteenth volume of the *Acta Mathematica* by Zorowski, who studied the case $n=2$ in detail, adding the complete computation of the Gaussian curvature and the most important differential parameters.

An extension of Lie's methods to the general case of n variables as far as the actual determination of invariants is concerned has, so far as I know, not yet been made; only the problem of determining the *number* of functionally independent invariants of a given order has been taken up. It seems that Lie's method is especially well adapted to this particular problem. In a paper in the *Atti del Reale Instituto Veneto* (1897), Levi-Civittà found a lower limit for the number of invariants of a given order. The actual number was determined by Haskins in the *Transactions of the American Mathematical Society*, vol. III, for the case of invariants proper (including also simultaneous invariants) and in vol. V, of differential parameters.

I am at the end of my paper. I have attempted to show, in a compendious way, what has been done in this attractive field of research which is so closely connected with various interesting parts of pure and applied mathematics. The number of problems that remain to be solved are numerous. Excepting the lowest cases as to the number of variables and the order of the invariants, not much more than the mere existence of the invariants is known, so that we have hardly the right to speak of a *theory* of these invariants.

When it comes to the question which of the different methods will be best adapted to a further systematical study of the subject, it seems probable that a combination of two or more of them will be the most promising one. But here, as always, it is the *man*, not the *method*, that solves the problem.

SHORT PAPERS

The Section of Algebra and Analysis attracted wide interest and caused many supplementary papers on various topics to be submitted. It is impossible to give a résumé of these, as their analytical nature demands that they be printed in full or not at all.

The first paper was presented by Professor G. A. Miller, of Leland Stanford Jr. University, on the "Bearing of Several Recent Theorems on Group Theory."

The second paper was read by Professor James Birney Shaw, of Milliken University, on "Linear Associative Algebra."

The third paper was presented by Professor M. W. Haskell, of the University of California, on "The Reduction of any Collineation to a Product of Perspective Collineations."

The fourth paper was presented by Professor M. B. Porter, of the University of Texas, "On Functions defined by an Infinite Series of Analytic Functions of a Complex Variable."

The fifth paper was presented by Professor Edward V. Huntington, of Harvard University, on "A Set of Postulates for Real Algebra comprising Postulates for a One Dimensional Continuum and for the Theory of Groups."

The sixth paper was presented by Professor J. I. Hutchinson, of Cornell University, on "Uniformizing of Algebraic Functions."

The seventh paper was read by Professor E. R. Hedrick, of the University of Missouri, on "Generalization of the Analytic Functions of a Complex Variable."

SECTION B—GEOMETRY

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(*Hall* 9, *September* 24, 10 *a. m.*)

CHAIRMAN: PROFESSOR M. W. HASKELL, University of California.
SPEAKERS: M. JEAN GASTON DARBOUX, Perpetual Secretary of the Academy of Sciences, Paris.
DR. EDWARD KASNER, Columbia University.
SECRETARY: PROFESSOR THOMAS J. HOLGATE, Northwestern University.

A STUDY OF THE DEVELOPMENT OF GEOMETRIC METHODS

BY M. JEAN GASTON DARBOUX

(*Translated from the French by Professor George Bruce Halsted, Kenyon College*)

[**Jean Gaston Darboux**, Perpetual Secretary Academy of Sciences, Paris; Doyen Honorary, Professor of Higher Geometry of the Faculty of Sciences, Paris. b. August 13, 1842, Nîmes, France. Dr.Sc., LL.D., University of Cambridge, University of Christiania, University of Heidelberg, *et al.* Professor of Special Mathematics, Lycée Louis le Grand, 1867–73; Master of Conferences in Superior Normal Schools, Paris, 1873–81; Professor Suppléant of Rational Mechanics and Higher Geometry, The Sorbonne, 1873–81; since 1881, Professor Titulaire of the Faculty of Sciences, and Doyen of the Faculty of Sciences since 1889; also Professor in Higher Normal School for Schools of Science; Member of Bureau des Longitudes; President of the First General Assembly of the International Association of Academies; and Honorary Vice-President for France of the Congress of Arts and Science; Member of Institute of France, Royal Society of London; Academies of Berlin, St. Petersburg, Rome, Amsterdam, Munich, Stockholm; American Philosophical Society, *et al.* Author of many publications and addresses on Mathematics, and editor of the *Bulletin of Science of Mathematics*.]

I

To appreciate the progress geometry has made during the century just ended, it is of advantage to cast a rapid glance over the state of mathematical science at the beginning of the nineteenth century.

We know that, in the last period of his life, Lagrange, fatigued by the researches in analysis and mechanics, which assured him, however, an immortal glory, neglected mathematics for chemistry (which, according to him, was easy as algebra), for physics, for philosophic speculations.

This mood of Lagrange we almost always find at certain moments of the life of the greatest savants. The new ideas which came to them in the fecund period of youth and which they introduced into the common domain have given them all they could have expected; they have fulfilled their task and feel the need of turning their mental activity towards wholly new subjects. This need, as we recognize, manifested itself with particular force at the epoch of Lagrange. At this moment, in fact, the programme of researches opened to geometers by the discovery of the infinitesimal calculus appeared very nearly finished up. Some differential equations more or less complicated to integrate, some chapters to add to the integral calculus, and one seemed about to touch the very outmost bounds of science.

Laplace had achieved the explanation of the system of the world and laid the foundations of molecular physics. New ways opened before the experimental sciences and prepared the astonishing development they received in the course of the century just ended. Ampère, Poisson, Fourier, and Cauchy himself, the creator of the theory of imaginaries, were occupied above all in studying the application of the analytic methods to molecular physics, and seemed to believe that outside this new domain, which they hastened to cover, the outlines of theory and science were finally fixed.

Modern geometry, a glory we must claim for it, came, after the end of the eighteenth century, to contribute in large measure to the renewing of all mathematical science, by offering to research a way new and fertile, and above all in showing us, by brilliant successes, that general methods are not everything in science, and that even in the simplest subject there is much for an ingenious and inventive mind to do.

The beautiful geometric demonstrations of Huygens, of Newton, and of Clairaut were forgotten or neglected. The fine ideas introduced by Desargues and Pascal had remained without development and appeared to have fallen on sterile ground.

Carnot, by his *Essai sur les transversales* and his *Géométrie de position,* above all Monge, by the creation of descriptive geometry and by his beautiful theories on the generation of surfaces, came to renew a chain which seemed broken. Thanks to them, the conceptions of the inventors of analytic geometry, Descartes and Fermat, retook alongside the infinitesimal calculus of Leibnitz and Newton the place they had lost, yet should never have ceased to occupy. With his geometry, said Lagrange, speaking of Monge, this demon of a man will make himself immortal.

And, in fact, not only has descriptive geometry made it possible to coördinate and perfect the procedures employed in all the arts where precision of form is a condition of success and of excellence for the work and its products; but it appeared as the graphic translation of a geometry, general and purely rational, of which numerous and important researches have demonstrated the happy fertility.

Moreover, beside the *Géométrie descriptive* we must not forget to place that other masterpiece, the *Application de l'analyse à la* *géométrie*; nor should we forget that to Monge are due the notion of lines of curvature and the elegant integration of the differential equation of these lines for the case of the ellipsoid, which, it is said, Lagrange envied him. To be stressed is this character of unity of the work of Monge.

The renewer of modern geometry has shown us from the beginning, what his successors have perhaps forgotten, that the alliance of geometry and analysis is useful and fruitful, that this alliance is perhaps for each a condition of success.

II

In the school of Monge were formed many geometers: Hachette, Brianchon, Chappuis, Binet, Lancret, Dupin, Malus, Gaultier de Tours, Poncelet, Chasles, *et al.* Among these Poncelet takes first rank. Neglecting, in the works of Monge, everything pertaining to the analysis of Descartes or concerning infinitesimal geometry, he devoted himself exclusively to developing the germs contained in the purely geometric researches of his illustrious predecessor.

Made prisoner by the Russians in 1813 at the passage of the Dnieper and incarcerated at Saratoff, Poncelet employed the leisure captivity left him in the demonstration of the principles which he has developed in the *Traité des propriétés projectives des figures*, issued in 1822, and in the great memoirs on reciprocal polars and on harmonic means, which go back nearly to the same epoch. So we may say the modern geometry was born at Saratoff.

Renewing the chain broken since Pascal and Desargues, Poncelet introduced at the same time homology and reciprocal polars, putting thus in evidence, from the beginning, the fruitful ideas on which the science has evolved during fifty years.

Presented in opposition to analytic geometry, the methods of Poncelet were not favorably received by the French analysts. But such were their importance and their novelty, that without delay they aroused, from divers sides, the most profound researches.

Poncelet had been alone in discovering the principles; on the contrary, many geometers appeared almost simultaneously to study them on all sides and to deduce from them the essential results which they implicitly contained.

At this epoch, Gergonne was brilliantly editing a periodical which has to-day for the history of geometry an inestimable value. The *Annales de Mathématiques*, published at Nîmes from 1810 to 1831, was during more than fifteen years the only journal in the entire world devoted exclusively to mathematical researches.

Gergonne, who, in many regards, was a model editor for a scientific journal, had the defects of his qualities; he collaborated, often against their will, with the authors of the memoirs sent him, rewrote them, and sometimes made them say more or less than they would have wished. Be that as it may, he was greatly struck by the originality and range of Poncelet's discoveries.

In geometry some simple methods of transformation of figures were already known; homology even had been employed in the plane, but without extending it to space, as did Poncelet, and especially without recognizing its power and fruitfulness. Moreover, all these transformations were *punctual;* that is to say, they made correspond a point to a point.

In introducing polar reciprocals, Poncelet was in the highest degree creative, because he gave the first example of a transformation in which to a point corresponded something other than a point.

Every method of transformation enables us to multiply the number of theorems, but that of polar reciprocals had the advantage of making correspond to a proposition another proposition of wholly different aspect. This was a fact essentially new. To put it in evidence, Gergonne invented the system, which since has had so much success, of memoirs printed in double columns with correlative propositions in juxtaposition; and he had the idea of substituting for Poncelet's demonstrations, which required an intermediary curve or surface of the second degree, the famous "principle of duality," of which the signification, a little vague at first, was sufficiently cleared up by the discussions which took place on this subject between Gergonne, Poncelet, and Pluecker.

Bobillier, Chasles, Steiner, Lamé, Sturm, and many others whose names escape me, were, at the same time as Pluecker and Poncelet, assiduous collaborators of the *Annales de Mathématiques*. Gergonne, having become rector of the Academy of Montpellier, was forced to suspend in 1831 the publication of his journal. But the success it had obtained, the taste for research it had contributed to develop, had commenced to bear their fruit. Quételet had established in Belgium the *Correspondance mathématique et physique*. Crelle, from 1826, brought out at Berlin the first sheets of his celebrated journal, where he published the memoirs of Abel, of Jacobi, of Steiner.

A great number of separate works began also to appear, wherein the principles of modern geometry were powerfully expounded and developed.

First came in 1827 the *Barycentrische Calcul* of Moebius, a work truly original, remarkable for the profundity of its conceptions, the elegance and the rigor of its exposition; then in 1828 the *Analytisch-geometrische Entwickelungen* of Pluecker, of which the second part appeared in 1831, and which was soon followed by the *System der analytischen Geometrie* of the same author, published at Berlin in 1835.

In 1832 Steiner brought out at Berlin his great work: *Systematische Entwickelung der Abhaengigkeit der geometrischen Gestalten von einander*, and, the following year, *Die geometrischen Konstruktionen ausgefuehrt mittels der geraden Linie und eines festen Kreises*, where was confirmed by the most elegant examples a proposition of Poncelet's relative to the employment of a single circle for the geometric constructions.

Finally, in 1830, Chasles sent to the Academy of Brussels, which happily inspired had offered a prize for a study of the principles of modern geometry, his celebrated *Aperçu historique sur l'origine et le développement des méthodes en géométrie*, followed by *Mémoire sur deux principes généraux de la science : la dualité et l'homographie*, which was published only in 1837.

Time would fail us to give a worthy appreciation of these beautiful works and to apportion the share of each. Moreover, to what would such a study conduct us, but to a new verification of the general laws of the development of science? When the times are ripe, when the fundamental principles have been recognized and enunciated, nothing stops the march of ideas; the same discoveries, or discoveries almost equivalent, appear at nearly the same instant, and in places the most diverse. Without undertaking a discussion of this sort, which, besides, might appear useless or become irritating, it is, however, of importance to bring out a fundamental difference between the tendencies of the great geometers, who, about 1830, gave to geometry a scope before unknown.

III

Some, like Chasles and Steiner, who consecrated their entire lives to research in pure geometry, opposed what they called *synthesis* to *analysis*, and, adopting in the *ensemble* if not in detail the tendencies of Poncelet, proposed to constitute an independent doctrine, rival of Descartes's analysis.

Poncelet could not content himself with the insufficient resources furnished by the method of projections; to attain imaginaries he created that famous *principle of continuity* which gave birth to such long discussions between him and Cauchy.

Suitably enunciated, this principle is excellent and can render great service. Poncelet was wrong in refusing to present it as a simple consequence of analysis; and Cauchy, on the other hand, was not willing to recognize that his own objections, applicable without doubt to certain transcendent figures, were without force in the applications made by the author of the *Traité des propriétés projectives*.

Whatever be the opinion of such a discussion, it showed at least in the clearest manner that the geometric system of Poncelet rested on an analytic foundation, and besides we know, by the untoward publication of the manuscripts of Saratoff, that by the aid of Descartes's analysis were established the principles which serve as foundation for the *Traité des propriétés projectives*.

Younger than Poncelet, who besides abandoned geometry for mechanics where his works had a preponderant influence, Chasles, for whom was created in 1847 a chair of *Géométrie supérieure* in the Faculty of Science of Paris, endeavored to constitute a geometric doctrine entirely independent and autonomous. He has expounded it in two works of high importance, the *Traité de géométrie supérieure*, which dates from 1852, and the *Traité des sections coniques*, unhappily unfinished and of which the first part alone appeared in 1865.

In the preface of the first of these works he indicates very clearly the three fundamental points which permit the new doctrine to share the advantages of analysis and which to him appear to mark an advance in the cultivation of the science. These are: (1) The introduction of the principle of signs, which simplifies at once the enunciations and the demonstrations, and gives to Carnot's analysis of transversals all the scope of which it is susceptible; (2) the introduction of imaginaries, which supplies the place of the principle of continuity and furnishes demonstrations as general as those of analytic geometry; (3) the simultaneous demonstration of propositions which are correlative, that is to say, which correspond in virtue of the principle of duality.

Chasles studies indeed in his work homography and correlation; but he avoids systematically in his exposition the employment of transformations of figures, which, he thinks, cannot take the place of direct demonstrations since they mask the origin and the true nature of the properties obtained by their means.

There is truth in this judgment, but the advance itself of the science permits us to declare it too severe. If it happens often that, employed without discernment, transformations multiply uselessly the number of theorems, it must be recognized that they often aid us to better understand the nature of the propositions even to which they have been applied. Is it not the employment of Poncelet's projection which has led to the so fruitful distinction between projective properties and metric properties, which has taught us also the high importance of that cross-ratio whose essential property is found already in Pappus, and of which the fundamental rôle has begun to appear after fifteen centuries only in the researches of modern geometry?

The introduction of the principle of signs was not so new as Chasles supposed at the time he wrote his *Traité de Géométrie supérieure*.

Moebius, in his *Barycentrische Calcul*, had already given issue to a *desideratum* of Carnot, and employed the signs in a way the largest and most precise, defining for the first time the sign of a segment and even that of an area.

Later he succeeded in extending the use of signs to lengths not laid off on the same straight line and to angles not formed about the same point.

Besides Grassmann, whose mind has so much analogy to that of Moebius, had necessarily employed the principle of signs in the definitions which serve as basis for his methods, so original, of studying the properties of space.

The second characteristic which Chasles assigns to his system of geometry is the employment of imaginaries. Here, his method was really new, and he illustrates it by examples of high interest. One will always admire the beautiful theories he has left us on homofocal surfaces of the second degree, where all the known properties and others new, as varied as elegant, flow from the general principle that they are inscribed in the same developable circumscribed to the circle at infinity.

But Chasles introduced imaginaries only by their symmetric functions, and consequently would not have been able to define the cross-ratio of four elements when these ceased to be real in whole or in part. If Chasles had been able to establish the notion of the cross-ratio of imaginary elements, a formula he gives in the *Géométrie supérieure* (p. 118 of the new edition) would have immediately furnished him that beautiful definition of angle as logarithm of a cross-ratio which enabled Laguerre, our regretted confrère, to give the complete solution, sought so long, of the problem of the transformation of relations which contain at the same time angles and segments in homography and correlation.

Like Chasles, Steiner, the great and profound geometer, followed the way of pure geometry; but he has neglected to give us a complete exposition of the methods upon which he depended. However, they may be characterized by saying that they rest upon the introduction of those elementary geometric forms which Desargues had already considered, on the development he was able to give to Bobillier's theory of polars, and finally on the construction of curves and surfaces of higher degrees by the aid of sheaves or nets of curves of lower orders. In default of recent researches, analysis would suffice to show that the field thus embraced has just the extent of that into which the analysis of Descartes introduces us without effort.

IV

While Chasles, Steiner, and, later, as we shall see, von Staudt, were intent on constituting a rival doctrine to analysis and set in some sort altar against altar, Gergonne, Bobillier, Sturm, and above all Pluecker, perfected the geometry of Descartes and constituted an analytic system in a manner adequate to the discoveries of the geometers. It is to Bobillier and to Pluecker that we owe the method called *abridged notation*. Bobillier consecrated to it some pages truly new in the last volumes of the *Annales* of Gergonne.

Pluecker commenced to develop it in his first work, soon followed by a series of works where are established in a fully conscious manner the foundations of the modern analytic geometry. It is to him that we owe tangential coördinates, trilinear coördinates, employed with homogeneous equations, and finally the employment of canonical forms whose validity was recognized by the method, so deceptive sometimes, but so fruitful, called the *enumeration of constants*.

All these happy acquisitions infused new blood into Descartes's analysis and put it in condition to give their full signification to the conceptions of which the geometry called *synthetic* had been unable to make itself completely mistress.

Pluecker, to whom it is without doubt just to adjoin Bobillier, carried off by a premature death, should be regarded as the veritable initiator of those methods of modern analysis where the employment of homogeneous coördinates permits treating simultaneously and, so to say, without the reader perceiving it, together with one figure all those deducible from it by homography and correlation.

V

Parting from this moment, a period opens brilliant for geometric researches of every nature.

The analysts interpret all their results and are occupied in translating them by constructions.

The geometers are intent on discovering in every question some general principle, usually undemonstrable without the aid of analysis, in order to make flow from it without effort a crowd of particular consequences, solidly bound to one another and to the principle whence they are derived. Otto Hesse, brilliant disciple of Jacobi, develops in an admirable manner that method of homogeneous coördinates to which Pluecker perhaps had not attached its full value. Boole discovers in the polars of Bobillier the first notion of a covariant; the theory of forms is created by the labors of Cayley, Sylvester, Hermite, Brioschi. Later Aronhold, Clebsch and Gordan, and other geometers still living, gave to it its final notation, established the fundamental theorem relative to the limitation of the number of covariant forms and so gave it all its amplitude.

The theory of surfaces of the second order, built up principally by the school of Monge, was enriched by a multitude of elegant properties, established principally by O. Hesse, who found later in Paul Serret a worthy emulator and continuer.

The properties of the polars of algebraic curves are developed by Pluecker and above all by Steiner. The study, already old, of curves of the third order is rejuvenated and enriched by a crowd of new elements. Steiner, the first, studies by pure geometry the double tangents of curves of the fourth order, and Hesse, after him, applies the methods of algebra to this beautiful question, as well as to that of points of inflection of curves of the third order.

The notion of *class* introduced by Gergonne, the study of a paradox in part elucidated by Poncelet and relative to the respective degrees of two curves reciprocal polars one of the other, give birth to the researches of Pluecker relative to the singularities called *ordinary* of algebraic plane curves. The celebrated formulas to which Pluecker is thus conducted are later extended by Cayley and by other geometers to algebraic skew curves, by Cayley again and by Salmon to algebraic surfaces.

The singularities of higher order are in their turn taken up by the geometers; contrary to an opinion then very widespread, Halphen demonstrates that each of these singularities cannot be considered as equivalent to a certain group of ordinary singularities, and his researches close for a time this difficult and important question.

Analysis and geometry, Steiner, Cayley, Salmon, Cremona, meet in the study of surfaces of the third order, and, in conformity with the anticipations of Steiner, this theory becomes as simple and as easy as that of surfaces of the second order.

The algebraic ruled surfaces, so important for applications, are studied by Chasles, by Cayley, of whom we find the influence and the mark in all mathematical researches, by Cremona, Salmon, La Gournerie; so they will be later by Pluecker in a work to which we must return.

The study of the general surface of the fourth order would seem to be still too difficult; but that of the particular surfaces of this order with multiple points or multiple lines is commenced, by Pluecker for the surface of waves, by Steiner, Kummer, Cayley, Moutard, Laguerre, Cremona, and many other investigators.

As for the theory of algebraic skew curves, grown rich in its elementary parts, it receives finally, by the labors of Halphen and of Noether, whom it is impossible for us here to separate, the most notable extensions.

A new theory with a great future is born by the labors of Chasles, of Clebsch, and of Cremona; it concerns the study of all the algebraic curves which can be traced on a determined surface.

Homography and correlation, those two methods of transformation which have been the distant origin of all the preceding researches, receive from them in their turn an unexpected extension; they are not the only methods which make a single element correspond to a single element, as might have shown a particular transformation briefly indicated by Poncelet in the *Traité des propriétés projectives.*

Pluecker defines the *transformation by reciprocal radii vectores* or *inversion,* of which Sir W. Thomson and Liouville hasten to show all the importance, as well for mathematical physics as for geometry.

A contemporary of Moebius and Pluecker, Magnus believed he had found the most general transformation which makes a point correspond to a point, but the researches of Cremona show us that the transformation of Magnus is only the first term of a series of birational transformations which the great Italian geometer teaches us to determine methodically, at least for the figures of plane geometry.

The Cremona transformations long retained a great interest, though later researches have shown us that they reduce always to a series of successive applications of the transformation of Magnus.

VI

All the works we have enumerated, others to which we shall return later, find their origin and, in some sort, their first motive in the conceptions of modern geometry; but the moment has come to indicate rapidly another source of great advances for geometric studies. Legendre's theory of elliptic functions, too much neglected by the French geometers, is developed and extended by Abel and Jacobi. With these great geometers, soon followed by Riemann and Weierstrass, the theory of Abelian functions which, later, algebra would try to follow solely with its own resources, brought to the geometry of curves and surfaces a contribution whose importance will continue to grow.

Already, Jacobi had employed the analysis of elliptic functions in the demonstration of Poncelet's celebrated theorems on inscribed and circumscribed polygons, inaugurating thus a chapter since enriched by a multitude of elegant results; he had obtained also, by methods pertaining to geometry, the integration of Abelian equations.

But it was Clebsch who first showed in a long series of works all the importance of the notion of *deficiency* (*Geschlecht, genre*) of a curve, due to Abel and Riemann, in developing a crowd of results and elegant solutions that the employment of Abelian integrals would seem, so simple was it, to connect with their veritable point of departure.

The study of points of inflection of curves of the third order, that of double tangents of curves of the fourth order, and, in general, the theory of osculation on which the ancients and the moderns had so often practiced, were connected with the beautiful problem of the division of elliptic functions and Abelian functions.

In one of his memoirs, Clebsch had studied the curves which are *rational* or of deficiency zero; this led him, toward the end of his too short life, to envisage what may be called also *rational* surfaces, those which can be simply represented by a plane. This was a vast field for research, opened already for the elementary cases by Chasles, and in which Clebsch was followed by Cremona and many other savants. It was on this occasion that Cremona, generalizing his researches on plane geometry, made known not indeed the totality of birational transformations of space, but certain of the most interesting among these transformations.

The extension of the notion of deficiency to algebraic surfaces is already commenced; already also works of high value have shown that the theory of integrals, simple or multiple, of algebraic differentials will find, in the study of surfaces as in that of curves, an ample field of important applications; but it is not proper for the reporter on geometry to dilate on this subject.

VII

While thus were constituted the mixed methods whose principal applications we have just indicated, the pure geometers were not inactive. Poinsot, the creator of the theory of couples, developed, by a method purely geometric, "that, where one never for a moment loses from view the object of the research," the theory of the rotation of a solid body that the researches of d'Alembert, Euler, and Lagrange seemed to have exhausted; Chasles made a precious contribution to kinematic by his beautiful theorems on the displacement of a solid body, which have since been extended by other elegant methods to the case where the motion has divers degrees of freedom. He made known those beautiful propositions on attraction in general, which figure without disadvantage beside those of Green and Gauss. Chasles and Steiner met in the study of the attraction of ellipsoids and showed thus once more that geometry has its designated place in the highest questions of the integral calculus.

Steiner did not disdain at the same time to occupy himself with the elementary parts of geometry. His researches on the contacts of circles and conics, on isoperimetric problems, on parallel surfaces, on the centre of gravity of curvature, excited the admiration of all by their simplicity and their depth.

Chasles introduced his principle of correspondence between two variable objects which has given birth to so many applications; but here analysis retook its place to study the principle in its essence, make it precise and generalize it.

It was the same concerning the famous theory of *characteristics* and the numerous researches of de Jonquières, Chasles, Cremona, and still others, which gave the foundations of a new branch of the science, *Enumerative Geometry*.

During many years, the celebrated postulate of Chasles was admitted without any objection: a crowd of geometers believed they had established it in a manner irrefutable.

But, as Zeuthen then said, it is very difficult to recognize whether, in demonstrations of this sort, there does not exist always some weak point that their author has not perceived; and, in fact, Halphen, after fruitless efforts, crowned finally all these researches by clearly indicating in what cases the postulate of Chasles may be admitted and in what cases it must be rejected.

VIII

Such are the principal works which restored geometric synthesis to honor and assured to it, in the course of the last century, the place belonging to it in mathematical research. Numerous and illustrious workers took part in this great geometric movement, but we must recognize that its chiefs and leaders were Chasles and Steiner. So brilliant were their marvelous discoveries that they threw into the shade, at least momentarily, the publications of other modest geometers, less preoccupied perhaps in finding brilliant applications, fitted to evoke love for geometry than to establish this science itself on an absolutely solid foundation. Their works have received perhaps a recompense more tardy, but their influence grows each day; it will assuredly increase still more. To pass them over in silence would be without doubt to neglect one of the principal factors which will enter into future researches. We allude at this moment above all to von Staudt. His geometric works were published in two books of great interest: the *Geometrie der Lage*, issued in 1847, and the *Beitrage zur Geometrie der Lage*, published in 1856, that is to say, four years after the *Géométrie supérieure*. Chasles, as we have seen, had devoted himself to constituting a body of doctrine independent of Descartes's analysis and had not completely succeeded. We have already indicated one of the criticisms that can be made upon this system: the imaginary elements are there defined only by their symmetric functions, which necessarily exclude them from a multitude of researches. On the other hand, the constant employment of cross-ratio, of transversals, and of involution, which requires frequent analytic transformations, gives to the *Géométrie supérieure* a character almost exclusively metric which removes it notably from the methods of Poncelet. Returning to these methods, von Staudt devoted himself to constituting a geometry freed from all metric relation and resting exclusively on relations of situation.

This is the spirit in which was conceived his first work, the *Geometrie der Lage* of 1847. The author there takes as point of departure the harmonic properties of the complete quadrilateral and those of homologic triangles, demonstrated uniquely by considerations of geometry of three dimensions, analogous to those of which the school of Monge made such frequent use.

In this first part of his work, von Staudt neglected entirely imaginary elements. It is only in the *Beitrage*, his second work, that he succeeds, by a very original extension of the method of Chasles, in defining geometrically an isolated imaginary element and distinguishing it from its conjugate.

This extension, although rigorous, is difficult and very abstract. It may be defined in substance as follows: Two conjugate imaginary points may always be considered as the double points of an involution on a real straight; and just as one passes from an imaginary to its conjugate by changing i into $-i$, so one may distinguish the two imaginary points by making correspond to each of them one of the two different senses which may be attributed to the straight. In this there is something a little artificial; the development of the theory erected on such foundations is necessarily complicated. By methods purely projective, von Staudt establishes a calculus of cross-ratios of the most general imaginary elements. Like all geometry, the projective geometry employs the notion of order and order engenders number; we are not astonished therefore that von Staudt has been able to constitute his calculus; but we must admire the ingenuity displayed in attaining it. In spite of the efforts of distinguished geometers who have essayed to simplify its exposition, we fear that this part of the geometry of von Staudt, like the geometry otherwise so interesting of the profound thinker Grassmann, cannot prevail against the analytical methods which have won to-day favor almost universal. Life is short; geometers know and also practice the principle of least action. Despite these fears, which should discourage no one, it seems to us that under the first form given it by von Staudt, projective geometry must become the necessary companion of descriptive geometry, that it is called to renovate this geometry in its spirit, its procedures, and its applications.

This has already been comprehended in many countries, and notably in Italy, where the great geometer Cremona did not disdain to write for the schools an elementary treatise on projective geometry.

IX

In the preceding articles, we have essayed to follow and bring out clearly the most remote consequences of the methods of Monge and Poncelet. In creating tangential coördinates and homogeneous coördinates, Pluecker seemed to have exhausted all that the method of projections and that of reciprocal polars give to analysis.

It remained for him, toward the end of his life, to return to his first researches to give them an extension enlarging to an unexpected degree the domain of geometry.

Preceded by innumerable researches on systems of straight lines, due to Poinsot, Moebius, Chasles, Dupin, Malus, Hamilton, Krummer, Transon, above all to Cayley, who first introduced the notion of the coördinates of the straight, researches originating perhaps in statics and kinematics, perhaps in geometrical optics, Pluecker's geometry of the straight line will always be regarded as the part of his work where are met the newest and most interesting ideas.

Pluecker first set up a methodic study of the straight line, which already is important, but that is nothing beside what he discovered. It is sometimes said that the principle of duality shows that the plane as well as the point may be considered as a space element. That is true; but in adding the straight line to the plane and point as possible space element, Pluecker was led to recognize that any curve, any surface, may also be considered as space element, and so was born a new geometry which already has inspired a great number of works, which will raise up still more in the future.

A beautiful discovery, of which we shall speak further on, has already connected the geometry of spheres with that of straight lines and permits the introduction of the notion of coördinates of a sphere.

The theory of systems of circles is already commenced; it will be developed without doubt when one wishes to study the representation, which we owe to Laguerre, of an imaginary point in space by an oriented circle.

But before expounding the development of these new ideas which have vivified the infinitesimal methods of Monge, it is necessary to go back to take up the history of branches of geometry that we have neglected until now.

X

Among the works of the school of Monge, we have hitherto confined ourselves to the consideration of those connected with *finite* geometry; but certain of the disciples of Monge devoted themselves above all to developing the new notions of infinitesimal geometry applied by their master to curves of double curvature, to lines of curvature, to the generation of surfaces, notions expounded at least in part in the *Application de l'Analyse à la Géométrie*. Among these we must cite Lancret, author of beautiful works on skew curves, and above all Charles Dupin, the only one perhaps who followed all the paths opened by Monge.

Among other works, we owe to Dupin two volumes Monge would not have hesitated to sign: *Les Développements de Géométrie pure*, issued in 1813, and *Les Applications de Géométrie et de Mécanique*, dating from 1822.

There we find the notion of *indicatrix*, which was to renovate, after Euler and Meunier, all the theory of curvature, that of conjugate tangents, of asymptotic lines which have taken so important a place in recent researches. Nor should we forget the determination of the surface of which all the lines of curvature are circles, nor above all the memoir on triple systems of orthogonal surfaces where is found, together with the discovery of the triple system formed by surfaces of the second degree, the celebrated theorem to which the name of Dupin will remain attached.

Under the influence of these works and of the renaissance of synthetic methods, the geometry of infinitesimals retook in all researches the place Lagrange had wished to take away from it forever.

Singular thing, the geometric methods thus restored were to receive the most vivid impulse in consequence of the publication of a memoir which, at least at first blush, would appear connected with the purest analysis; we mean the celebrated paper of Gauss, *Disquisitiones generales circa superficies curvas*, which was presented in 1827 to the Göttingen Society, and whose appearance marked, one may say, a decisive date in the history of infinitesimal geometry.

From this moment, the infinitesimal method took in France a free scope before unknown.

Frenet, Bertrand, Molins, J. A. Serret, Bouquet, Puiseux, Ossian Bonnet, Paul Serret, develop the theory of skew curves. Liouville, Chasles, Minding, join them to pursue the methodic study of the memoir of Gauss.

The integration made by Jacobi of the differential equation of the geodesic lines of the ellipsoid started a great number of researches. At the same time the problems studied in the *Application de l'Analyse* of Monge were greatly developed.

The determination of all the surfaces having their lines of curvature plane or spheric completed in the happiest manner certain partial results already obtained by Monge.

At this moment, one of the most penetrating of geometers, according to the judgment of Jacobi, Gabriel Lamé, who, like Charles Sturm, had commenced with pure geometry and had already made to this science contributions the most interesting by a little book published in 1817 and by memoirs inserted in the *Annales* of Gergonne, utilized the results obtained by Dupin and Binet on the system of confocal surfaces of the second degree, and, rising to the idea of curvilinear coördinates in space, became the creator of a wholly new theory destined to receive in mathematical physics the most varied applications.

XI

Here again, in this infinitesimal branch of geometry are found the two tendencies we have pointed out *à propos* of the geometry of finite quantities.

Some, among whom must be placed J. Bertrand and O. Bonnet, wish to constitute an independent method resting directly on the employment of infinitesimals. The grand *Traité de Calcul différentiel*, of Bertrand, contains many chapters on the theory of curves and of surfaces, which are, in some sort, the illustration of this conception.

Others follow the usual analytic ways, being only intent to clearly recognize and put in evidence the elements which figure in the first plan. Thus did Lamé in introducing his theory of *differential parameters*. Thus did Beltrami in extending with great ingenuity the employment of these differential invariants to the case of two independent variables, that is to say, to the study of surfaces.

It seems that to-day is accepted a mixed method whose origin is found in the works of Ribaucour, under the name *périmorphie*. The rectangular axes of analytic geometry are retained, but made mobile and attached as seems best to the system to be studied. Thus disappear most of the objections which have been made to the method of coördinates. The advantages of what is sometimes called *intrinsic* geometry are united to those resulting from the use of the regular analysis. Besides, this analysis is by no means abandoned; the complications of calculation which it almost always carries with it, in its applications to the study of surfaces and rectilinear coördinates, usually disappear if one employs the notion on the invariants and the covariants of quadratic powers of differentials which we owe to the researches of Lipschitz and Christoffel, inspired by Riemann's studies on the non-Euclidean geometry.

XII

The results of so many labors were not long in coming. The notion of geodesic curvature which Gauss already possessed, but without having published it, was given by Bonnet and Liouville; the theory of surfaces of which the radii of curvature are functions one of the other, inaugurated in Germany by two propositions which would figure without disadvantage in the memoir of Gauss, was enriched by Ribaucour, Halphen, S. Lie, and others, with a multitude of propositions, some concerning these surfaces envisaged in a general manner; others applying to particular cases where the relation between the radii of curvature takes a form particularly simple; to minimal surfaces for example, and also to surfaces of constant curvature, positive or negative.

The minimal surfaces were the object of works which make of their study the most attractive chapter of infinitesimal geometry. The integration of their partial differential equation constitutes one of the most beautiful discoveries of Monge; but because of the imperfection of the theory of imaginaries, the great geometer could not get from its formulas any mode of generation of these surfaces, nor even any particular surface. We will not here retrace the detailed history which we have presented in our *Leçons sur la théorie des surfaces ;* but it is proper to recall the fundamental researches of Bonnet which have given us, in particular, the notion of *surfaces associated with a given surface,* the formulas of Weierstrass which establish a close bond between the minimal surfaces and the functions of a complex variable, the researches of Lie by which it was established that just the formulas of Monge can to-day serve as foundation for a fruitful study of minimal surfaces.

In seeking to determine the minimal surfaces of smallest classes or degrees, we were led to the notion of double minimal surfaces which is dependent on *analysis situs.*

Three problems of unequal importance have been studied in this theory.

The first, relative to the determination of minimal surfaces inscribed along a given contour in a developable equally given, was solved by celebrated formulas which have led to a great number of propositions. For example, every straight traced on such a surface is an axis of symmetry.

The second, set by S. Lie, concerns the determination of all the algebraic minimal surfaces inscribed in an algebraic developable, without the curve of contact being given. It also has been entirely elucidated.

The third and the most difficult is what the physicists solve experimentally, by plunging a closed contour into a solution of glycerine. It concerns the determination of the minimal surface passing through a given contour.

The solution of this problem evidently surpasses the resources of geometry. Thanks to the resources of the highest analysis, it has been solved for particular contours in the celebrated memoir of Riemann and in the profound researches which have followed or accompanied this memoir.

For the most general contour, its study has been brilliantly begun; it will be continued by our successors.

After the minimal surfaces, the surfaces of constant curvature attracted the attention of geometers. An ingenious remark of Bonnet connects with each other the surfaces of which one or the other of the two curvatures, mean curvature or total curvature, is constant.

Bour announced that the partial differential equation of surfaces of constant curvature could be completely integrated. This result has not been secured; it would seem even very doubtful if we consider a research where S. Lie has essayed in vain to apply a general method of integration of partial differential equations to the particular equation of surfaces of constant curvature.

But, if it is impossible to determine in finite terms all these surfaces, it has at least been possible to obtain certain of them, characterized by special properties, such as that of having their lines of curvature plane or spheric; and it has been shown, by employing a method which succeeds in many other problems, that from every surface of constant curvature may be derived an infinity of other surfaces of the same nature, by employing operations clearly defined which require only quadratures.

The theory of the deformation of surfaces in the sense of Gauss has been also much enriched. We owe to Minding and to Bour the detailed study of that special deformation of ruled surfaces which leaves the generators rectilineal. If we have not been able, as has been said, to determine the surfaces applicable on the sphere, other surfaces of the second degree have been attacked with more success, and, in particular, the paraboloid of revolution.

The systematic study of the deformation of general surfaces of the second degree is already entered upon; it is one of those which will give shortly the most important results.

The theory of infinitesimal deformation constitutes to-day one of the most finished chapters of geometry. It is the first somewhat extended application of a general method which seems to have a great future.

Being given a system of differential or partial differential equations, suitable to determine a certain number of unknowns, it is advantageous to associate with it a system of equations which we have called *auxiliary system*, and which determines the systems of solutions infinitely near any given system of solutions. The auxiliary system being necessarily linear, its employment in all researches gives precious light on the properties of the proposed system and on the possibility of obtaining its integration.

The theory of lines of curvature and of asymptotic lines has been notably extended. Not only have been determined these two series of lines for particular surfaces such as the tetrahedral surfaces of Lamé; but also, in developing Moutard's results relative to a particular class of linear partial differential equations of the second order, it proved possible to generalize all that had been obtained for surfaces with lines of curvature plane or spheric, in determining completely all the classes of surfaces for which could be solved the problem of *spheric representation*.

Just so has been solved the correlative problem relative to asymptotic lines in making known all the surfaces of which the infinitesimal deformation can be determined in finite terms. Here is a vast field for research whose exploration is scarcely begun.

The infinitesimal study of rectilinear congruences, already commenced long ago by Dupin, Bertrand, Hamilton, Kummer, has come to intermingle in all these researches. Ribaucour, who has taken in it a preponderant part, studied particular classes of rectilinear congruences and, in particular, the congruences called *isotropes*, which intervene in the happiest way in the study of minimal surfaces.

The triply orthogonal systems which Lamé used in mathematical physics have become the object of systematic researches. Cayley was the first to form the partial differential equation of the third order on which the general solution of this problem was made to depend.

The system of homofocal surfaces of the second degree has been generalized and has given birth to that theory of general *cyclides* in which may be employed at the same time the resources of metric geometry, of projective geometry, and of infinitesimal geometry. Many other orthogonal systems have been made known. Among these it is proper to signalize the *cyclic* systems of Ribaucour, for which one of the three families admits circles as orthogonal trajectories and the more general systems for which these orthogonal trajectories are simply plane curves.

The systematic employment of imaginaries, which we must be careful not to exclude from geometry, has permitted the connection of all these determinations with the study of the finite deformation of a particular surface.

Among the methods which have permitted the establishment of all these results, it is proper to note the systematic employment of linear partial differential equations of the second order and of systems formed of such equations. The most recent researches show that this employment is destined to renovate most of the theories.

Infinitesimal geometry could not neglect the study of the two fundamental problems set it by the calculus of variations.

The problem of the shortest path on a surface was the object of masterly studies by Jacobi and by Ossian Bonnet. The study of geodesic lines has been followed up; we have learned to determine them for new surfaces. The theory of *ensembles* has come to permit the following of these lines in their course on a given surface.

The solution of a problem relative to the representation of two surfaces one on the other has greatly increased the interest of discoveries of Jacobi and of Liouville relative to a particular class of surfaces of which the geodesic lines could be determined. The results concerning this particular case led to the examination of a new question: to investigate all the problems of the calculus of variations of which the solution is given by curves satisfying a given differential equation.

Finally, the methods of Jacobi have been extended to space of three dimensions and applied to the solution of a question which presented the greatest difficulties: the study of properties of minimum appertaining to the minimal surface passing through a given contour.

XIII

Among the inventors who have contributed to the development of infinitesimal geometry, Sophus Lie distinguishes himself by many capital discoveries which place him in the first rank.

He was not one of those who show from infancy the most characteristic aptitudes, and at the moment of quitting the University of Christiania in 1865, he still hesitated between philology and mathematics.

It was the works of Pluecker which gave him for the first time full consciousness of his true calling.

He published in 1869 a first work on the interpretation of imaginaries in geometry, and from 1870 he was in possession of the directing ideas of his whole career. I had at this time the pleasure of seeing him often, of entertaining him at Paris, where he had come with his friend F. Klein.

A course by M. Sylow followed by Lie had revealed to him all the importance of the theory of substitutions; the two friends studied this theory in the great treatise of C. Jordan; they were fully conscious of the important rôle it was called on to play in so many branches of mathematical science where it had not yet been applied.

They have both had the good fortune to contribute by their works to impress upon mathematical studies the direction which to them appeared the best.

In 1870, Sophus Lie presented to the Academy of Sciences of Paris a discovery extremely interesting. Nothing bears less resemblance to a sphere than a straight line, and yet Lie had imagined a singular transformation which made a sphere correspond to a straight line, and permitted, consequently, the connecting of every proposition relative to straight lines with a proposition relating to spheres, and *vice versa*.

In this so curious method of transformation, each property relative to the lines of curvature of a surface furnishes a proposition relative to the asymptotic lines of the surface attained.

The name of Lie will remain attached to these deep-lying relations which join to one another the straight line and the sphere, those two essential and fundamental elements of geometric research. He developed them in a memoir full of new ideas which appeared in 1872.

The works which followed this brilliant début of Lie fully confirmed the hopes it had aroused. Pluecker's conception relative to the generation of space by straight lines, by curves or surfaces arbitrarily chosen, opens to the theory of algebraic forms a field which has not yet been explored, which Clebsch scarcely began to recognize and settle the boundaries of. But, from the side of infinitesimal geometry, this conception has been given its full value by Sophus Lie. The great Norwegian geometer was able to find in it first the notion of congruences and complexes of curves, and afterward that of *contact transformations* of which he had found, for the case of the plane, the first germ in Pluecker. The study of these transformations led him to perfect, at the same time with M. Mayer, the methods of integration which Jacobi had instituted for partial differential equations of the first order; but above all it threw the most brilliant light on the most difficult and the most obscure parts of the theories relative to partial differential equations of higher order. It permitted Lie, in particular, to indicate all the cases in which the method of characteristics of Monge is fully applicable to equations of the second order with two independent variables.

In continuing the study of these special transformations, Lie was led to construct progressively his masterly theory of continuous groups of transformations and to put in evidence the very important rôle that the notion of group plays in geometry. Among the essential elements of his researches, it is proper to signalize the infinitesimal transformations, of which the idea belongs exclusively to him.

Three great books published under his direction by able and devoted collaborators contain the essential part of his works and their applications to the theory of integration, to that of complex units and to the non-Euclidean geometry.

XIV

By an indirect way I have arrived at that non-Euclidean geometry the study of which takes in the researches of geometers a place which grows greater each day.

If I were the only one to talk with you about geometry, I should take pleasure in recalling to you all that has been done on this subject since Euclid or at least from Legendre to our days.

Envisaged successively by the greatest geometers of the last century, the question has progressively enlarged.

It commenced with the celebrated *postulatum* relative to parallels; it ends with the totality of geometric axioms.

The *Elements* of Euclid, which have withstood the action of so many centuries, will have at least the honor before ending of arousing a long series of works admirably enchained which will contribute, in the most effective way, to the progress of mathematics, at the same time that they furnish to the philosophers the most precise and the most solid points of departure for the study of the origin and of the formation of our cognitions.

I am assured in advance that my distinguished collaborator will not forget, among the problems of the present time, this one, which is perhaps the most important, and with which he has occupied himself with so much success; and I leave to him the task of developing it with all the amplitude which it assuredly merits.

I have just spoken of the elements of geometry. They have received in the last hundred years extensions which must not be forgotten. The theory of polyhedrons has been enriched by the beautiful discoveries of Poinsot on the star polyhedrons and those of Moebius on polyhedrons with a single face. The methods of transformation have enlarged the exposition. We may say to-day that the first book contains the theory of translation and of symmetry, that the second amounts to the theory of rotation and of displacement, that the third rest on homothety and inversion. But it must be recognized that it is due to analysis that the *Elements* have been enriched by their most beautiful propositions.

It is to the highest analysis that we owe the inscription of regular polygons of seventeen sides and analogous polygons. To it we owe the demonstrations, so long sought, of the impossibility of the quadrature of the circle, of the impossibility of certain geometric constructions with the aid of the ruler and the compasses; and to it finally we owe the first rigorous demonstrations of the properties of maximum and of minimum of the sphere. It will belong to geometry to enter upon this ground where analysis has preceded it.

What will be the elements of geometry in the course of the century which has just commenced? Will there be a single elementary book of geometry? It is perhaps America, with its schools free from all programme and from all tradition, which will give us the best solution of this important and difficult question.

Von Staudt has sometimes been called the *Euclid of the nineteenth century;* I would prefer to call him the *Euclid of projective geometry;* but is projective geometry, interesting though it may be, destined to furnish the unique foundation of the future elements?

XV

The moment has come to close this over-long recital, and yet there is a crowd of interesting researches that I have been, so to say, forced to neglect.

I would have loved to talk with you about those geometries of any number of dimensions of which the notion goes back to the first days of algebra, but of which the systematic study was commenced only sixty years ago by Cayley and by Cauchy. This kind of researches has found favor in your country and I need not recall that our illustrious president, after having shown himself the worthy successor of Laplace and Le Verrier, in a space which he considers with us as being endowed with three dimensions, has not disdained to publish, in the *American Journal,* considerations of great interest on the geometries of n dimensions.

A single objection can be made to studies of this sort, and was already formulated by Poisson: the absence of all real foundation, of all *substratum* permitting the presentation, under aspects visible and in some sort palpable, of the results obtained.

The extension of the methods of descriptive geometry, and above all the employment of Pluecker's conceptions on the generation of space, will contribute to take away from this objection much of its force.

I would have liked to speak to you also of the method of equipollences, of which we find the germ in the posthumous works of Gauss, of Hamilton's quaternions, of Grassmann's methods, and in general of systems of complex units, of the *analysis situs*, so intimately connected with the theory of functions, of the geometry called *kinematic*, of the theory of abaci, of geometrography, of the applications of geometry to natural philosophy or to the arts. But I fear, if I branched out beyond measure, some analyst, as has happened before, would accuse geometry of wishing to monopolize everything.

My admiration for analysis, grown so fruitful and so powerful in our time, would not permit me to conceive such a thought. But if some reproach of this sort could be formulated to-day, it is not to geometry, it is to analysis it would be proper, I believe, to address it. The circle in which the mathematical studies appeared to be inclosed at the beginning of the nineteenth century has been broken on all sides.

The old problems present themselves to us under a new form, new problems offer themselves, whose study occupies legions of workers.

The number of those who cultivate pure geometry has become prodigiously restricted. Therein is a danger against which it is important to provide. We must not forget that, if analysis has acquired means of investigation which it lacked heretofore, it owes them in great part to the conceptions introduced by the geometers. Geometry must not remain in some sort entombed in its triumph. It is in its school we have learned; our successors must learn never to be blindly proud of methods too general, to envisage the questions in themselves and to find, in the conditions particular to each problem, perhaps a direct way towards a solution, perhaps the means of applying in an appropriate manner the general procedures which every science should gather.

As Chasles said at the beginning of the *Aperçu historique*, "The doctrines of pure geometry offer often, and in a multitude of questions, that simple and natural way which, penetrating to the very source of the truths, lays bare the mysterious chain which binds them to each other and makes us know them individually in the way most luminous and most complete."

Cultivate therefore geometry, which has its own advantages, without wishing, on all points, to make it equal to its rival.

For the rest, if we were tempted to neglect it, it would soon find in the applications of mathematics, as it did once before, means to rise up again and develop itself anew. It is like the giant Antæus who recovered his strength in touching the earth.

THE PRESENT PROBLEMS OF GEOMETRY

BY DR. EDWARD KASNER

[**Edward Kasner,** Instructor in Mathematics, Columbia University. **b.** New York City, 1877. B.S. College of the City of New York, 1896; A.M. Columbia University, 1899; Ph.D. *ibid.* 1899. Post-graduate, Fellow in Mathematics, Columbia University, 1897–99; Student, University of Göttingen, 1899–1900; Tutor in Mathematics, Columbia University, 1900–05; Instructor, 1905; Member American Mathematical Society; Fellow American Association for Advancement of Science. **Associate editor,** *Transactions American Mathematical Society.*]

In spite of the richness and power of recent geometry, it is noticeable that the geometer himself has become more modest. It was the ambition of Descartes and Leibnitz to discover universal methods, applicable to all conceivable questions; later, the *Ausdehnungslehre* of Grassmann and the quaternion theory of Hamilton were believed by their devotees to be ultimate geometric analyses; and Chasles attributed to the principles of duality and homography the same rôle in the domain of pure space as that of the law of gravitation in celestial mechanics. To-day, the mathematician admits the existence and the necessity of many theories, many geometries, each appealing to certain interests, each to be developed by the most appropriate methods; and he realizes that, no matter how large his conceptions and how powerful his methods, they will be replaced before long by others larger and more powerful.

Aside from the conceivability of other spaces with just as self-consistent properties as those of the so-called ordinary space, such diverse theories arise, in the first place, on account of the variety of objects demanding consideration, — curves, surfaces, congruences and complexes, correspondences, fields of differential elements, and so on in endless profusion. The totality of configurations is indeed not thinkable in the sense of an ordinary assemblage, since the totality itself would have to be admitted as a configuration, that is, an element of the assemblage.

However, more essential in most respects than the diversity in the material treated is the diversity in the points of view from which it may be regarded. Even the simplest figure, a triangle or a circle, has an infinity of properties — indeed, recalling the unity of the physical world, the complete study of a single figure would involve its relations to all other figures and thus not be distinguishable from the whole of geometry. For the past three decades the ruling thought in this connection has been the principle (associated with the names of Klein and Lie) that the properties which are deemed of interest in the various geometric theories may be classified according to the groups of transformations which leave those properties unchanged. Thus almost all discussions on algebraic curves are connected with the group of displacements (more properly the so-called principal group), or the group of projective transformations, or the group of birational transformations; and the distinction between such theories is more fundamental than the distinction between the theories of curves, of surfaces, and of complexes.

Historically, the advance has been, in general, from small to larger groups of transformations. The change thus produced may be likened to the varying appearance of a painting, at first viewed closely in all its details, then at a distance in its significant features. The analogy also suggests the desirability of viewing an object from several standpoints, of studying geometric configurations with respect to various groups. It is indeed true, though in a necessarily somewhat vague sense, that the more essential properties are those invariant under the more extensive groups; and it is to be expected that such groups will play a predominating rôle in the not far distant future.

The domain of geometry occupies a position, as indicated in the programme of the Congress, intermediate between the domain of analysis on the one hand and of mathematical physics on the other; and in its development it continually encroaches upon these adjacent fields. The concepts of transformation and invariant, the algebraic curve, the space of n dimensions, owe their origin primarily to the suggestions of analysis; while the null-system, the theory of vector fields, the questions connected with the applicability and deformation of surfaces, have their source in mechanics. It is true that some mathematicians regard the discussion of point sets, for example, as belonging exclusively to the theory of functions, and others look upon the composition of displacements as a part of mechanics. While such considerations show the difficulty, if not impossibility, of drawing strict limits about any science, it is to be observed that the consequent lack of definiteness, deplored though it be by the formalist, is more than compensated by the fact that such overlapping is actually the principal means by which the different realms of knowledge are bound together.

If a mathematician of the past, an Archimedes or even a Descartes, could view the field of geometry in its present condition, the first feature to impress him would be its lack of concreteness. There are whole classes of geometric theories which proceed, not merely without models and diagrams, but without the slightest (apparent) use of the spatial intuition. In the main this is due, of course, to the power of the analytic instruments of investigation as compared with the purely geometric. The formulas move in advance of thought, while the intuition often lags behind; in the oft-quoted words of d'Alembert, "L'algèbre est généreuse, elle donne souvent plus qu'on lui demande." As the field of research widens, as we proceed from the simple and definite to the more refined and general, we naturally cease to picture our processes and even our results. It is often necessary to close our eyes and go forward blindly if we wish to advance at all. But admitting the inevitableness of such a change in the spirit of any science, one may still question the attitude of the geometer who rests content with his blindness, who does not at least strive to intensify and enlarge the intuition. Has not such an intensification and enlargement been the main contribution of geometry to the race, its very *raison d'être* as a separate part of mathematics, and is there any ground for regarding this service as completed?

From the point of view here referred to, a problem is not to be regarded as completely solved until we are in position to construct a model of the solution, or at least to conceive of such a construction. This requires the interpretation, not merely of the results of a geometric investigation, but also, as far as possible, of the intermediate processes — an attitude illustrated most strikingly in the works of Lie. This duty of the geometer, to make the ground won by means of analysis really geometric, and as far as possible concretely intuitive, is the source of many problems of to-day, a few of which will be referred to in the course of this address.

The tendency to generalization, so characteristic of modern geometry, is counteracted in many cases by this desire for the concrete, in others by the desire for the exact, the rigorous (not to be confused with the rigid). The great mathematicians have acted on the principle "Devinez avant de démontrer," and it is certainly true that almost all important discoveries are made in this fashion. But while the demonstration comes after the discovery, it cannot therefore be disregarded. The spirit of rigor, which tended at first to the arithmetization of all mathematics and now tends to its exhibition in terms of pure logic, has always been more prominent in analysis than in geometry. Absolute rigor may be unattainable, but it cannot be denied that much remains to be done by the geometers, judging even by elementary standards. We need refer only to the loose proofs based upon the invaluable but insufficient enumeration of constants, the so-called principle of the conservation of number, and the discussions which confine themselves to the "general case." Examples abound in every field of geometry. The theorem announced by Chasles concerning the number of conics satisfying five arbitrary conditions was proved by such masters as Clebsch and Halphen before examples invalidating the result were devised. Picard recently called attention to the need of a new proof of Noether's theorem that upon the general algebraic surface of degree greater than three every algebraic curve is a complete intersection with another algebraic surface. The considerations given by Noether render the result highly probable, but do not constitute a complete proof; while the exact meaning of the term general can be determined only from the context.

The reaction against such loose methods is represented by Study [1] in algebraic geometry, and Hilbert in differential geometry. The tendency of a considerable portion of recent work is towards the exhaustive treatment of definite questions, including the consideration of the special or degenerate cases ordinarily passed over as unimportant. Another aspect of the same tendency is the discussion of converses of familiar problems, with the object of obtaining conditions at once necessary and sufficient, that is, completely characteristic results.[2]

Another set of problems is suggested by the relation of geometry to physics. It is the duty of the geometer to abstract from the physical sciences those domains which may be expressed in terms of pure space, to study the geometric foundations (or, as some would put it, the skeletons) of the various branches of mechanics and physics. Most of the actual advance, it is true, has hitherto come from the physicists themselves, but undoubtedly the time has arrived for more systematic discussions by the mathematicians. In addition to the importance which is due to possible applications of such work, it is to be noticed that we meet, in this way, configurations as interesting and remarkable as those created by the geometer's imagination. Even in this field, one is tempted to remark, truth is stranger than fiction.

We have now considered, briefly and inadequately, some of the leading ideals and influences which are at work towards both the widening and the deepening of geometry in general; and turn to our proper topic, a survey of the leading problems or groups of problems in certain selected (but it is hoped representative) fields of contemporaneous investigation.

Foundations

The most striking development of geometry during the past decade relates to the critical revision of its foundations, more precisely, its *logical* foundations. There are, of course, other points of view, for example, the physical, the physiological, the psychological, the metaphysical, but the interest of mathematicians has been confined to the purely logical aspect. The main results in this direction are due to Peano and his co-workers; but the whole field was first brought prominently to the attention of the mathematical world by the appearance, five years ago, of Hilbert's elegant *Festschrift*.

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[1] "[Es ist eine] tief eingewurzelte Gewohnheit vieler Geometer, Satze zu formulieren, die 'im allgemeinen' gelten sollen, d. h. einen klaren Sinn *überhaupt nicht* haben, zudem noch häufig als *allgemein* gültig hingestellt oder mangelhaft begründet werden. [Dies Verfahren wird], trotz etwanigen Verweisungen auf Träger sehr berühmter Namen, späteren Geschlechtern sicher als ganz unzulässig erscheinen, scheint aber in unserem 'kritischen' Zeitalter von vielen als eine *berechtigte Eigentümlichkeit der Geometrie* betrachtet zu werden . . ." *Jahr. Deut. Math.-Ver.*, vol. XI (1902), p. 100.

[2] As an example may be mentioned the theorem of Malus and Dupin, known for almost a century, that the rays emanating from a point are converted, by any refraction, into a normal congruence. Quite recently, Levi-Civittà succeeded in showing that this property is characteristic; that is, any normal congruence may be refracted into a bundle.

The central problem is to lay down a system of primitive (undefined) concepts or symbols and primitive (unproved) propositions or postulates, from which the whole body of geometry (that is, the geometry considered) shall follow by purely deductive processes. No appeal to intuition is then necessary. "We might put the axioms into a reasoning apparatus like the logical machine of Stanley Jevons, and see all geometry come out of it" (Poincaré). Such a system of concepts and postulates may be obtained in a great (indeed endless) variety of ways: the main question, at present, concerns the comparison of various systems, and the possibility of imposing limitations so as to obtain a unique and perhaps simplest basis.

The first requirement of a system is that it shall be consistent. The postulates must be compatible with one another. No one has yet deduced contradictory results from the axioms of Euclid, but what is our guarantee that this will not happen in the future? The only method of answering this question which has suggested itself is the exhibition of some object (whose existence is admitted) which fulfills the conditions imposed by the postulates. Hilbert succeeded in constructing such an ideal object out of numbers; but remarks that the difficulty is merely transferred to the field of arithmetic. The most far-reaching result is the definition of number in terms of logical classes as given by Pieri and Russell; but no general agreement is yet to be expected in these discussions. Will the ultimate conclusion be the impossibility of a direct proof of compatibility?

More accessible is the question concerning the independence of postulates (and the analogous question of the irreducibility of concepts). Most of the work of the last few years has been concentrated on this point. In Hilbert's original system the various groups of axioms (relating respectively to combination, order, parallels, congruence, and continuity) are shown to be independent, but the discussion is not carried out completely for the individual axioms. In Dr. Veblen's recently published system of twelve postulates, each is proved independent of the remaining eleven.[1] This marks an advance, but, of course, it does not terminate the problem. In what respect does a group of propositions differ from what is termed a single proposition? Is it possible to define the notion of an absolutely simple postulate? The statement that any two points determine a straight line involves an infinity of statements, and its fulfillment for certain pairs of points may necessitate its fulfillment for all pairs. If in Euclid's system the postulate of parallels is replaced by the postulate concerning the sum of the angles of a triangle, a well-known example of such a reduction is obtained; for it is sufficient to assume the new postulate for a single triangle, the general result being then deducible. As other examples we may mention Peano's reduction of the Euclidean definition of the plane; and the definition of a collineation which demands, instead of the conversion of *all* straight lines into straight lines, the existence of four simply infinite systems of such straight lines.[1]

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[1] *Trans. Amer. Math. Soc.*, vol. v (1904).

These examples illustrate the difficulty, if not the impossibility, of formulating a really fundamental, that is, absolute standard of independence and irreducibility. It is probable that the guiding ideas will be obtained in the discussion of simpler deductive theories, in particular, the systems for numbers and groups.

Two features are especially prominent in the actual development of the body of geometry from its fundamental system. First, the consideration of what may be termed the collateral geometries, which arise by replacing one of the original postulates by its opposite, or otherwise varying the system. Such theories serve to show the limitation of that point of view which restricts the term general geometry (pangeometry) to the Euclidean and non-Euclidean geometries. The variety of possible abstract geometries is, of course, inexhaustible; this is the central fact brought to light by the exhibition of such systems as the non-Archimedean and the non-arguesian. In the second place, much valuable work is being done in discussing the various methods by which the same theorem may be deduced from the postulates, the ideal being to use as few of the postulates as possible. Here again the question of *simplicity* (simplest proof), though it baffles analysis, forces itself upon the attention.

Among the minor problems in this field, it is sufficient to consider that concerning the relation of the theory of volume to the axiom of continuity. This axiom need not be used in establishing the theory of areas of polygons; but after Dehn and others had proved the existence of polyhedra having the same volume though not decomposable into mutually congruent parts (even after the addition of congruent polyhedra), it was stated by Hilbert, and deemed evident generally, that reference to continuity could not be avoided in three dimensions. In a recent announcement [2] of Vahlen's forthcoming *Abstrakte Geometrie* this conclusion is declared unsound. It seems probable, however, that the difference is merely one concerning the interpretation to be given to the term continuity.

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[1] Together with certain continuity assumptions. Cf. *Bull. Amer. Math. Soc.*, vol. IX (1903), p. 545.

[2] *Jahr. Deut. Math.-Ver.*, vol. XIII (1904), p. 395.

The work on logical foundations has been confined almost entirely to the Euclidean and projective geometries. It is desirable, however, that other geometric theories should be treated in a similar deductive fashion. In particular, it is to be hoped that we shall soon have a really systematic foundation for the so-called inversion geometry, dealing with properties invariant under circular transformations. This theory is of interest, not only for its own sake and for its applications in function theory, but also because its study serves to free the mind from what is apt to become, without some check, slavery to the projective point of view.

The Curve Concept — Analysis Situs

Although curves and surfaces have constituted the almost exclusive material of the geometric investigation of the thirty centuries of which we have record, it can hardly be claimed that the concepts themselves have received their final analysis. Certain vague notions are suggested by the naïve intuition. It is the duty of mathematicians to create perfectly precise concepts which agree more or less closely with such intuitions, and at the same time, by the reaction of the concepts, to refine the intuition. The problem, evidently, is not at all determinate. It would be of interest to trace the evolution which has actually produced several distinct curve concepts defining more or less extensive classes of curves, agreeing in little beyond the possession of an infinite number of points.

The more familiar special concepts or classes of curves are defined in terms of the corresponding equation $y=f(x)$ or function $f(x)$. Such are, for example: (1) algebraic curves; (2) analytic curves; (3) graphs of functions possessing derivatives of all orders; (4) the curves considered in the usual discussions of infinitesimal geometry, in which the existence of first and second derivatives is assumed; (5) the so-called regular curves with a continuously turning tangent (except for a finite number of corners); (6) the so-called ordinary curves possessing a tangent and having only a finite number of oscillations (maxima and minima) in any finite interval; (7) curves with tangents; (8) the graphs of continuous functions.

How far are such distinctions accessible to the intuition? Of course there are limitations. For over two centuries, from Descartes to the publication of Weierstrass's classic example, the intuition of mathematicians declared the classes (7) and (8) to be identical. Still later it was found that such extraordinary (pathological or crinkly) curves may present themselves in class (7). However, even here partially successful attempts to connect with intuition have been made by Wiener, Hilbert, Schoenflies, Moore, and others.

Let us consider a simpler extension in the field of ordinary curves. If the function $f(x)$ is continuous except for a certain value of x

where there is an ordinary discontinuity, this is indicated by a break in the graph; if f is continuous, but the derivative f' has such a discontinuity, this shows itself by a sharp turn in the curve; if the discontinuity is only in the second derivative, there is a sudden change in the radius of curvature, which is, however, relatively difficult to observe from the figure; finally, if the third derivative is discontinuous, the effect upon the curve is no longer apparent. Does this mean that it is impossible to picture it? Does it not rather indicate a limitation in the usual geometric training which goes only as far as relations expressible in terms of tangency and curvature? For the interpretation of the third derivative it is necessary to consider say the osculating parabola at each point of the curve: in the case referred to, as we pass over the critical point, the tangent line and osculating circle change continuously, but there is a sudden change in the osculating parabola. If in fact our intuition were trained to picture osculating algebraic curves of all orders, it would detect a discontinuity in a derivative of any order. A partial equivalent would be the ability to picture the successive evolutes of a given curve; a complete equivalent would be the picturing of the successive slope curves $y=f'(x)$, $y=f''(x)$, etc. All this requires, evidently, only an increase in the intensity of our intuition, not a change in its nature.

This, however, would not apply to all questions. There are functions which, while possessing derivatives of all orders (then necessarily continuous), are not analytic (that is, not expressible by power series). What is it that distinguishes the analytic curves among this larger class? Is it possible to put the distinction in a form capable of assimilation by an idealized intuition? In short, what is the really geometric definition of an analytic curve?[1]

Much recent work in function theory has had for its point of departure a more general basis than the theory of curves, namely, the theory of sets or assemblages of points, with special reference to the notions of derived set and the various contents or areas. The geometry of point sets must indeed be regarded as one of the most important and promising in the whole field of mathematics. It receives its distinctive character, as compared with the general abstract theory of assemblages (*Mengenlehre*), from the fact that it operates not with all one-to-one correspondences, but with the group of *analysis situs*, the group of *continuous* one-to-one correspondences. From the point of view of the larger group, there is no distinction between a one-dimensional and a two- or many-dimensional continuum (Cantor). This is still the case if the correspondence is continuous but not one-to-one (Peano, 1890). In the domain of continuous one-to-one correspondence, however, spaces of different dimensions are not equivalent (Jürgens, 1899).

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[1] One method of attack would be the interpretation of Pringsheim's conditions; this requires not merely the individual derivative curves, but the limit of the system.

An important class of curves, much more general than those referred to above, consists of those point sets which are equivalent (in the sense of *analysis situs*) to the straight line or segment of a straight line. This is Hurwitz's simple and elegant geometric formulation of the concept originally treated analytically by Jordan, the most fundamental curve concept of to-day. The closed Jordan curves are defined in analogous fashion as equivalent to the perimeter of a square (or the circumference of a circle).

A curve of this kind divides the remaining points of the plane into two simply connected continua, an inside and an outside. The necessity for proof of this seemingly obvious result is seen from the fact that the Jordan class includes such extraordinary types as the curve with positive content constructed recently by Osgood.[1] Such a separation of the plane may, however, be thought about by other than Jordan curves: the concept of the boundary of a connected region gives perhaps the most extensive class of point sets which deserve to be called curve. Schoenflies proposes a definition for the idea of a simple closed curve which makes it appear as the natural extension, in a certain sense, of the polygon: a perfect set of points P which separates the plane into an exterior region E and an interior region I such that any E point can be connected with any I point by a path (*Polygonstrecke*) having only one point in common with P. This is in effect a converse of Jordan's theorem, and shows precisely how the Jordan curve is distinguished from other types of boundaries of connected regions.

These discussions are mentioned here simply as aspects of a really fundamental problem: the revision of the concepts and results of that division of geometry which has been variously termed *analysis situs*, theory of connection, topology, geometry of situation — a revision to be carried out in the light of the theory of assemblages.[2]

Algebraic Surfaces and Birational Transformations

After the demonstration of the power of the methods based upon projective transformation, — the chief contribution due to the geometers of the first half of the nineteenth century, — attempts were made to introduce other types of one-to-one correspondence or transformation into algebraic geometry; in particular the inversion of William Thomson and Liouville, and the quadratic transformation of Magnus. The general theory of such Cremona transformations was inaugurated by the Italian geometer in his memoir *Sulle tras-*

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[1] *Trans. Amer. Math. Soc.*, vol. IV (1903), p. 107.

[2] Cf. Schoenflies, *Math. Annalen*, vols. LVIII, LIX (1903, 1904).

formazioni geometriche delle figure piane, published in 1863. Within a few years, Clifford, Noether, and Rosanes, working independently, established the remarkable result that every Cremona transformation in a plane can be decomposed into a succession of quadratic transformations, thus bringing to light the fact that there are at bottom only two types of algebraic one-to-one correspondence, the homographic and the quadratic.[1]

The development of a corresponding theory in space has been one of the chief aims of the geometers of Italy, Germany, and England for the last thirty years, but the essential question of decomposition still remains unanswered. Is it possible to reduce the general Cremona transformation of space to a finite number of fundamental types?

In its application to the study of the properties of algebraic curves and surfaces, the theory of the Cremona transformation is usually merged in the more general theory of the birational transformation. By means of the latter, a correspondence is established which is one-to-one for the points of the particular figure considered and the transformed figure, but not for all the points of space. In the plane theory an important result is that a curve with the most complicated singularities can, by means of Cremona transformations, be converted into a curve whose only singularities are multiple points with distinct tangents (Noether); furthermore, by means of birational transformations, the singularities may be reduced to the very simplest type, ordinary double points (Bertini). The known theory of space curves is also, in this aspect, quite complete. The analogous problem of the reduction of higher singularities of a surface has been considered by Noether, Del Pezzo, Segre, Kobb, and others, but no ultimate conclusion has yet been obtained.

One principal source of difficulty is that, while in case of two birationally equivalent curves the correspondence is one-to-one without exception, on the other hand, in the case of two surfaces, there may be isolated points which correspond to curves, and just such irregular phenomena escape the ordinary methods. Again, not only singular points require consideration, as is the case in the plane theory, but also singular lines, and the points may be isolated or superimposed on the lines. Most success is to be expected from further application of the method of projection from a higher space due to Clifford and Veronese. In this direction the most important result hitherto obtained is the theorem, of Picard and Simart, that any algebraic surface (in ordinary space) can be regarded as the projection of a surface free from singularities situated in five-dimensional space.

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[1] Segre recently called attention to a case where the usual methods of discussion fail to apply; the proof has been completed by Castelnuovo. Cf. *Atti di Torino*, vol. XXXVI (1901).

A question which awaits solution even in the case of the plane is that relating to the invariants of the group of Cremona transformations proper. The genus and the moduli of a curve are unaltered by all birational transformations, but the problem arises: Are there properties of curves which remain unchanged by Cremona, although not by other birational transformations? From the fact that birationally equivalent curves need not be equivalent under the Cremona group, it would seem that such invariants — Cremona invariants proper — do exist, but no actual examples have yet been obtained. The problem may be restated in the form: What are the necessary and sufficient conditions which must be fulfilled by two curves if they are to be equivalent with respect to Cremona transformations? Equality of genera and moduli, as already remarked, is necessary but not sufficient.

The invariant theory of birational transformations has for its principal object the study of the linear systems of point groups on a given algebraic curve, that is, the point groups cut out by linear systems of curves. Its foundations were implicitly laid by Riemann in his discussion of the equivalent theory of algebraic functions on a Riemann surface, though the actual application to curves is due to Clebsch. Most of the later work has proceeded along the algebraic-geometric lines developed by Brill and Noether, the promising purely geometric treatment inaugurated by Segre being rather neglected.

The extension of this type of geometry to space, that is, the development of a systematic geometry on a fundamental algebraic surface (especially as regards the linear systems of curves situated thereon), is one of the main tasks of recent mathematics. The geometric treatment is given in the memoirs of Enriques and Castelnuovo, while the corresponding functional aspect is the subject of the treatise of Picard and Simart on algebraic functions of two variables, at present in course of publication.

The most interesting feature of the investigations belonging in this field is the often unexpected light which they throw on the inter-relations of distinct fields of mathematics, and the advantage derived from such relations. For example, Picard (as he himself relates on presenting the second volume of his treatise to the Paris Academy a few months ago) [1] for a long time was unable to prove directly that the integrals of algebraic total differentials can be reduced, in general, to algebraic-logarithmic combinations, until finally a method for deciding the matter was suggested by a theorem on surfaces which Noether had stated some twenty years earlier. Again, in the enumeration of the double integrals of the second species, Picard arrived at a certain result, which was soon noticed to be essentially equivalent to one obtained by Castelnuovo in his investigations on linear systems; and thus there was established a connection between the so-called numerical and linear genera of a surface, and the number of distinct double integrals.[1]

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[1] *Comptes Rendus*, February 1, 1904.

A closely related set of investigations, originating with Clebsch's theorems on intersections and Liouville's on confocal quadrics, may be termed the "geometry of Abel's theorem." As later applications we can merely mention Humbert's memoirs on certain metric properties of curves, and Lie's determination of surfaces of translation.

Investigations in analysis have often suggested the introduction of new types of configurations into geometry. The field of algebraic surfaces is especially fruitful in this respect. Thus, while in the case of curves (excluding the rational) there always exist integrals everywhere finite, this holds for only a restricted class of surfaces; their determination depends on the solution of a partial differential equation which has been discussed in a few special cases.

In addition to such relations between analysis and geometry, important relations arise between various fields of geometry. Just as an algebraic function of one variable is pictured by either a plane curve or a Riemann surface (according as the independent and dependent variables are taken to be real or complex), so an algebraic function of two independent variables may be represented by either a surface in ordinary space or a Riemannian four-dimensional manifold in space of five dimensions. In the case of one variable, the single invariant number (deficiency or genus p) which arises is capable of definition in terms of the characteristics of the curve or the connectivity of the Riemann surface. In passing to two variables, however, it is necessary to consider several arithmetical invariants — just how many is an unsettled question. For the algebraic surface we have, for instance, the geometric genus of Clebsch, the numerical genus of Cayley, and the so-called second genus, each of which may be regarded as a generalization, from a certain point of view, of the single genus of a curve; all are invariant with respect to birational transformation.

The other geometric interpretation, by means of a Riemannian manifold, has rendered necessary the study of the *analysis situs* of higher spaces. The connection of such a manifold is no longer expressed by a single number as in the case of an ordinary surface, but by a set of two or more, the so-called numbers of Betti and Riemann. The detailed theory of these connectivities, difficult and delicate because it must be derived with little aid from the intuition, has been made the subject of an extensive series of memoirs by Poincaré.

From the point of view of analysis, the chief interest in these investigations is the fact that the connectivities are related to the number of integrals of certain types. The chief problem for the geometer, however, is the discovery of the precise relations between the connectivities of the Riemann manifold and the various genera of the algebraic surface. That relations do exist between such diverse geometries — the one operating with all *continuous*, the other with the *algebraic*, one-to-one correspondence — is one of the most striking results of recent mathematics.

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[1] *Comptes Rendus*, February 22, 1904.

Geometry of Multiple Forms

For some time after its origin, the linear invariant theory of Boole, Cayley, and Sylvester confined itself to forms containing a single set of variables. The needs of both analysis and geometry, however, have emphasized the importance and the necessity of further development of the theory of forms containing two or more sets of variables (of the same or different type), so-called multiple forms.

In the plane we have both point coördinates (x) and line coördinates (u). A form in x corresponds to a point curve (locus), a form in u to a line curve (envelope), and a form involving both x and u to a connex. The latter was introduced into geometry, some thirty years ago, by Clebsch, the suggestion coming from the fact that, even in the study of a simple form in x, covariants in x and u present themselves, so that it seemed desirable to deal with such forms *ab initio*.

Passing to space, we meet three simple elements, the point (x), the plane (u), and the line (p). Forms in a single set of variables represent, respectively, a surface as point locus, a surface as plane envelope, and a complex of lines. The compound elements composed of two simple elements are the point-plane, the point-line, and the plane-line. The first type, leading to point-plane connexes, has been studied extensively during the past few years; the second to a more limited degree; the third is merely the dual of the second. To complete the series, the case of the point-line-plane as element, or forms involving x, u, and p, requires investigation.

In the corresponding n-dimensional theory it is necessary to take account of n simple elements and the various compound elements formed by their combinations.

The importance of such work is twofold: First, on account of connection with the algebra of invariants. A fundamental theorem of Clebsch states that, in the investigation of complete systems of comitants, it is sufficient to consider forms involving not more than one set of variables of each type: if in the given forms the types are involved in any manner, it is possible to find an equivalent reduced system of the kind described. On the other hand, it is impossible to reduce the system further, so that the introduction of the n types of variables is necessary for the algebraically complete discussion. Geometry must accordingly extend itself to accommodate the configurations defined by the new elements.

Second, on account of connection with the theory of differential equations. The ordinary plane connex in x, u, assigns to each point of the plane a certain number of directions (represented by the tangents drawn to the corresponding curve), and thus gives rise to an (algebraic) differential equation of the first order in two variables; the point-plane connex in space, associating with each point a single infinity of incident planes, defines a partial differential equation of the first order; the point-line connex yields a Monge equation. The point-line-plane case has not yet been interpreted from this point of view.

One special problem in this field deserves mention, on account of its many applications. This is the study of the system composed of a quadric form in any number of variables and a bilinear form in contragredient variables, that is, a quadric manifold and an arbitrary (not merely automorphic) collineation in n-space. For $n=6$, for example, this corresponds to the general linear transformation of line or sphere coördinates.

In addition to forms containing variables of different types, the forms involving several sets of variables of the same type require consideration. Forms in two sets of line coördinates present themselves in connection with the pfaffian problem of differential systems. The main interest attaches, however, to forms in sets of point coördinates, since it is these which occur in the theory of contact transformations and of multiple correspondences. For example, while the ordinary homography on a line is represented by a bilinear form in binary variables, the trilinear form in similar variables gives rise to a new geometric variety, the so-called homography of the second class (associating with any two points a unique third point), which has applications to the generation of cubic surfaces and to the constructions at the basis of photogrammetry. The theory of multilinear forms in general deserves more attention than it has yet received.

Other important problems, connected with the geometric phases of linear invariant theory, can merely be mentioned: (1) The general geometric interpretation of what appears algebraically as the simplest projective relation, namely, apolarity. (2) The invariant discussion of the simpler discontinuous varieties, for example, the polygon considered as n-point or as n-line.[1] (3) The establishment of a system of forms corresponding to the general space curve. (4) The study of the properties and the groups of the configurations corresponding in hyperspace to the simpler systems of invariants. (5) Complete systems of orthogonal or metric invariants for the simpler curves.[1]

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[1] Cf. F. Morley "On the geometry whose element is the 3-point of a plane," *Trans. Amer. Math. Soc.*, vol. v (1904). E. Study in his *Geometrie der Dynamen* develops a new foundation for kinematics by employing as element the *Soma* or trirectangular trihedron.

Transcendental Curves

To reduce to systematic order the chaos of non-algebraic curves has been the aspiration of many a mathematician; but, despite all efforts, we have no theory comparable with that of algebraic curves. The very vagueness and apparent hopelessness of the question is apt to repel the modern mathematician, to cause him to return to the more familiar field. The resulting concentration has led to the powerful methods, already referred to, for studying algebraic varieties. In the transcendental domain, on the other hand, we have a multitude of interesting but particular geometric forms, — some suggested by mechanics and physics, others derived from their relation to algebraic curves, or by the interpretation of analytic results — a few thousands of which have been considered of sufficient importance to deserve specific names.[2] The problem at issue is then a practical one (comparable with corresponding discussions in natural history): to formulate a principle of classification which will apply, not to all possible curves, but to as many as possible of the usual important transcendental curves.

The most fruitful suggestion hitherto applied has come from the consideration of differential equations: almost all the important transcendental curves satisfy algebraic differential equations, and these in the great majority of cases are of the first order. Hence the need of a systematic discussion of the curves defined by any algebraic equation $F(x, y, y') = 0$, the so-called *panalgebraic* curves of Loria. If F is of degree n in y' and of degree ν in x, y, the curve is said to belong to a system with the characteristics (n, ν), and we thus have an important basis for classification. Closely related is the theory of the Clebsch connex; this figure, it is true, is considered as belonging to algebraic geometry, but it defines (by means of its principal coincidence) a system of usually transcendental panalgebraic curves.

Both points of view appear to characterize certain systems of curves rather than individual curves. The following interpretation may serve as a simple geometric definition of the curves considered.

With any plane curve C we may associate a space curve in this way: at each point of C erect a perpendicular to the plane whose length represents the slope of the curve at that point; the locus of the end points of these perpendiculars is the associated space curve

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[1] Here would belong in particular the theory of algebraic curves based on linkages. Little advance has been made beyond the existence theorems of Kempe and Koenigs. An important unsolved problem is the determination of the linkage with minimum number of pieces by which a given curve can be described.

[2] Cf. Loria, *Spezielle Kurven*, Leipzig, 1902.

C'. Not every space curve is obtained in this way, but only those whose tangents belong to a certain linear complex. If C is algebraic so is C', and then an infinite number of algebraic surfaces may be passed through the latter. If C is transcendental, so is C', and usually no algebraic surface can be passed through it. Sometimes, however, one such algebraic surface F exists. (If there were two, C' and C would be algebraic.) It is precisely in this case that the curve C is panalgebraic in the sense of Loria's theory. That such a curve belongs to a definite system is seen from the fact that while the surface F is unique, it contains a singly infinite number of curves whose tangents belong to the linear complex mentioned, and the orthogonal projections of these curves constitute the required system.

The principal problems in this field which require treatment are: first, the exhaustive discussion of the simplest systems, corresponding to small values of the characteristics n and ν; second, the study of the general case in connection with (1) algebraic differential equations, (2) connexes, and (3) algebraic surfaces and linear complexes.

Natural or Intrinsic Geometry

In spite of the immediate triumph of the Cartesian system at the time of its introduction into mathematics, rebellion against what may be termed the tyranny of extraneous coördinates, first expressed in the *Characteristica geometrica* of Leibnitz, has been an ever-present though often subdued influence in the development of geometry. Why should the properties of a curve be expressed in terms of x's and y's which are defined not by the curve itself, but by its relation to certain arbitrary elements of reference? The same curve in different positions may have unlike equations, so that it is not a simple matter to decide whether given equations represent really distinct or merely congruent curves. The idea of the so-called natural or intrinsic coördinates had its birth during the early years of the nineteenth century, but it is only the systematic treatment of recent years which has created a new field of geometry.

For a plane curve there is at each point the arc s measured from some fixed point on the curve, and the radius of curvature ρ; these intrinsic coördinates are connected by a relation $\rho=f(s)$ which is precisely characteristic of the curve, that is, the curves corresponding to the equation differ only in position. There is, however, still something arbitrary in the point taken as origin. This is eliminated by taking as coördinates ρ and its derivative δ taken with respect to the arc; so that the final intrinsic equation is of the form $\delta=F(\rho)$. There is no difficulty in extending the method to space curves. The two natural equations necessary are here $\tau=\phi(\rho)$, $\delta=\psi(\rho)$, where ρ and τ are the radii of first and second curvature and δ is the arc derivative of ρ.

The application to surfaces is not so evident. Thus, in Cesaro's standard work, while the discussion of curves is consistently intrinsic, this is true to only a slight extent in the treatment of surfaces. The natural geometry of surfaces is in fact only in process of formation. Bianchi proposes as intrinsic the familiar representation by means of the two fundamental quadratic differential forms; but, although it is true that the surfaces corresponding to a given pair of forms are necessarily congruent, there is the disadvantage, arising from the presence of arbitrary parameters, that the same surface may be represented by distinct pairs of forms. One way of overcoming this difficulty is to introduce the common feature of all pairs corresponding to a surface, that is, the invariants of the forms: in this direction we may cite Ricci's principle of covariant differentiation and Maschke's recent application of symbolic methods.

The basis of natural geometry is, essentially, the theory of differential invariants. Under the group of motions, a given configuration assumes ∞^r positions, where r is in general 6, but may be smaller in certain cases. The r parameters which thus enter in the analytic representation may be eliminated by the formation of differential equations. The aim of natural geometry is to express these differential equations in terms of the simplest geometric elements of the given configuration.

The beginning of such a discussion of surfaces was given by Sophus Lie in 1896 and his work has been somewhat simplified by Scheffers. As natural coördinates we may take the principal radii of curvature R_1, R_2, at a point of the surface, and their derivatives

$$\partial_{11}=\frac{dR_1}{ds_2} \qquad \partial_{12}=\frac{dR_1}{ds_2} \qquad \partial_{21}=\frac{dR_2}{ds_1} \qquad \partial_{22}=\frac{dR_2}{ds_2}$$

taken in the principal directions. For a given surface (excluding the Weingarten class) the radii are independent, and there are four relations of the form

$$\partial_{11}=f_{11}(R_1, R_1), \quad \partial_{12}=f_{12}(R_1, R_2), \quad \partial_{21}=f_{21}(R_1, R_2),$$
$$\partial_{22}=f_{22}(R_1, R_2).$$

Conversely, these equations are not satisfied by any surfaces except those congruent or symmetric to the given surface.

It is to be noticed that four equations thus appear to be necessary to define a surface, although two are sufficient for a twisted curve. If a single equation in the above-mentioned natural coördinates is considered, it is not, as in the case of ordinary coördinates, characteristic: surfaces not congruent or symmetric to the given surface would satisfy the equation. The apparent inconsistency which arises is removed, however, by the fact that the four natural equations are dependent.[1] It is just this that makes the subject difficult as compared with the theory of curves, in which the defining equations are entirely arbitrary. The questions demanding treatment fall under these two headings: first, the derivation of the natural equations of the familiar types of surfaces, and second, the study of the new types that correspond to equations of simple form. The natural geometry of the Weingarten class of surfaces requires a distinct basis.

The fact that intrinsic coördinates are, at bottom, differential invariants with respect to the group of motions, suggests the extension of the same idea to the other groups. Thus in the projective geometry of arbitrary (algebraic or transcendental) curves, coördinates are required which, unlike the distances and angles ordinarily used, are invariant under projection. These might, for example, be introduced as follows. At each point of the general curve C, there is a unique osculating cubic and a unique osculating W (self-projective) curve. Connected with each of these osculating curves is an absolute projective invariant defined as an anharmonic ratio. These ratios may then be taken as natural projective coördinates γ and ω, and the natural equation on the curve is of the form $\gamma = f(\omega)$. The principal advantage of such a representation is that the necessary and sufficient condition for the equivalence of two curves under projective transformations is simply the identity of the corresponding equations.

Returning to the theory of surfaces, natural coördinates may be introduced so as to fit into the so-called geometry of a flexible but inextensible surface, originated by Gauss, in which the criterion of equivalence is applicability, or, according to the more accurate phraseology of Voss, isometry. Intrinsic coördinates must then be invariant with respect to bending (*Biegungsinvariante*). This property is fulfilled, for example, by the Gaussian curvature κ and the differential parameters connected with it $\lambda = \Delta\ (\kappa, \kappa)$, $\mu = \Delta(\kappa, \lambda)$, $\nu = \Delta(\lambda, \lambda)$, all capable of simple geometric interpretation. The intrinsic equations are then of the form $\mu = \phi(\kappa, \lambda)$, $\nu = \phi(\kappa, \lambda)$.

A pair of equations of this kind thus represent, not so much a single surface S, as the totality of all surfaces applicable on S (or into which S may be bent) — a totality which is termed a complete group G, since no additional surfaces are obtained when the same process is applied to any member of the totality. The discussion of such groups is ordinarily based on the first fundamental form (representing the squared element of length), since this is the same for isometric surfaces; though of course it changes on the introduction of new parameters.

The simplest example of a complete isometric group is the group typified by the plane, consisting of all the developable surfaces. In this case the equations of the group may be obtained explicitly, in terms of eliminations, differentiations, and quadratures. This is, however, quite exceptional; thus, even in the case of the surfaces applicable on the unit sphere (surfaces of constant Gaussian curvature +1), the differential equation of the group has not been integrated explicitly. In fact, until the year 1866, not a single case analogous to that of the developable surfaces was discovered. Weingarten, by means of his theory of evolutes, then succeeded in determining the complete group of the catenoid and of the paraboloid of revolution, and, some twenty years later, a fourth group defined in terms of minimal surfaces.

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[1] The three relations connecting the functions $f_{11}, f_{12}, f_{21}, f_{22}$ have been worked out recently by S. Heller, *Math. Annalen*, vol. LVIII (1904).

During the past decade, the French geometers have concentrated their efforts in this field mainly on the arbitrary paraboloid (and to some extent on the arbitrary quadric). The difficulties even in this extremely restricted and apparently simple case are great, and are only gradually being conquered by the use of almost the whole wealth of modern analysis and the invention of new methods which undoubtedly have wider fields of application. The results obtained exhibit, for example, connections with the theories of surfaces of constant curvature, isometric surfaces, Backlund transformations, and motions with two degrees of freedom. The principal workers are Darboux, Goursat, Bianchi, Thybaut, Cosserat, Servant, Guichard, and Raffy.

Geometry im Grossen

The questions we have just been considering, in common with almost all the developments of general or infinitesimal geometry, deal with the properties of the figure studied *im Kleinen*, that is, in the sufficiently small neighborhood of a given point. Algebraic geometry, on the other hand, deals with curves and surfaces in their entirety. This distinction, however, is not inherent in the subject-matter, but is rather a subjective one due to the limitations of our analysis: our results being obtained by the use of power series are valid only in the region of convergence. The properties of a curve or surface (assumed analytic) considered as a whole are represented not by means of function elements, but by means of the entire functions obtained say by analytic continuation.

Only the merest traces of such a transcendental geometry *im Grossen* are in existence, but the interest of many investigators is undoubtedly tending in this direction. The difficulty of the problems which arise (in spite of their simple and natural character) and the delicacy of method necessary in their treatment may be compared to the corresponding problems and methods of celestial mechanics. The calculation of the ephemeris of a planet for a limited time is a problem *im Kleinen*, while the discovery of periodic orbits and the theory of the stability of the solar system are typical problems *im Grossen.*

The principal problems in this field of geometry are connected with closed curves and surfaces. Of special importance are the investigations relating to the closed geodesic lines which can be drawn on a given surface, since these are apt to lead to the invention of methods applicable to the wider field of dynamics. Geodesics may in fact be defined dynamically as trajectories of a particle constrained to the surface and acted upon either by no force or by a force due to a force function U whose first differential parameter is expressible in terms of U. The few general theorems known in this connection are due in the main to Hadamard (*Journal de Mathématiques*, 1897, 1898). Thus, on a closed surface whose curvature is everywhere positive, a point describing a geodesic must cross any existing closed geodesic an infinite number of times, so that, in particular, two closed geodesics necessarily intersect.[1] On a surface of negative curvature, under certain restrictions, there exist closed geodesics of various topological types, as well as geodesics which approach these asymptotically.

As regards surfaces all of whose geodesics are closed, the investigations have been confined entirely to the case of surfaces of revolution, the method employed being that suggested by Darboux in the *Cours de Mécanique* of Despeyrons. Last year Zoll[2] succeeded in determining such a surface (beyond the obvious sphere) which differs from the other known solutions in not having any singularities. Analogous problems in connection with closed lines of curvature and asymptotic lines will probably soon secure the consideration they deserve.

A problem of different type is the determination of applicability criteria valid for entire surfaces. The ordinary conditions (in terms of differential parameters) assert, for example, the applicability of any surface of constant positive curvature upon a sphere; but the bending is actually possible only for a sufficiently small portion of the surface. A spherical surface as a whole cannot be applied on any other surface, that is, cannot be bent without extension or tearing. This result is analogous to the theorem known to Euclid, although first proved by Cauchy, that a closed convex polyhedral surface is necessarily rigid. Lagrange, Minding, and Jellet stated the result for all closed convex surfaces, but the complete discussion is due to H. Liebmann.[3] The theory of the deformation of concave surfaces is far more complicated, and awaits solution even in the case of polyhedral surfaces.

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[1] In a paper read before the St. Louis meeting of the American Mathematical Society, Poincaré stated reasons which make very probable the existence of at least three closed geodesics on a surface of this kind.

[2] *Math. Annalen*, vol. LVII (1903).

[3] *Göttingen Nachrichten*, 1899; *Math. Annalen*, vols. LIII, LIV

Beltrami's visualization of Lobachevsky's geometry by picturing the straight lines of the Lobachevsky plane as geodesics on a surface of constant negative curvature is well known. However, since the known surfaces of this kind, like the pseudosphere, have singular lines, this method really depicts only part of the plane. In fact Hilbert (*Transactions of the American Mathematical Society* for 1900), by very refined considerations, has shown that an analytic surface of constant negative curvature which is everywhere regular does not exist, so that the entire Lobachevsky plane cannot be depicted by any analytic surface.[1] There remains undecided the possibility of a complete representation by means of a non-analytic surface. The partial differential equation of the surfaces of negative constant curvature is of the hyperbolic type and hence does admit non-analytic solutions.[2] (This is not true for surfaces of positive curvature, since the equation is then of elliptic type.) The discussion of non-analytic curves and surfaces will perhaps be one of the really new features of future geometry, but it is not yet possible to indicate the precise direction of such a development.[3]

Other theories belonging essentially to geometry *im Grossen* are the questions of *analysis situs*, or topology, to which reference has been made on several occasions, and the properties of the very general convex surfaces introduced by Minkowski in connection with his *Geometrie der Zahlen*.

Systems of Curves — Differential Equations

Although projective geometry has for its domain the investigation of all properties unaltered by collineation, attention has been confined almost exclusively to the algebraic configuration, so that projective is often confused with algebraic geometry. To the more general projective geometry belong, for example, the ideas of osculating conic of an arbitrary curve and the asymptotic lines of an arbitrary surface, and Mehmke's theorem which asserts that when two surfaces touch each other, the ratio of their Gaussian curvatures at the point of contact is an (absolute) projective invariant. The field for investigation in this direction is of course very extensive, but we may mention as a problem of special importance the deriva-

[1] The entire projective plane, on the other hand, can be so depicted on a surface devised by W. Boy (*Inaugural Dissertation*, Göttingen, 1901).

[2] According to Bernstein (*Math. Annalen*, vol. LIX, 1904, p. 72), the proof given by Lütkemeyer (*Inaugural Dissertation*, Göttingen, 1902) is not valid, though the conclusion is correct.

[3] Lebesgue (*Comptes Rendus*, 1900, and *Thèse*, 1902) has examined the theory of surfaces applicable on a plane without assuming the existence of derivatives for the defining functions, and thereby obtains an example of a non-ruled developable.

tion of the conditions for the projective equivalence of surfaces in terms of their fundamental quadratic forms.

Coördinate with what has just been stated, that general configurations may be studied from the projective point of view, is the fact that algebraic configurations may be studied in relation to general transformation theory. One may object that, with respect to the group of all (analytic) point transformations, the algebraic configurations do not form a *body*,[1] that is, are not converted into algebraic configurations; but such a *body* is obtained by adjoining to the algebraic all those transcendental configurations which are equivalent to algebraic. As this appears to have been overlooked, it seems desirable to give a few concrete instances, of interest in showing the effect of looking at familiar objects from a new and more general point of view.

As a first example, consider the idea of a linear system of plane curves. In algebraic geometry, a linear system is understood to be one represented by an equation of the form

$$F_0+\lambda_1 F_1+\lambda_2 F_2+\cdots+\lambda_\kappa F_\kappa=0,$$

where the λ's are parameters and the F's are polynomials in x,y. On the other hand, in general (infinitesimal) geometry, a system is defined to be linear when it can be reduced (by the introduction of new parameters) to the same form where the F's are arbitrary functions. The first definition is invariant under the projective group; the second, under the group of all point transformations. If now we apply the second definition to algebraic curves, the result does not coincide with that given by the first definition. Thus, every one-parameter system is linear in the general sense, while only pencils of curves are linear in the projective sense. The first case of real importance is, however, the two-parameter system, since here each point of view gives restricted, though not identical, types. An example in point is furnished by the vertical parabolas tangent to a fixed line, the equation of the system being $y=(ax+b)^2$. From the algebraic or projective point of view, this is a quadratic system since the parameters are involved to the second degree; but the system is linear from the general point of view since its equation may be written $ax+b-\sqrt{y}=0$. This suggests the problem: Determine the systems of algebraic curves which are linear in the general sense.

As a second example, consider, from both points of view, the equivalence of pencils of straight lines in the plane. By means of collineations any two pencils may be converted into any other two;

[1] The most extensive group for which the algebraic configurations form a body consists of all algebraic transformations. It is rather remarkable that even this theory has received no development.

[2] Halphen, Laguerre, Forsyth. This theory has been extended to simultaneous equations and applied geometrically by E. J. Wilczynski (*Trans. Amer. Math. Soc.*, 1901–1904).

but if three pencils are given, it is necessary to distinguish the case where the three base points are in a straight line from the case where they are not so situated. We thus have two projectively distinct cases, which may be represented canonically by: (1) x=const., y=const., $x+y$=const., and (2) x=const., y=const., y/x=const. The first type may, however, be converted into the second by the transcendental transformation $x_1=e^x$, $y_1=e^y$, so that, in the general group of point transformations, all sets of three pencils are equivalent. The discussion for four or more pencils yields the rather surprising result that the projective classification remains valid for the larger group.

Dropping these special considerations on algebraic systems, let us pass to the theory of arbitrary systems of curves, or, what is equivalent, the geometry of differential equations. While belonging to the cycle of theories due primarily to Sophus Lie, it has received little development in the purely geometric direction. Most attention has been devoted to special classes of differential equations with respect to special groups of transformations. Thus there is an extensive theory of the homogeneous linear equations with respect to the group $x_1=\xi(x)$, $y_1=y\eta(x)$ which leaves the entire class invariant.[1] A special theory which deserves development is that of equations of the first order with respect to the infinite group of conformal transformations.

As regards the general group of all point transformations, all equations of the first order are equivalent, so that the first case of interest is the theory of the two-parameter systems. The invariants of the differential equation of second order have been discussed most completely in the prize essay of A. Tresse (submitted to the Jablonowski Gesellschaft in 1896), with application to the equivalence problem. A specially important class, treated earlier by Lie and R. Liouville, consists of the equations of cubic type

$$y''=Ay'^3+By'^2+Cy'+D,$$

where the coefficients are functions of x, y. It includes, in particular, the general linear system and all systems capable of representing the geodesics of any surface. While the analytical conditions which characterize these subclasses are known, little advance has been made in their geometric interpretation.

Perhaps the simplest configuration belonging to the field considered, that is, having properties invariant under all point transformations, is that composed of three simply infinite systems of curves, which may be represented analytically by an equation of third degree in y' with one-valued functions of x, y for coefficients. In the case of equations of the fourth and higher degree in y', certain invariants may be found immediately from the fact that when x and y undergo an arbitrary transformation, the derivative y' undergoes a fractional linear transformation (of special type). The invariants found from this algebraic principle are, however, in a sense, trivial, and the real problem remains almost untouched: to determine the essential invariants due to the differential relations connecting the coefficients in the linear transformation of the derivative.

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[1] The elementary (metric) theory of curve systems has been too much neglected; it may be compared in interest and extent with the usual theory of surfaces.

General Theory of Transformations

Closely connected with the geometry of differential equations that we have been considering is the geometry of point transformations. In the former theory the transformations enter only as instruments, in the latter these instruments are made the subject-matter of the investigation. The distinction is parallel to that which occurs in projective geometry between the theory of projective properties of curves and surfaces and the properties of collineations. (It may be remarked, however, that although a transformation is generally regarded as dynamic and a configuration as static, the distinction is not at all essential. Thus a point transformation or correspondence between the points of a plane may be viewed as simply a double infinity of point pairs; on the other hand, a curve in the plane may be regarded as the equivalent of a correspondence between the points of two straight lines.[1])

We consider first two problems concerning the general (analytic) point transformation which are of interest and importance from the theoretic standpoint. The one relates to the discussion of the character of such a transformation in the neighborhood of a given point. Transon's theorem states that the effect of any analytic transformation upon an infinitesimal region is the same as that of a projective transformation. This is true, however, only in general; it ceases to hold when the derivatives of the defining functions vanish at the point considered. What is the character of the transformation in the neighborhood of such singular points ?

A more fundamental problem relates to the theory of equivalence. Consider a transformation T which puts in correspondence the points P and Q of a plane. Let the entire plane be subjected to a transformation S which converts P into P' and Q into Q'. We thus obtain a new transformation T' in which P' and Q' are corresponding points. This is termed the transform of T by means of S, the relation being expressed symbolically by $T'=S^{-1}TS$. The question then arises whether all transformations are equivalent, that is, can any one be converted into any other in the manner defined. The answer depends on certain functional equations which also arise in connection with the question whether an arbitrary transformation belongs to a continuous group. The problem deserves treatment not merely for the analytic transformations, but also for the algebraic and for the continuous transformations.[1]

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[1] Geometry on a straight line, in its entirety, is as rich as geometry in a plane or in space of any number of dimensions.

Aside from such fundamental questions, further development is desirable both in the study of the general properties (associated curve systems and contact relations) of an arbitrary transformation, and in the introduction of new special types of transformation, for instance, those which may be regarded as natural extensions of familiar types.

The main problems in the theory of point transformation are connected with certain fields of application which we now pass in review.

1. *Cartography.* A map may be regarded, abstractly, as the point by point representation of one surface upon another, the case of especial practical importance being, of course, the representation of a spherical or spheroidal surface upon the plane. As it is impossible to map any but the developable surfaces without distortion upon a plane, the chief types of available representation are characterized by the invariance of certain elements, as angles or areas, or the simple depiction of certain curves, as of geodesics by straight lines. Most attention has been devoted to the conformal type, but the question proposed by Gauss remains unsolved: what is the *best* conformal representation of a given surface on the plane, that is, the one accompanied by the minimum distortion? The answer, of course, depends on the criterion adopted for measuring the degree of distortion, and it is in this direction that progress is to be expected.

2. *Mathematical theory of elasticity.* As a geometric foundation for the mechanics of continua, it is necessary to study the most general deformation of space, defined say by putting x_1, y_1, z_1 equal to arbitrary functions of x, y, z. The most elegant analytical representation, as given for instance in the memoir of E. and F. Cosserat (*Annales de Toulouse*, volume 10), is obtained by introducing the elements of length ds and ds_1 before and after deformation, and the related quadratic differential form $ds_1^2 - ds^2 = 2\epsilon_1 dx^2 + 2\epsilon_2 dy^2 + 2\epsilon_3 dz^2 + 2\gamma_1\, dydz + 2\gamma_2\, dxdz + 2\gamma_3\, dxdy$. The theory is thus seen to be analogous to though of course more complicated than the usual theory of surfaces. The six functions of x, y, z which appear as coefficients in this form are termed the components of the deformation. Their

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[1] This problem is not to be confused with the similar (but simpler) question connected with Lie's division of (analytic) groups into *demokratisch* and *aristokratisch*. In those of the first kind all the infinitesimal transformations are equivalent, in those of the second there exist non-equivalent infinitesimal transformations. Lie shows that all finite groups are *aristokratisch*, while the groups of all (analytic) point and contact transformations are *demokratisch*. Cf. *Leipziger Berichte*, vol. XLVII (1895), p. 271.

importance is due to the fact that they vanish only when the transformation is a rigid displacement, so that two deformations have the same components when, and only when, they differ by a displacement. The case where the components are constants leads to the homogeneous deformation (or affine transformation of the geometers), the type considered almost exclusively in the usual discussions of elasticity. It would seem desirable to study in detail the next case which presents itself, namely, that in which the components are linear functions of x, y, z.

In the general deformation, the six components are not independent, but are connected by nine differential equations analogous to those of Codazzi. The fact that a transformation is defined by three independent functions indicates, however, that there should be only three distinct relations between the components. This means that the nine equations of condition which occur in the standard theory are themselves interdependent; but their relations (analogous to syzygies among syzygies in the algebra of forms) do not appear to have been worked out.

3. *Vector fields.* From its beginning in the Faraday-Maxwell theory of electricity until the present day, the course which the discussion of vector fields has followed has been guided almost entirely by external considerations, namely, the physical applications. While this is advantageous in many respects, it cannot be denied that it has led to lack of symmetry and generality. The time seems to be ripe for a more systematic mathematical development. The vector field deserves to be introduced as a standard form into geometry.

Abstractly, such a field is equivalent to a point transformation of space, since each is represented by three scalar relations in six variables. Instead of taking these variables as the coördinates of corresponding points, it is more convenient to consider three as the coördinates x, y, z of a particle and the other three as components u, v, w of its velocity; we thus picture the set of functional relations by means of the steady motion of a hypothetical space-filling fluid. This image should be of service even in abstract analysis; for its rôle is analogous to that of the curve in dealing with a single relation between two variables. The streaming of a material fluid is, of course, not sufficiently general for such a purpose, since, in virtue of the equation of continuity, it images only a particular class of vector fields.

In addition to the ordinary vector fields, physics makes use of so-called hypervector fields, which, geometrically, lead to configurations consisting of a triply infinite system of quadric surfaces, one for each point of space. In the special case of interest in hydrodynamics (irrotational motion), the configuration simplifies in that the quadrics are ellipsoids about the corresponding points as centres.

This is equivalent to the *tensor* field which arises in studying the moments of inertia of an arbitrary distribution of mass. The more general case actually arises in Maxwell's theory of magnetism.

4. As a final domain of application we mention the class of questions which have received systematic treatment, under the title of *nomography,* only during the past few years. This subject deals with the methods of representing graphically, in a plane, functional relations containing any number of variables. Thus a function of two independent variables, $z=f(x, y)$, may be represented by the system of plane curves $f(x, y)=c$, each marked with the corresponding value of the parameter. This "parametered" system is then a cartesian graphical table, which is the simplest type of abacus or nomogram.

By means of any point transformation, one nomogram is converted into another which may serve to represent the same functional relation. The importance of this process of conversion (the so-called *anamorphosis* of Lalanne and Massau) depends on the fact that it may replace a complicated table by a simpler. The problems which arise (for example, the determination of all relations between three variables which can be represented by a nomogram composed of three systems of straight lines[1]) are of both practical and theoretical interest. The literature is scattered through the French, Italian, and German technological journals, but a systematic presentation of the main results is to be found in the *Traité de Nomographie* of d'Ocagne (Paris, 1899).

We return to the abstract theory of transformations. The type of transformation we have been considering, converting point into point, is only a special case of more general types. The most important extension hitherto made depends upon the introduction of differential elements. Thus the lineal element or directed point (x, y, y') leads to transformations which in general convert a point into a system of elements; when the latter form a curve, every curve is converted into a curve and the result is termed a contact transformation. Backlund has shown that no extension results from the elements of second or higher order: osculation transformations are necessarily contact transformations. The discussion of elements of infinitely high order, defined by an infinite set of coördinates $(x, y, y', y'', \cdots)$, may perhaps lead to a real extension. The question may be put in this form: Are there transformations (in addition to ordinary contact transformations) which convert analytic curves into analytic curves in such a way that contact is an invariant relation? The idea of curve transformation in general will probably be worked out in the near future: what is the most general mode of setting up a correspondence which associates with every Jordan curve another Jordan curve? Such discussions are aspects of geometry with an infinite number of dimensions.

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[1] The case of three systems of circles has also been discussed. See d'Ocagne, *Journal de l'Ecole Polytechnique*, 1902.

After a review of the kind given in this paper, one is tempted to ask: What is it which influences the mathematician in selecting *certain* (out of an infinite number of equally conceivable) problems for investigations? It is true, of course, that his subject is ideal, self-created, and that "Das Wesen der Mathematik liegt in ihrer Freiheit." Georg Cantor would indeed replace the term *pure* mathematics by *free* mathematics. This freedom, however, is not entirely caprice. The investigators of each age have always felt it their duty to deal with the unsolved questions and to generalize the results and conceptions inherited from the past, to correlate with other fields of contemporaneous thought, to keep in contact, as far as possible, with the whole body of truth. This is not all, however. The influence of æsthetic considerations, though less subject to analysis, has been, and still is, of at least equal importance in guiding the course of mathematical development.

SHORT PAPERS

The Section of Geometry was very fully attended and productive of extended discussion and a number of supplementary papers. For the same reason as in the Section of Algebra and Analysis it is impossible to give a satisfactory résumé of the short papers on this subject owing to their close technical reasoning.

The first paper was presented by Professor Harris Hancock, of the University of Cincinnati, on "Algebraic Minimal Surfaces."

The second paper was presented by Professor H. T. Blichfeldt, of Leland Stanford Jr. University, on the subject "Concerning some Geometrical Properties of Surfaces of Revolution."

The third paper was presented by Professor George Bruce Halsted, of Kenyon College, on "Non-Euclidean Spherics."

The fourth paper was presented by Professor Arnold Emch, of the University of Colorado, on "The Configuration of the Points of Inflection of a Plane Cubic and their Harmonic Polars."

The fifth paper was presented by Professor H. P. Manning, of Brown University, on "Representation of Complex Variables in Space of Four Dimensions."

The sixth paper was read by Professor G. A. Bliss, of the University of Missouri, on "Concerning Calcidus of Variations."

The seventh paper was presented by Professor L. W. Dowling, of the University of Wisconsin, on "Certain Universal Curves."

SECTION C—APPLIED MATHEMATICS

SECTION C — APPLIED MATHEMATICS

(*Hall* 7, *September* 24, 3 *p. m.*)

CHAIRMAN: PROFESSOR ARTHUR G. WEBSTER, Clark University, Worcester, Mass.
SPEAKERS: PROFESSOR LUDWIG BOLTZMANN, University of Vienna.
PROFESSOR HENRI POINCARÉ, The Sorbonne; Member of the Institute of France.
SECRETARY: PROFESSOR HENRY T. EDDY, University of Minnesota.

THE RELATIONS OF APPLIED MATHEMATICS

BY LUDWIG BOLTZMANN

(*Translated from the German by Professor S. Epsteen, University of Chicago*)

[**Ludwig Boltzmann,** Professor of Physics, University of Vienna, since 1902. **b.** Vienna, Austria, 1840. Studied, Vienna, Heidelberg, and Berlin. Professor of Physico-Mathematics, University of Gratz, 1869–73; Professor of Mathematics, University of Vienna, 1873–76; Professor of Experimental Physics, University of Gratz, 1876–90; Professor of Theoretical Physics, University of Munich, 1891–95; *ibid.* University of Vienna, 1895–1900; Professor of Physics, University of Leipzig, 1900–02. **Author of** *Vorlesungen über Maxwell's Theorie der Elekt rizitat und des Lichts; Vorlesungen über Kinetische Gastheorie; Vorlesungen über die Prinzipe der Mechanik.*]

MY present lecture has been put under the heading of applied mathematics, while my activity as a teacher and investigator belongs to the science of physics. The immense gap which divides the latter science into two distinct camps has almost nowhere been so sharply emphasized as in the division of the lecture material of this scientific congress, which covers such an enormous range of subjects that one may designate it as a flood, or, to preserve local coloring, as a Niagara of scientific lectures. I speak of the division of physics into theoretical and experimental. Although I have been assigned, as representative of theoretical physics, to "A.—Normative Science," experimental physics appears much later under "C.—Physical Science." Between them lie history, science of language, literature, art, and science of religion. Over all this, however, the theoretical physicist must extend his hand to the experimental physicist. We shall therefore not be able to avoid entirely the question of the justification of dividing physics into two parts and, in particular, into theoretical and experimental.

Let us listen first of all to an investigator of a time when natural science had not yet grown beyond its first beginnings, to Emmanuel Kant. Kant requires of each science that it should be developed logically from unified principles and firmly established theories. Natural science seems to him a primary science only in so far as it rests on a mathematical basis. Thus, he does not reckon the chemistry of his day among the sciences, because it rests merely upon an empirical basis and lacks a unified, regulative principle.

From this point of view theoretical physics is preferred to experimental physics, and occupies, in a sense, a higher rank. Experimental physics was merely to gather the material, but it remained for the theoretical physics to form the structure.

But the succession in the order of rank becomes reversed when we take into account the acquisitions of the last decades as well as the progress which is to be expected in the immediate future. The chain of experimental discoveries of the last century received a fitting completion with the discovery of the Röntgen rays. Connected with these there appear in the present century a multitude of new rays, with the most enigmatical properties, which have the profoundest effects upon our conceptions of nature. The more enigmatical these newly discovered facts are, and the more they seem at first to contradict our present conceptions, the greater the successes which they promise for the future. But this is not the occasion for the discussion of these experimental triumphs. I must leave to the representatives of experimental physics at this Congress the prolific problem of portraying all of the fruits which have hitherto been gathered in this domain, one might almost say, daily, and those which are to be expected.

The representative of theoretical physics scarcely finds himself in an equally fortunate position. Great activity does indeed prevail in this domain. One could almost say that it is in process of revolution. Only how much less tangible are the results here attained in comparison with those in experimental physics! It appears here that in a certain sense experimentation deserves precedence over all theory. An immediate fact is at once comprehensible. Its fruits may become evident in the shortest time, such as the various applications of the Röntgen rays and the utilization of the Hertz waves in wireless telegraphy. The battle which the theories have to fight is, however, an infinitely wearisome one; indeed, it seems as if certain disputed questions which existed from the beginning will live as long as the science.

Every firmly established fact remains forever unchangeable; at most, it may be generalized, completed, additions may be made, but it cannot be completely upset. Thus it is explained why the development of experimental physics is continuously progressive, never making a sudden jump, and never visited by great tremblings and revolutions. It occurs only in rare instances that something which was regarded as a fact turns out afterwards to have been an error, and in such cases the explanations of the errors follow soon, and they are not of great influence on the structure of the science as a whole.

It is, indeed, strongly emphasized that every established and logically recognized truth must remain incontrovertible. Although this cannot be doubted, experience teaches that the structure of our theories is by no means composed entirely of such incontrovertibly established truths. They are composed rather of many arbitrary pictures of the connections between phenomena, of so-called hypotheses.

Without some departure, however slight, from direct observation, a theory or even an intelligibly connected practical description for predicting the facts of nature cannot exist. This is equally true of the old theories whose foundations have become questionable, and of the most modern ones, which are resigning themselves to a great illusion if they regard themselves as free from hypotheses.

The hypotheses may perhaps be indefinite, or may be in the shape of mathematical formulae, or the thought may be equivalent to the latter, but expressed in words. In the latter cases the agreement with given data may be checked step by step; a complete revolution of that previously constructed is indeed not absolutely impossible, as, for example, if the law of the conservation of energy should turn out to be incorrect. But such a revolution will be exceedingly rare and highly improbable.

Such an indefinite, slightly specialized theory might serve as a guiding thread for experiments whose purpose is a detailed development of knowledge previously acquired and which is proceeding in barren channels; beyond this its usefulness does not reach.

In contradistinction to these are the hypotheses which give the imagination room for play and by boldly going beyond the material at hand afford continual inspiration for new experiments, and are thus pathfinders for the most unexpected discoveries. Such a theory will indeed be subject to change, a very complicated mass of information will be brought together and will then be replaced by a new and more comprehensive theory in which the old one will be the picture of a limited type of phenomena. Examples of this are the theory of emission in regard to the description of the phenomena of catoptrics and dioptrics, the hypothesis of an elastic ether in the representation of the phenomena of interference and refraction of light, and the notion of the electric fluid in the description of the phenomena of electrostatics.

Moreover the theories which proudly designate themselves as free from hypotheses are not exempt from great revolutions; thus, no one will doubt that the so-called theory of energy will have completely to alter its form if it desires to remain effective.

The accusation has been made that physical hypotheses have sometimes proved injurious and have delayed the progress of the science. This accusation is based chiefly upon the rôle which the hypothesis of the electric fluid has played in the development of the theory of electricity. This hypothesis was brought to a high stage of perfection by Wilhelm Weber, and the general recognition which his works found in Germany did indeed stand in the road of the theory of Maxwell. In a similar manner Newton's emanation theory stood in the way of the theory of undulations. But such inconveniences can scarcely be entirely avoided in the future. It will always be the tendency to complete as far as possible the prevailing view, and to make it self-sufficient whenever such a theory is self-consistent and does not in any way lead to a contradiction, whether it consist of mechanical models, of geometrical pictures, or of mathematical formulas. It will always be possible that a new theory will arise which has not yet been tested by experiment and which will represent a much larger field of phenomena. In such cases the older theory will count the largest following until this field of phenomena is brought into the range of experiment, and decisive tests demonstrate the superiority of the newer one. It is certainly useful, if the theory of Weber be always held up as a warning example, that one should bear in mind the essential progressiveness of the intellect. The services of Weber are not decreased by this, however; Maxwell himself speaks of his theory with the greatest wonder. Indeed, this instance cannot be taken into consideration against the usefulness of hypotheses, since Maxwell's theory contained as much of the hypothetical as any other. And this was eliminated only after it became generally known through Hertz, Poynting, and others.

The accusation has also been raised against hypotheses in physics that the creation and development of mathematical methods for the computation of the hypothetical molecular motions has been useless and even harmful. This accusation I cannot recognize as substantiated. Were it so, the theme selected for my present lecture would be an unfortunate one; and this fact may excuse me for having lingered on this much-discussed subject and for having sought to justify the use of hypotheses in physics.

I have not chosen for the thesis of my present lecture the entire development of physical theory. Several years ago I treated this subject at the German *Naturforscherversammlung* in Munich, and although some new developments have taken place since then, I should have to repeat myself a great deal. Moreover, one who has committed himself to one faction is not in a position to judge the other factions in a completely objective manner. I do not refer to a criticism of its value; my lecture shall not criticise, but shall judge. I am also convinced of the value of the views of my opponents and only arise to repel them when they attempt to belittle mine. But one can scarcely give as complete an account according to subject-matter, and an exposition of the inter-relations of all ideas in the views of another, as in his own.

I shall therefore select as the goal of my lecture to-day not merely the kinetic theory of molecules, but, moreover, a highly specialized branch of it. Far from denying that it contains hypotheses, I must rather characterize it as a bold advance beyond the facts of observation. And I nevertheless do not consider it unworthy of this occasion; this much faith do I have in hypotheses which present certain peculiarities of observation in a new light or which bring forth relations among them which cannot be reached by other methods. We must indeed be mindful of the fact that hypotheses require and are capable of continuous development, and are only then to be abandoned when all the relations which they represent can be better understood in some other manner.

To the above-mentioned problems, which are as old as the science and still unsolved, belongs the one if matter is continuous, or if it is to be considered as made up of discrete parts, of very many, but not in the mathematical sense infinite, individuals. This is one of the difficult questions which form the boundary of philosophy and physics.

Even some decades ago, scientists felt very shy of going deeply into the discussion of such questions. The one before us is too real to be entirely avoided; but one cannot discuss it without touching on some profounder still, such as upon the nature of the law of causation, of matter, of force, and so forth. The latter are the ones of which it was said that they did not trouble the scientist, that they belonged entirely to philosophy. To-day the situation is different, there is evident a tendency among scientists to consider philosophic questions, and properly so. One of the first rules of science is never to trust blindly to the instrument with which one works, but to test it in all directions. How, then, are we to trust blindly to inherited and historically developed conceptions, particularly when there are instances known where they led us into error? But in the examination of even the simplest elements, where is the boundary between science and philosophy at which we should pause?

I hope that none of the philosophers present will take offense or perceive an accusation, if I say boldly that by assigning this question to philosophy the resulting success has been rather meagre. Philosophy has done noticeably little toward the explanation of these questions, and from her own one-sided point of view she can do so just as little as natural science can from hers. If real progress is possible, it is only to be expected by coöperation of both of these sciences. May I therefore be pardoned if I touch slightly upon these questions although not a specialist; their connection with the aim of my lecture is very intimate.

Let us consult the famous thinker already quoted, Emmanuel Kant, on the question if matter is continuous, or if it is composed of atoms. He treats of this in his *Antimonies*. Of all the questions there raised, he shows that both the *pro* and *con* can be logically demonstrated. It can be shown rigorously that there is no limit to the divisibility of matter while an infinite divisibility contradicts the laws of logic. Kant shows likewise that a beginning and end of time, a boundary where space ceases, are as inconceivable as absolutely endless duration, absolutely endless extension.

This is by no means the sole instance where philosophical thought becomes tangled in contradictions; indeed, one finds them at every step. The ordinary things of philosophy are sources of insolvable riddles; to explain our perceptions it invents the concept of matter and then finds that it is altogether unsuited to possess perception itself or to generate perception in a spirit. With consummate acumen it constructs the concept of space, or of time, and finds that it is absolutely impossible that things should exist in this space, that events should occur during this time. It finds insurmountable difficulties in the relation of cause to effect, of body and soul, in the possibility of consciousness, in short, everywhere and in everything. Indeed, it finally finds it inexplicable and self-contradictory that anything can exist at all, that something originated and is capable of continuing, that everything can be classified according to our categories, nor that there is a quite perfect classification. Such a classification will always be a variable one and adapted to the requirements of the moment. Also the breaking up of physics into theoretical and experimental is merely a consequence of the prevalent division of methods and will not last forever.

My present thesis is quite different from the one that certain questions are beyond the boundary of human comprehension. For according to the latter, there is a deficiency, an incompleteness in the human intelligence, while I consider the existence of these questions, these problems, as an illusion. By superficial consideration it seems astonishing, after this illusion is recognized, that the impulse to answer those questions does not cease. Habit of thought is much too powerful to release us.

It is here as with the ordinary illusion which continues operative after its cause is recognized. In consequence of this is the feeling of uncertainty, of want of satisfaction which the scientist feels when he philosophizes. These illusions will yield but very slowly and gradually, and I consider it as one of the chief problems of philosophy to set forth clearly the uselessness of reaching beyond the limits of our habits of thought and to strive, in the choice and combination of concepts and words, to give the most useful expression of facts in a manner which is independent of our inherited habits. Then all these complications and contradictions must vanish. It must be made clear what is stone in the structure of our thoughts and what is mortar, and the oppressive sentiment, that the simplest things are the most inexplicable and the most trivial are the most mysterious, becomes mere imagination-change.

To call upon logic seems to me as if one were to put on for a trip into the mountains a long flowing robe, which always wrapped itself about the feet so that one fell at the first steps while on the level. The source of this kind of logic is the immoderate trust in the so-called laws of thought. It is certain that we could not gather experience did we not have certain forms of connecting phenomena, that is to say, of thought, innate. If we wish to call these laws of thought, they are indeed *a priori* to the extent that they accompany every experience in our soul, or if we prefer, in our brain. Only nothing seems to me less reasonable than the conclusion from the reasoning in this sense to certainty, to infallibility. These laws of thought have been developed according to the same laws of evolution as the optical apparatus of the eye, the acoustic apparatus of the ear, and the pumping arrangements of the heart. In the course of human development everything useless was eliminated, and thus a unity and finish arose which might be mistaken for infallibility. Thus the perfection of the eye, of the ear, of the arrangement of the heart excite our admiration, without the absolute perfection of these organs being emphasized, however. Just so little should the laws of thought be regarded as absolutely infallible. They are the very ones which have developed with regard to seizing that which is most necessary and practically useful in the maintenance of life. With these, the results of experimental investigation show more relation than the examination of the mechanism of thought. We should, therefore, not be surprised that the customary forms of thought for the abstract are not entirely suited to practical applications in far removed problems of philosophy, and that they have not become applicable since the days of Thales. Therefore the simplest things seem to be the most puzzling to the philosopher. And he finds everywhere contradictions; these are nothing more, however, than useless, incorrect facsimiles of that which is given us through our thoughts. In facts there can be no contradictions. As soon as contradictions seem unavoidable we must test, extend, and seek to modify that which we call laws of thought, but which are only inherited, customary representations, preserved for aeons, for the description of practical needs. Just as to the inherited discoveries of the cylinder, the carriage, the plow, numerous artificial ones have been consciously added, so must we improve, artificially and consciously, our likewise inherited concepts. Our problem cannot be to quote facts before the judgment seat of our laws of thought, but to fit our mental representations and concepts to the facts. Since we attempt to express with clearness such complicated processes merely by words, written, spoken, or inwardly thought, it might also be said that we should combine the words in such wise as to give the most appropriate expression of the facts, that the relations indicated by our words should be most adequate for the relations among the actualities. When the problem is enunciated in this fashion, its appropriate solution may still offer the greatest difficulties, but one knows then the end in view and will not stumble on self-made difficulties.

Much that is useless in the usage and in the bearing of the nature of life is brought forth by a method of treatment which, being useful in most cases, becomes through habit a second nature, until one cannot set it aside when it becomes inapplicable somewhere. I say that the adaptability goes beyond the point aimed at. This happens frequently in the commonplaces of thought, and becomes the source of apparent contradictions between the laws of thought and the world, as well as between the laws of thought themselves.

Thus, the regularity of the phenomena of nature is the fundamental condition for all cognition; thus comes the habit of inquiring of everything the cause, the non-resisting compulsion, and we inquire also concerning the cause, why everything must have a cause. In fact people strove for a long time to determine if cause and effect is a necessary bond or merely an accidental sequence, and if it did or did not have a unique meaning to say that a certain particular phenomenon was connected with, and a necessary consequence of, a definite group of other phenomena.

Similarly, something is said to be useful, valuable, if it satisfies the needs of the individual or of humanity; but we go beyond the mark if we ask concerning the value of life itself, if such it seem to have, because it has no purpose outside of itself. The same happens when we strive vainly to explain the simplest concepts, out of which all others are built, by means of simpler ones still, to explain the simplest fundamental laws.

We should not attempt to deduce nature from our concepts, but should adapt the latter to the former. We should not believe our inherited rules of thought to be conditions preceding our more complicated experiences, for they are not so for the simplest essentials. They arose slowly in connection with simple experiences and passed, by heredity, to the more highly organized being. Thus is explained how synthetic judgments arise which were formed by our ancestors and were born in us, and are in this sense *a priori*. Their great power is also seen in this way, but not their infallibility.

In saying that such judgments as "everything is red or is not red" are results of experience, I do not mean that every person checks this empty truth by experience, but that he learns that his parents called everything either red or not red and that he preserves this nomenclature.

It might seem as if we had gone somewhat deeply into philosophical questions, but I believe that the views we have reached could not have been attained in a shorter and simpler manner. For we have reached an impartial judgment how the question of the atomistic structure of matter is to be viewed. We shall not invoke the law of thought that there is no limit to the divisibility of matter. This law is of no more value than if a naïve person were to say that no matter where he went upon the earth the plumb-line directions seemed always to be parallel and therefore there were no antipodes.

On the one hand we shall start from facts only, and on the other we shall take nothing into consideration except the effort to attain to the most adequate expression of these facts.

Regarding the first point, the numerous facts of the theory of heat, of chemistry, of crystallography, show that bodies which are apparently continuous do not by any means fill the entire volume indistinguishably and uniformly with matter. Indeed, it appears that the space which they occupy is filled with innumerably many individuals, molecules, and atoms, which are extraordinarily small, but not infinitely small in the mathematical sense. Their sizes can be computed in different manners and always with the same result.

The fruitfulness of this line of thought has been verified in the most recent time. All the phenomena which are observed with the cathode rays, the Becquerel rays, etc., indicate that we are dealing with diminutive, moving particles, electrons. After a vigorous battle, this view vanquished completely the opposing explanation of these phenomena by the theory of undulations. Not only did the former theory give a better explanation of the previously known facts, it inspired new experiments and permitted the prediction of unknown phenomena, and thus it developed into an atomistic theory of electricity. If it continue to develop with the same success as in past years, if phenomena, such as the one observed by Ramsay on the transmutation of radium into helium, do not remain isolated, this theory promises deductions concerning the nature and structure of atoms as yet undreamed of. Computation shows that electrons are much smaller than the atoms of ponderable matter; and the hypothesis that the atoms are built up of many elements, as well as various interesting views on the character and structure of this composition, is to-day on every tongue. The word atom should not lead us into error, it comes from a past time; no physicist ascribes indivisibility to the atoms.

It is not my intention to confine the thought merely to the above facts and their resulting consequences; these are not sufficient to carry through the question as to the finite or infinite divisibility of matter. If we are going to think of the atoms of chemistry as made up of electrons, what would hinder us from considering the electrons as particles filled with rarefied, continuous matter? We shall adhere faithfully to the previously developed philosophical principles and shall examine in the most unhampered manner the concepts themselves in order to express them in a consistent and most useful form.

It appears now, that we are unable to define the infinite in any other way except as the limit of continually increasing magnitudes, at least no one has hitherto been able to set up any other intelligible conception of the infinite. Should we desire a verbal picture of the continuum, we must first think of a large finite number of particles which are endowed with certain properties and study the totality of these particles. Certain properties of this totality may approach a definite limit as the number of particles is increased, and their size decreased. It can be asserted, concerning these properties, that they belong to the continuum, and it is my opinion that this is the only self-consistent definition of a continuum which is endowed with certain properties.

The question if matter is composed of atoms or is continuous becomes then the question if the observed properties are accurately satisfied by the assumption of an exceedingly great number of such particles or, by increasing number, their limit. We have not indeed answered the old philosophical question, but we are cured of the effort to answer it in a senseless and hopeless manner. The thought-process, that we must investigate the properties of a finite totality and then let the number of members of this totality increase greatly, remains the same in both cases. It is nothing other than the abbreviated expression in algebraic symbols of exactly the same thought when, as often happens, differential equations are made the basis of a mathematical-physical theory.

The members of the totality which we select as the picture of the material body cannot be thought of as absolutely at rest, for there would then be no motion of any kind, nor can the members be thought of as relatively at rest in one and the same body, for in this case it would be impossible to account for the fluids. No effort has been made to subject them to anything more than to the general laws of mechanics. In order to explain nature we shall therefore select a totality of an exceedingly large number of very minute fundamental individuals which are constantly in motion, and which are subject to the laws of mechanics. But an objection is raised that will be an appropriate introduction to the final considerations of this lecture. The fundamental equations of mechanics do not alter their form in the slightest way when the algebraic sign of the time is changed. All pure mechanical events can therefore occur equally well in one sense as in its opposite, that is, in the sense of increasing time or of diminishing time. We remark, however, that in ordinary life future and past do not coincide as completely as the directions right and left, but that the two are distinctly different.

This becomes still more definite by means of the second law of the mechanical theory of heat, which asserts that when an arbitrary system of bodies is left to itself, uninfluenced by other bodies, the sense in which changes of condition occur can be assigned. A certain function of the condition of all the bodies, the entropy can be determined, which is such that every change that occurs must be in the sense of carrying with it an increase of this function; thus, with increasing time the entropy increases. This law is indeed an abstraction, just as the principle of Galileo; for it is impossible, in strict rigor, to isolate a system of bodies from all others. But since it has given correct results hitherto, in connection with all the other laws, we assume it to be correct, just as in the case of the principle of Galileo.

It follows from this law that every closed system of bodies must tend toward a definite final condition for which the entropy is a maximum. The outcome of this law, that the universe must come to a final state in which nothing more can occur, has caused astonishment; but this outcome is only comprehensible on the assumption that the universe is finite and subject to the second law of the mechanical theory of heat. If one regards the universe as infinite, the above-mentioned difficulties of thought arise again if one does not consider the infinite as a mere limit of the finite. Since there is nothing analogous to the second law in the differential equations of mechanics, it follows that it can be represented mechanically only by the initial conditions. In order to find the assumptions suitable for this purpose, we must reflect that, to explain the apparent continuity of bodies, we had to assume that every family of atoms, or more generally, of mechanical individuals, existed in incredibly many different initial positions. In order to treat this assumption mathematically, a new science was founded whose problem is, not the study of the motion of a single mechanical system, but of the properties of complexes of very many mechanical systems which begin with a great variety of initial conditions. The task of systematizing this science, of compiling it into a large book, and of giving it a characteristic name, was executed by one of the greatest American scholars, and in regard to abstract thinking, purely theoretic investigation, perhaps the greatest, Willard Gibbs, the recently deceased professor at Yale University. He called this science statistical mechanics, and it falls naturally into two parts. The first investigates the conditions under which the outwardly visible properties of a complex of very many mechanical individuals is not in any wise altered; this first part I shall call statistical statics. The second part investigates the gradual changes of these outwardly visible properties when those conditions are not fulfilled; it may be called statistical dynamics. At this point we may allude to the broad view which is opened by applying this science to the statistics of animated beings, of human society, of sociology, etc., and not merely upon mechanical particles. A development of the details of this science would only be possible in a series of lectures and by means of mathematical formulas. Apart from mathematical difficulties it is not free from difficulties of principle. It is based upon the theory of probabilities. The latter is as exact as any other branch of mathematics if the concept of equal probabilities, which cannot be deduced from the other fundamental notions, is assumed. It is here as in the method of least squares which is only free from objection when certain definite assumptions are made concerning the equal probability of elementary errors. The existence of this fundamental difficulty explains why the simplest result of statistical statics, the proof of Maxwell's speed law among the molecules of a gas, is still being disputed.

The theorems of statistical mechanics are rigorous consequences of the assumptions and will always remain valid, just as all well-founded mathematical theorems. But its application to the events of nature is the prototype of a physical hypothesis. Starting from the simplest fundamental assumption of the equal probabilities, we find that aggregates of very many individuals behave quite analogously as experience shows of the material world. Progressive or visible rotary motion must always go over into invisible motion of the minutest particles, into heat, as Helmholtz characteristically says: ordered motion tends always to go over into not ordered motion; the mixture of different substances as well as of different temperatures, the points of greater or less intense molecular motion, must always tend toward homogeneity. That this mixture was not complete from the start, that the universe began in such an improbable state, belongs to the fundamental hypotheses of the entire theory; and it may be said that the reason for this is as little known as the reason why the universe is just so and not otherwise. But we may take a different point of view. Conditions of great mixture and great differences in temperature are not absolutely impossible according to the theory but are very highly improbable. If the universe be considered as large enough there will be, according to the laws of probability, here and there places of the size of fixed stars, of altogether improbable distributions. The development of such a spot would be one-sided both in its structure and subsequent dissolution. Were there thinking beings at such a spot their impressions of time would be the same as ours, although the course of events in the universe as a whole would not be one-sided. The above-developed theory does indeed go boldly beyond our experience, but it has the merit which every such theory should have of showing us the facts of experience in an entirely new light and of inspiring us to new thought and reflection. In contradistinction to the first fundamental law, the second one is merely based on probability, as Gibbs pointed out in the '70's of the last century.

I have not avoided philosophical questions, in the firm hope that coöperation between philosophy and natural science will give new sustenance to both; indeed, that only in this manner a consistent argument can be carried through. I agree with Schiller when he says to the scientists and philosophers of his day, "Let there be strife between you, and the union will come speedily;" I believe that the time for this union has now arrived.

THE PRINCIPLES OF MATHEMATICAL PHYSICS

BY JULES HENRI POINCARÉ

(Translated from the French by George Bruce Halsted, Kenyon College)

[**Jules Henri Poincaré**, Professor University of Paris, and the Polytechnic School; Member of Bureau of Longitude. b. Nancy, April 29, 1854. D.Sc. August 3, 1879; D.Sc. Cambridge and Oxford, 1879; Charge of the Course of the Faculty of Sciences at Caen; Master of Conference of the Faculty of Sciences of Paris, 1881; Professor of the same Faculty, 1886; Member of the Institute of France, 1887; Corresponding Member of the National Academy of Washington; Philosophical Society of Philadelphia; the Academies of Berlin, London, St. Petersburg, Vienna, Rome, Munich, Göttingen, Bologna, Turin, Naples, Venice, Amsterdam, Copenhagen, Stockholm, etc. Written books and numerous articles for reviews and periodicals.]

WHAT is the actual state of mathematical physics? What are the problems it is led to set itself? What is its future? Is its orientation on the point of modifying itself?

Will the aim and the methods of this science appear in ten years to our immediate successors in the same light as to ourselves; or, on the contrary, are we about to witness a profound transformation? Such are the questions we are forced to raise in entering to-day upon our investigation.

If it is easy to propound them, to answer is difficult.

If we feel ourselves tempted to risk a prognostication, we have, to resist this temptation, only to think of all the stupidities the most eminent savants of a hundred years ago would have uttered, if one had asked them what the science of the nineteenth century would be. They would have believed themselves bold in their predictions, yet after the event how very timid we should have found them.

Mathematical physics, we know, was born of celestial mechanics, which engendered it at the end of the eighteenth century, at the moment when the latter was attaining its complete development. During its first years especially, the infant resembled in a striking way its mother.

The astronomic universe is formed of masses, very great without doubt, but separated by intervals so immense that they appear to us only as material points. These points attract each other in the inverse ratio of the square of the distances, and this attraction is the sole force which influences their movements. But if our senses were sufficiently subtle to show us all the details of the bodies which the physicist studies, the spectacle we should there discover would scarcely differ from what the astronomer contemplates. There also we should see material points, separated one from another by intervals enormous in relation to their dimensions, and describing orbits following regular laws.

These infinitesimal stars are the atoms. Like the stars properly so called, they attract or repel each other, and this attraction or this repulsion directed following the straight line which joins them, depends only on the distance. The law according to which this force varies as function of the distance is perhaps not the law of Newton, but it is an analogous law; in place of the exponent — 2, we have probably a different exponent, and it is from this change of exponent that springs all the diversity of physical phenomena, the variety of qualities and of sensations, all the world colored and sonorous which surrounds us, — in a word, all nature.

Such is the primitive conception in all its purity. It only remains to seek in the different cases what value should be given to this exponent in order to explain all the facts. It is on this model that Laplace, for example, constructed his beautiful theory of capillarity; he regards it only as a particular case of attraction, or as he says of universal gravitation, and no one is astonished to find it in the middle of one of the five volumes of the *Mécanique céleste.*

More recently Briot believed he had penetrated the final secret of optics in demonstrating that the atoms of ether attract each other in the inverse ratio of the sixth power of the distance; and does not Maxwell himself say somewhere that the atoms of gases repel each other in the inverse ratio of the fifth power of the distance? We have the exponent — 6, or — 5 in place of the exponent — 2, but it is always an exponent.

Among the theories of this period, one alone is an exception, that of Fourier; in it are indeed atoms, acting at a distance one upon the other; they mutually transmit heat, but they do not attract, they never budge. From this point of view, the theory of Fourier must have appeared to the eyes of his contemporaries, even to Fourier himself, as imperfect and provisional.

This conception was not without grandeur; it was seductive, and many among us have not finally renounced it; we know that we shall attain the ultimate elements of things only by patiently disentangling the complicated skein that our senses give us; that it is necessary to advance step by step, neglecting no intermediary; that our fathers were wrong in wishing to skip stations; but we believe that when we shall have arrived at these ultimate elements, there again will be found the majestic simplicity of celestial mechanics.

Neither has this conception been useless; it has rendered us an inestimable service, since it has contributed to make precise in us the fundamental notion of the physical law.

I will explain myself; how did the ancients understand law? It was for them an internal harmony, static, so to say, and immutable;

or it was like a model that nature constrained herself to imitate. A law for us is not that at all; it is a constant relation between the phenomenon of to-day and that of to-morrow; in a word, it is a differential equation.

The ideal form of physical law is the law of Newton which first covered it; and then how has one to adapt this form to physics? by copying as much as possible this law of Newton, that is, in imitating celestial mechanics.

Nevertheless, a day arrived when the conception of central forces no longer appeared sufficient, and this is the first of those crises of which I just now spoke.

Then investigators gave up trying to penetrate into the detail of the structure of the universe, to isolate the pieces of this vast mechanism, to analyze one by one the forces which put them in motion, and were content to take as guides certain general principles which have precisely for their object the sparing us this minute study.

How so? Suppose that we have before us any machine; the initial wheel-work and the final wheel-work alone are visible, but the transmission, the intermediary wheels by which the movement is communicated from one to the other are hidden in the interior and escape our view; we do not know whether the communication is made by gearing or by belts, by connecting-rods or by other dispositives.

Do we say that it is impossible for us to understand anything about this machine so long as we are not permitted to take it to pieces? You know well we do not, and that the principle of the conservation of energy suffices to determine for us the most interesting point. We easily ascertain that the final wheel turns ten times less quickly than the initial wheel, since these two wheels are visible; we are able thence to conclude that a couple applied to the one will be balanced by a couple ten times greater applied to the other. For that there is no need to penetrate the mechanism of this equilibrium and to know how the forces compensate each other in the interior of the machine; it suffices to be assured that this compensation cannot fail to occur.

Well, in regard to the universe, the principle of the conservation of energy is able to render us the same service. This is also a machine, much more complicated than all those of industry, and of which almost all the parts are profoundly hidden from us; but in observing the movement of those that we can see, we are able, by aid of this principle, to draw conclusions which remain true whatever may be the details of the invisible mechanism which animates them.

The principle of the conservation of energy, or the principle of Mayer, is certainly the most important, but it is not the only one;

there are others from which we are able to draw the same advantage. These are:

The principle of Carnot, or the principle of the degradation of energy.

The principle of Newton, or the principle of the equality of action and reaction.

The principle of relativity, according to which the laws of physical phenomena should be the same, whether for an observer fixed, or for an observer carried along in a uniform movement of translation; so that we have not and could not have any means of discerning whether or not we are carried along in such a motion.

The principle of the conservation of mass, or principle of Lavoisier.

I would add the principle of least action.

The application of these five or six general principles to the different physical phenomena is sufficient for our learning of them what we could reasonably hope to know of them.

The most remarkable example of this new mathematical physics is, beyond contradiction, Maxwell's electro-magnetic theory of light.

We know nothing of the ether, how its molecules are disposed, whether they attract or repel each other; but we know that this medium transmits at the same time the optical perturbations and the electrical perturbations; we know that this transmission should be made conformably to the general principles of mechanics, and that suffices us for the establishment of the equations of the electromagnetic field.

These principles are results of experiments boldly generalized; but they seem to derive from their generality itself an eminent degree of certitude.

In fact the more general they are, the more frequently one has the occasion to check them, and the verifications, in multiplying themselves, in taking forms the most varied and the most unexpected, finish by no longer leaving place for doubt.

Such is the second phase of the history of mathematical physics, and we have not yet emerged from it.

Do we say that the first has been useless? that during fifty years science went the wrong way, and that there is nothing left but to forget so many accumulated efforts as vicious conceptions condemned in advance to non-success?

Not the least in the world; the second phase could not have come into existence without the first?

The hypothesis of central forces contained all the principles; it involved them as necessary consequences; it involved both the conservation of energy and that of masses, and the equality of action and reaction; and the law of least action, which would appear, it is true, not as experimental verities, but as theorems, and of which the enunciation would have at the same time a something more precise and less general than under their actual form.

It is the mathematical physics of our fathers which has familiarized us little by little with these divers principles; which has taught us to recognize them under the different vestments in which they disguise themselves. One has to compare them to the data of experience, to find how it was necessary to modify their enunciation so as to adapt them to these data; and by these processes they have been enlarged and consolidated.

So we have been led to regard them as experimental verities; the conception of central forces became then a useless support, or rather an embarrassment, since it made the principles partake of its hypothetical character.

The frames have not therefore broken, because they were elastic; but they have enlarged; our fathers, who established them, did not work in vain, and we recognize in the science of to-day the general traits of the sketch which they traced.

Are we about to enter now upon the eve of a second crisis? Are these principles on which we have built all about to crumble away in their turn? For some time, this may well have been asked.

In hearing me speak thus, you think without doubt of radium, that grand revolutionist of the present time, and in fact I will come back to it presently; but there is something else.

It is not alone the conservation of energy which is in question; all the other principles are equally in danger, as we shall see in passing them successively in review.

Let us commence with the principle of Carnot. This is the only one which does not present itself as an immediate consequence of the hypothesis of central forces; more than that, it seems, if not directly to contradict that hypothesis, at least not to be reconciled with it without a certain effort.

If physical phenomena were due exclusively to the movements of atoms whose mutual attraction depended only on the distance, it seems that all these phenomena should be reversible; if all the initial velocities were reversed, these atoms, always subjected to the same forces, ought to go over their trajectories in the contrary sense, just as the earth would describe in the retrograde sense this same elliptic orbit which it describes in the direct sense, if the initial conditions of its movement had been reversed. On this account, if a physical phenomenon is possible, the inverse phenomenon should be equally so, and one should be able to reascend the course of time.

But it is not so in nature, and this is precisely what the principle of Carnot teaches us; heat can pass from the warm body to the cold body; it is impossible afterwards to make it reascend the inverse way and reëstablish differences of temperature which have been effaced.

Motion can be wholly dissipated and transformed into heat by friction; the contrary transformation can never be made except in a partial manner.

We have striven to reconcile this apparent contradiction. If the world tends toward uniformity, this is not because its ultimate parts, at first unlike, tend to become less and less different, it is because, shifting at hazard, they end by blending. For an eye which should distinguish all the elements, the variety would remain always as great, each grain of this dust preserves its originality and does not model itself on its neighbors; but as the blend becomes more and more intimate, our gross senses perceive no more than the uniformity. Behold why, for example, temperatures tend to a level, without the possibility of turning backwards.

A drop of wine falls into a glass of water; whatever may be the law of the internal movements of the liquid, we soon see it colored to a uniform rosy tint, and from this moment, however well we may shake the vase, the wine and the water do not seem capable of further separation. Observe what would be the type of the reversible physical phenomenon: to hide a grain of barley in a cup of wheat is easy; afterwards to find it again and get it out is practically impossible.

All this Maxwell and Boltzmann have explained; the one who has seen it most clearly, in a book too little read because it is a little difficult to read, is Gibbs, in his *Elementary Principles of Statistical Mechanics*.

For those who take this point of view, the principle of Carnot is only an imperfect principle, a sort of concession to the infirmity of our senses; it is because our eyes are too gross that we do not distinguish the elements of the blend; it is because our hands are too gross that we cannot force them to separate; the imaginary demon of Maxwell, who is able to sort the molecules one by one, could well constrain the world to return backward. Can it return of itself? That is not impossible; that is only infinitely improbable.

The chances are that we should long await the concourse of circumstances which would permit a retrogradation, but soon or late they would be realized, after years whose number it would take millions of figures to write.

These reservations, however, all remained theoretic and were not very disquieting, and the principle of Carnot retained all its practical value.

But here the scene changes.

The biologist, armed with his microscope, long ago noticed in his preparations disorderly movements of little particles in suspension: this is the Brownian movement; he first thought this was a vital phenomenon, but he soon saw that the inanimate bodies danced with no less ardor than the others; then he turned the matter over to the physicists. Unhappily, the physicists remained long uninterested in this question; the light is focused to illuminate the microscopic preparation, thought they; with light goes heat; hence inequalities of temperature and interior currents produce the movements in the liquid of which we speak.

M. Gouy, however, looked more closely, and he saw, or thought he saw, that this explanation is untenable, that the movements become more brisk as the particles are smaller, but that they are not influenced by the mode of illumination.

If, then, these movements never cease, or rather are reborn without ceasing, without borrowing anything from an external source of energy, what ought we to believe? To be sure, we should not renounce our belief in the conservation of energy, but we see under our eyes now motion transformed into heat by friction, now heat changed inversely into motion, and that without loss since the movement lasts forever. This is the contrary of the principle of Carnot.

If this be so, to see the world return backward, we no longer have need of the infinitely subtle eye of Maxwell's demon; our microscope suffices us. Bodies too large, those, for example, which are a tenth of a millimeter, are hit from all sides by moving atoms, but they do not budge, because these shocks are very numerous and the law of chance makes them compensate each other: but the smaller particles receive too few shocks for this compensation to take place with certainty and are incessantly knocked about. And thus already one of our principles is in peril.

We come to the principle of relativity: this not only is confirmed by daily experience, not only is it a necessary consequence of the hypothesis of central forces, but it is imposed in an irresistible way upon our good sense, and yet it also is battered.

Consider two electrified bodies; though they seem to us at rest, they are both carried along by the motion of the earth; an electric charge in motion, Rowland has taught us, is equivalent to a current; these two charged bodies are, therefore, equivalent to two parallel currents of the same sense and these two currents should attract each other. In measuring this attraction, we measure the velocity of the earth; not its velocity in relation to the sun or the fixed stars, but its absolute velocity.

I know it will be said that it is not its absolute velocity that is measured, but its velocity in relation to the ether. How unsatisfactory that is! Is it not evident that from a principle so understood we could no longer get anything? It could no longer tell us anything just because it would no longer fear any contradiction.

If we succeed in measuring anything, we should always be free to say that this is not the absolute velocity in relation to the ether, it might always be the velocity in relation to some new unknown fluid with which we might fill space.

Indeed, experience has taken on itself to ruin this interpretation of the principle of relativity; all attempts to measure the velocity of the earth in relation to the ether have led to negative results. This time experimental physics has been more faithful to the principle than mathematical physics; the theorists, to put in accord their other general views, would not have spared it; but experiment has been stubborn in confirming it.

The means have been varied in a thousand ways and finally Michelson has pushed precision to its last limits; nothing has come of it. It is precisely to explain this obstinacy that the mathematicians are forced to-day to employ all their ingenuity.

Their task was not easy, and if Lorentz has gotten through it, it is only by accumulating hypotheses.

The most ingenious idea has been that of local time.

Imagine two observers who wish to adjust their watches by optical signals; they exchange signals, but as they know that the transmission of light is not instantaneous, they take care to cross them.

When the station B perceives the signal from the station A, its clock should not mark the same hour as that of the station A at the moment of sending the signal, but this hour augmented by a constant representing the duration of the transmission. Suppose, for example, that the station A sends its signal when its clock marks the hour 0, and that the station B perceives it when its clock marks the hour t. The clocks are adjusted if the slowness equal to t represents the duration of the transmission, and to verify it the station B sends in its turn a signal when its clock marks 0; then the station A should perceive it when its clock marks t. The time-pieces are then adjusted. And in fact, they mark the same hour at the same physical instant, but on one condition, namely, that the two stations are fixed. In the contrary case the duration of the transmission will not be the same in the two senses, since the station A, for example, moves forward to meet the optical perturbation emanating from B, while the station B flies away before the perturbation emanating from A. The watches adjusted in that manner do not mark, therefore, the true time; they mark what one may call the *local time*, so that one of them goes slow on the other. It matters little, since we have no means of perceiving it. All the phenomena which happen at A, for example, will be late, but all will be equally so, and the observer who ascertains them will not perceive it, since his watch is slow; so, as the principle of relativity would have it, he will have no means of knowing whether he is at rest or in absolute motion.

Unhappily, that does not suffice, and complementary hypotheses are necessary; it is necessary to admit that bodies in motion undergo a uniform contraction in the sense of the motion. One of the diameters of the earth, for example, is shrunk by $\frac{1}{200,000,000}$ in consequence of the motion of our planet, while the other diameter retains its normal length. Thus, the last little differences find themselves compensated. And then there still is the hypothesis about forces. Forces, whatever be their origin, gravity as well as elasticity, would be reduced in a certain proportion in a world animated by a uniform translation; or, rather, this would happen for the components perpendicular to the translation; the components parallel would not change.

Resume, then, our example of two electrified bodies; these bodies repel each other, but at the same time if all is carried along in a uniform translation, they are equivalent to two parallel currents of the same sense which attract each other. This electro-dynamic attraction diminishes, therefore, the electro-static repulsion, and the total repulsion is more feeble than if the two bodies were at rest. But since to measure this repulsion we must balance it by another force, and all these other forces are reduced in the same proportion, we perceive nothing.

Thus, all is arranged, but are all the doubts dissipated?

What would happen if one could communicate by non-luminous signals whose velocity of propagation differed from that of light? If, after having adjusted the watches by the optical procedure, one wished to verify the adjustment by the aid of these new signals, then would appear divergences which would render evident the common translation of the two stations. And are such signals inconceivable, if we admit with Laplace that universal gravitation is transmitted a million times more rapidly than light?

Thus, the principle of relativity has been valiantly defended in these latter times, but the very energy of the defense proves how serious was the attack.

Let us speak now of the principle of Newton, on the equality of action and reaction.

This is intimately bound up with the preceding, and it seems indeed that the fall of the one would involve that of the other. Thus we should not be astonished to find here the same difficulties.

Electrical phenomena, we think, are due to the displacements of little charged particles, called electrons, immersed in the medium that we call ether. The movements of these electrons produce perturbations in the neighboring ether; these perturbations propagate themselves in every direction with the velocity of light, and in turn other electrons, originally at rest, are made to vibrate when the perturbation reaches the parts of the ether which touch them.

The electrons, therefore, act upon one another, but this action is not direct, it is accomplished through the ether as intermediary.

Under these conditions can there be compensation between action and reaction, at least for an observer who should take account only of the movements of matter, that is to say, of the electrons, and who should be ignorant of those of the ether that he could not see? Evidently not. Even if the compensation should be exact, it could not be simultaneous. The perturbation is propagated with a finite velocity; it, therefore, reaches the second electron only when the first has long ago entered upon its rest.

This second electron, therefore, will undergo, after a delay, the action of the first, but certainly it will not react on this, since around this first electron nothing any longer budges.

The analysis of the facts permits us to be still more precise. Imagine for example, a Hertzian generator, like those employed in wireless telegraphy; it sends out energy in every direction; but we can provide it with a parabolic mirror, as Hertz did with his smallest generators, so as to send all the energy produced in a single direction.

What happens, then, according to the theory? It is that the apparatus recoils as if it were a gun and as if the energy it has projected were a bullet; and that is contrary to the principle of Newton, since our projectile here has no mass, it is not matter, it is energy.

It is still the same, moreover, with a beacon light provided with a reflector, since light is nothing but a perturbation of the electromagnetic field. This beacon light should recoil as if the light it sends out were a projectile. What is the force that this recoil should produce? It is what one has called the Maxwell-Bartholdi pressure. It is very minute, and it has been difficult to put it into evidence even with the most sensitive radiometers; but it suffices that it exists.

If all the energy issuing from our generator falls on a receiver, this will act as if it had received a mechanical shock, which will represent in a sense the compensation of the recoil of the generator; the reaction will be equal to the action, but it will not be simultaneous; the receiver will move on but not at the moment when the generator recoils. If the energy propagates itself indefinitely without encountering a receiver, the compensation will never be made.

Do we say that the space which separates the generator from the receiver and which the perturbation must pass over in going from the one to the other is not void, that it is full not only of ether, but of air; or even in the interplanetary spaces of some fluid subtle but still ponderable; that this matter undergoes the shock like the receiver at the moment when the energy reaches it, and recoils in its turn when the perturbation quits it? That would save the principle of Newton, but that is not true.

If energy in its diffusion remained always attached to some material substratum, then matter in motion would carry along light with it, and Fizeau has demonstrated that it does nothing of the sort, at least for air. This is what Michelson and Morley have since confirmed.

One may suppose also that the movements of matter, properly so called, are exactly compensated by those of the ether; but that would lead us to the same reflections as just now. The principle so extended would explain everything, since whatever might be the visible movements, we should always have the power of imagining hypothetical movements which compensated them.

But if it is able to explain everything, this is because it does not permit us to foresee anything; it does not enable us to decide between different possible hypotheses, since it explains everything beforehand. It therefore becomes useless.

And then the suppositions that it would be necessary to make on the movements of the ether are not very satisfactory.

If the electric charges double, it would be natural to imagine that the velocities of the divers atoms of ether double also, and for the compensation, it would be necessary that the mean velocity of the ether quadruple.

This is why I have long thought that these consequences of theory, contrary to the principle of Newton, would end some day by being abandoned, and yet the recent experiments on the movements of the electrons issuing from radium seem rather to confirm them.

I arrive at the principle of Lavoisier on the conservation of masses: in truth this is one not to be touched without unsettling all mechanics.

And now certain persons believe that it seems true to us only because we consider in mechanics merely moderate velocities, but that it would cease to be true for bodies animated by velocities comparable to that of light. These velocities, it is now believed, have been realized; the cathode rays or those of radium may be formed of very minute particles or of electrons which are displaced with velocities smaller no doubt than that of light, but which might be its one tenth or one third.

These rays can be deflected, whether by an electric field, or by a magnetic field, and we are able by comparing these deflections, to measure at the same time the velocity of the electrons and their mass (or rather the relation of their mass to their charge). But when it was seen that these velocities approached that of light, it was decided that a correction was necessary.

These molecules, being electrified, could not be displaced without agitating the ether; to put them in motion it is necessary to overcome a double inertia, that of the molecule itself and that of the ether. The total or apparent mass that one measures is composed, therefore, of two parts: the real or mechanical mass of the molecule and the electro-dynamic mass representing the inertia of the ether.

The calculations of Abraham and the experiments of Kaufmann have then shown that the mechanical mass, properly so called, is null, and that the mass of the electrons, or, at least, of the negative electrons, is of exclusively electro-dynamic origin. This forces us to change the definition of mass; we cannot any longer distinguish mechanical mass and electro-dynamic mass, since then the first would vanish; there is no mass other than electro-dynamic inertia. But in this case the mass can no longer be constant, it augments with the velocity, and it even depends on the direction, and a body animated by a notable velocity will not oppose the same inertia to the forces which tend to deflect it from its route, as to those which tend to accelerate or to retard its progress.

There is still a resource; the ultimate elements of bodies are electrons, some charged negatively, the others charged positively. The negative electrons have no mass, this is understood; but the positive electrons, from the little we know of them, seem much greater. Perhaps they have, besides their electro-dynamic mass, a true mechanical mass. The veritable mass of a body would, then, be the sum of the mechanical masses of its positive electrons, the negative electrons not counting; mass so defined could still be constant.

Alas, this resource also evades us. Recall what we have said of the principle of relativity and of the efforts made to save it. And it is not merely a principle which it is a question of saving, such are the indubitable results of the experiments of Michelson.

Lorentz has been obliged to suppose that all the forces, whatever be their origin, were affected with a coefficient in a medium animated by a uniform translation; this is not sufficient; it is still necessary, says he, that *the masses of all the particles be influenced by a translation to the same degree as the electro-magnetic masses of the electrons.*

So the mechanical masses will vary in accordance with the same laws as the electro-dynamic masses; they cannot, therefore, be constant.

Need I point out that the fall of the principle of Lavoisier involves that of the principle of Newton? This latter signifies that the centre of gravity of an isolated system moves in a straight line; but if there is no longer a constant mass, there is no longer a centre of gravity, we no longer know even what this is. This is why I said above that the experiments on the cathode rays appeared to justify the doubts of Lorentz on the subject of the principle of Newton.

From all these results, if they are confirmed, would arise an entirely new mechanics, which would be, above all, characterized by this fact, that no velocity could surpass that of light, any more than any temperature could fall below the zero absolute, because bodies would oppose an increasing inertia to the causes, which would tend to accelerate their motion; and this inertia would become infinite when one approached the velocity of light.

Nor for an observer carried along himself in a translation he did not suspect could any apparent velocity surpass that of light; there would then be a contradiction, if we recall that this observer would not use the same clocks as a fixed observer, but, indeed, clocks marking "local time."

Here we are then facing a question I content myself with stating. If there is no longer any mass, what becomes of the law of Newton?

Mass has two aspects, it is at the same time a coefficient of inertia and an attracting mass entering as factor into Newtonian attraction. If the coefficient of inertia is not constant, can the attracting mass be? That is the question.

At least, the principle of the conservation of energy yet remains to us, and this seems more solid. Shall I recall to you how it was in its turn thrown into discredit? This event has made more noise than the preceding and it is in all the records.

From the first works of Becquerel, and, above all, when the Curies had discovered radium, one saw that every radio-active body was an inexhaustible source of radiations. Its activity would seem to subsist without alteration throughout the months and the years. This was already a strain on the principles; these radiations were in fact energy, and from the same morsel of radium this issued and forever issued. But these quantities of energy were too slight to be measured; at least one believed so and was not much disquieted.

The scene changed when Curie bethought himself to put radium into a calorimeter; it was seen then that the quantity of heat incessantly created was very notable.

The explanations proposed were numerous; but in so far as no one of them has prevailed over the others, we cannot be sure there is a good one among them.

Sir William Ramsay has striven to show that radium is in process of transformation, that it contains a store of energy enormous but not inexhaustible.

The transformation of radium, then, would produce a million times more of heat than all known transformations; radium would wear itself out in 1250 years; you see that we are at least certain to be settled on this point some hundreds of years from now. While waiting our doubts remain.

In the midst of so many ruins what remains standing? The principle of least action has hitherto remained intact, and Larmor appears to believe that it will long survive the others; in reality, it is still more vague and more general.

In presence of this general ruin of the principles, what attitude will mathematical physics take?

And first, before too much perplexity, it is proper to ask if all this is really true. All these apparent contradictions to the principles are encountered only among infinitesimals; the microscope is necessary to see the Brownian movement; electrons are very light; radium is very rare, and no one has ever seen more than some milligrams of it at a time.

And, then, it may be asked if, beside the infinitesimal seen, there be not another infinitesimal unseen counterpoise to the first.

So, there is an interlocutory question, and, as it seems, only experiment can solve it. We have, therefore, only to hand over the matter to the experimenters, and, while waiting for them to determine the question finally, not to preoccupy ourselves with these disquieting problems, but quietly continue our work, as if the principles were still uncontested. We have much to do without leaving the domain where they may be applied in all security; we have enough to employ our activity during this period of doubts.

And as to these doubts, is it indeed true that we can do nothing to disembarrass science of them? It may be said, it is not alone experimental physics that has given birth to them; mathematical physics has well contributed. It is the experimenters who have seen radium throw out energy, but it is the theorists who have put in evidence all the difficulties raised by the propagation of light across a medium in motion; but for these it is probable we should not have become conscious of them. Well, then, if they have done their best to put us into this embarrassment, it is proper also that they help us to get out of it.

They must subject to critical examination all these new views I have just outlined before you, and abandon the principles only after having made a loyal effort to save them.

What can they do in this sense? That is what I will try to explain.

Among the most interesting problems of mathematical physics, it is proper to give a special place to those relating to the kinetic theory of gases. Much has already been done in this direction, but much still remains to be done. This theory is an eternal paradox. We have reversibility in the premises and irreversibility in the conclusions; and between the two an abyss. Statistic considerations, the law of great numbers, do they suffice to fill it? Many points still remain obscure to which it is necessary to return, and doubtless many times. In clearing them up, we shall understand better the sense of the principle of Carnot and its place in the *ensemble* of dynamics, and we shall be better armed to interpret properly the curious experiment of Gouy, of which I spoke above.

Should we not also endeavor to obtain a more satisfactory theory of the electro-dynamics of bodies in motion? It is there especially, as I have sufficiently shown above, that difficulties accumulate. Evidently we must heap up hypotheses, we cannot satisfy all the principles at once; heretofore, one has succeeded in safeguarding some only on condition of sacrificing the others; but all hope of obtaining better results is not yet lost. Let us take, therefore, the theory of Lorentz, turn it in all senses, modify it little by little, and perhaps everything will arrange itself.

Thus in place of supposing that bodies in motion undergo a contraction in the sense of the motion, and that this contraction is the same whatever be the nature of these bodies and the forces to which they are otherwise submitted, could we not make an hypothesis more simple and more natural?

We might imagine, for example, that it is the ether which is modified when it is in relative motion in reference to the material medium which it penetrates, that when it is thus modified, it no longer transmits perturbations with the same velocity in every direction. It might transmit more rapidly those which are propagated parallel to the medium, whether in the same sense or in the opposite sense, and less rapidly those which are propagated perpendicularly. The wave surfaces would no longer be spheres, but ellipsoids, and we could dispense with that extraordinary contraction of all bodies.

I cite that only as an example, since the modifications one might essay would be evidently susceptible of infinite variation.

It is possible also that the astronomer may some day furnish us data on this point; he it was in the main who raised the question in making us acquainted with the phenomenon of the aberration of light. If we make crudely the theory of aberration, we reach a very curious result. The apparent positions of the stars differ from their real positions because of the motion of the earth, and as this motion is variable, these apparent positions vary. The real position we cannot know, but we can observe the variations of the apparent position. The observations of the aberration show us, therefore, not the movement of the earth, but the variations of this movement; they cannot, therefore, give us information about the absolute motion of the earth. At least this is true in first approximation, but it would be no longer the same if we could appreciate the thousandths of a second. Then it would be seen that the amplitude of the oscillation depends not alone on the variation of the motion, variation which is well known, since it is the motion of our globe on its elliptic orbit, but on the mean value of this motion; so that the constant of aberration would not be altogether the same for all the stars, and the differences would tell us the absolute motion of the earth in space.

This, then, would be, under another form, the ruin of the principle of relativity. We are far, it is true, from appreciating the thousandths of a second, but after all, say some, the total absolute velocity of the earth may be much greater than its relative velocity with respect to the sun. If, for example, it were 300 kilometers per second in place of 30, this would suffice to make the phenomena observable.

I believe that in reasoning thus we admit a too simple theory of aberration. Michelson has shown us, I have told you, that the physical procedures are powerless to put in evidence absolute motion; I am persuaded that the same will be true of the astronomic procedures, however far one pushes precision.

However that may be, the data astronomy will furnish us in this regard will some day be precious to the physicist. While waiting, I believe the theorists, recalling the experience of Michelson, may anticipate a negative result, and that they would accomplish a useful work in constructing a theory of aberration which would explain this in advance.

But let us come back to the earth. There also we may aid the experimenters. We can, for example, prepare the ground by studying profoundly the dynamics of electrons; not, be it understood, in starting from a single hypothesis, but in multiplying hypotheses as much as possible. It will be, then, for the physicists to utilize our work in seeking the crucial experiment to decide between these different hypotheses.

This dynamics of electrons can be approached from many sides, but among the ways leading thither is one which has been somewhat neglected, and yet this is one of those which promise us most of surprises. It is the movements of the electrons which produce the line of the emission spectra; this is proved by the phenomenon of Zeemann; in an incandescent body, what vibrates is sensitive to the magnet, therefore electrified. This is a very important first point, but no one has gone farther; why are the lines of the spectrum distributed in accordance with a regular law?

These laws have been studied by the experimenters in their least details; they are very precise and relatively simple. The first study of these distributions recalled the harmonics encountered in acoustics; but the difference is great. Not only the numbers of vibrations are not the successive multiples of one number, but we do not even find anything analogous to the roots of those transcendental equations to which so many problems of mathematical physics conduct us: that of the vibrations of an elastic body of any form, that of the Hertzian oscillations in a generator of any form, the problem of Fourier for the cooling of a solid body.

The laws are simpler, but they are of wholly other nature, and to cite only one of these differences, for the harmonics of high order the number of vibrations tends toward a finite limit, instead of increasing indefinitely.

That has not yet been accounted for, and I believe that there we have one of the most important secrets of nature. Lindemann has made a praiseworthy attempt, but, to my mind, without success; this attempt should be renewed. Thus we shall penetrate, so to say, into the inmost recess of matter. And from the particular point of view which we to-day occupy, when we know why the vibrations of incandescent bodies differ from ordinary elastic vibrations, why the electrons do not behave themselves like the matter which is familiar to us, we shall better comprehend the dynamics of electrons and it will be perhaps more easy for us to reconcile it with the principles.

Suppose, now, that all these efforts fail, and after all I do not believe they will, what must be done? Will it be necessary to seek to mend the broken principles in giving what we French call a *coup de pouce?* That is evidently always possible, and I retract nothing I have formerly said.

Have you not written, you might say if you wished to seek a quarrel with me, have you not written that the principles, though of experimental origin, are now unassailable by experiment because they have become conventions? And now you have just told us the most recent conquests of experiment put these principles in danger. Well, formerly I was right and to-day I am not wrong.

Formerly I was right, and what is now happening is a new proof of it. Take, for example, the calorimeter experiment of Curie on radium. Is it possible to reconcile that with the principle of the conservation of energy?

It has been attempted in many ways; but there is among them one I should like you to notice.

It has been conjectured that radium was only an intermediary, that it only stored radiations of unknown nature which flashed through space in every direction, traversing all bodies, save radium, without being altered by this passage and without exercising any action upon them. Radium alone took from them a little of their energy and afterward gave it out to us in divers forms.

What an advantageous explanation, and how convenient! First, it is unverifiable and thus irrefutable. Then again it will serve to account for any derogation whatever to the principle of Mayer; it responds in advance not only to the objection of Curie, but to all the objections that future experimenters might accumulate. This new and unknown energy would serve for everything. This is just what I have said, and we are thereby shown that our principle is unassailable by experiment.

And after all, what have we gained by this *coup de pouce ?*

The principle is intact, but thenceforth of what use is it?

It permitted us to foresee that in such or such circumstance we could count on such a total quantity of energy; it limited us; but now where there is put at our disposition this indefinite provision of new energy, we are limited by nothing; and as I have written elsewhere, if a principle ceases to be fecund, experiment, without contradicting it directly, will be likely to condemn it.

This, therefore, is not what would have to be done, it would be necessary to rebuild anew.

If we were cornered down to this necessity, we should moreover console ourselves. It would not be necessary to conclude that science can weave only a Penelope's web, that it can build only ephemeral constructions, which it is soon forced to demolish from top to bottom with its own hands.

As I have said, we have already passed through a like crisis. I have shown you that in the second mathematical physics, that of the principles, we find traces of the first, that of the central forces; it will be just the same if we must learn a third.

When an animal exuviates, and breaks its too narrow carapace to make itself a fresh one, we easily recognize under the new envelope the essential traits of the organism which have existed.

We cannot foresee in what way we are about to expand; perhaps it is the kinetic theory of gases which is about to undergo development and serve as model to the others. Then, the facts which first appeared to us as simple, thereafter will be merely results of a very great number of elementary facts which only the laws of chance make coöperate for a common end. Physical law will then take an entirely new aspect; it will no longer be solely a differential equation, it will take the character of a statistical law.

Perhaps, likewise, we should construct a whole new mechanics, of which we only succeed in catching a glimpse, where inertia increasing with the velocity, the velocity of light would become an impassable limit.

The ordinary mechanics, more simple, would remain a first approximation, since it would be true for velocities not too great, so that we should still find the old dynamics under the new.

We should not have to regret having believed in the principles, and even, since velocities too great for the old formulas would always be only exceptional, the surest way in practice would be still to act as if we continued to believe in them. They are so useful, it would be necessary to keep a place for them. To determine to exclude them altogether would be to deprive one's self of a precious weapon. I hasten to say in conclusion we are not yet there, and as yet nothing proves that the principles will not come forth from the combat victorious and intact.

SHORT PAPERS

Three short papers were read in the Section of Applied Mathematics, the first by Professor Henry T. Eddy, of the University of Minnesota, on "The Electromagnetic Theory and the Velocity of Light."

The second paper was presented by Professor Alexander Macfarlane, of Chatham, Ontario, "On the Exponential Notation in Vector-analysis."

The third paper was presented by Professor James McMahon, of Cornell University, "On the Use of N-fold Riemann Spaces in Applied Mathematics."

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